Teaching Multiplication and Division Realistically in Indonesian Primary Schools: A Prototype of Local Instructional Theory

Dian Armanto

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This dissertation is dedicated to:
my wife (Erna Lindiawaty),
my two sons (Haqim and Hilmy),
and my family.

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If I have seen further, it is by standing on the shoulder of giants
(Isaac Newton)

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CHAPTER 1

ORIGINS OF THIS STUDY

The challenging parts of implementing a new mathematics teaching approach in Indonesia are to construct a new formal curriculum, to influence teacher's beliefs about what is good mathematics teaching, and to change the pupils' habits and attitude toward learning. The struggle to meet these challenges is an important thread in this study. This chapter describes the rationale of those intriguing aspects in the Indonesian contexts as well as the aims of this study. It means that this book explores the drawbacks of the old mode of teaching mathematics (teaching by telling) and the inherent usefulness of a new teaching approach (teaching mathematics realistically) for the sake of teachers, pupils, and the community in the future. This book also explains the cyclic process of front-end analysis, expert reviews, teaching experiments, and reflections to the local instructional sequences process for developing, implementing, and evaluating the RME prototypical exemplary materials. The organization of the chapters is presented as well.

1.1 INTRODUCTION

This study is about developing, implementing, and evaluating prototypical instructional materials for teaching multiplication and division of multi-digit numbers in Indonesian primary schools. It is believed that this is an alternative strategy to improve the Indonesian mathematics education in primary schools; that is in a deep annoyance nowadays (see Haji, 1994; Jailani, 1990; and TIMSS Report, 1997). Particular sets of the instructional materials provide teachers opportunities to practice their teaching approach, to enlighten their knowledge, and to improve their competencies (Feiter & Van den Akker, 1995; Louck-Horsley, et al., 1996). The instructional materials are developed based on the RME approach.

RME (Realistic Mathematics Education) is a theory that has been evolving for about three decades in the developmental research on teaching and learning mathematics. It is rooted in Freudenthal's interpretation of mathematics as a human activity (De Lange, 1994; Freudenthal, 1973; and Gravemeijer, 1994). Mathematics ought not to be associated a well-organized deductive system, but to an activity of doing mathematics: an activity the greater part of which consists of organizing or mathematizing subject matter. The subject matter can be taken from reality and must be organized in relation with to mathematical patterns when solving problems from reality.

Freudenthal also argues that mathematics should never be presented to pupils as a ready-made product and that pupils should reinvent mathematics. This mean that pupils should not learn mathematics starting within the formal system of mathematics, but those concepts appearing in reality should be the source of concept formation (extracting the appropriate concept from a concrete situation). This theory has been developed in the Netherlands over the last thirty years (De Lange, 1994; Gravemeijer, 1997; Treffers, 1987; and Treffers & Goffree, 1985;). It dominates the Dutch teaching practice nowadays (Treffers, 1991), and has influenced the work of mathematics educators in many parts of the world (Becker & Selter, 1996; Cobb, Wood & Yackel, 1991; Romberg, 1994; Selter, 1995; and Wittman, 1991).

This study builds upon the RME theory in which a cyclic process of front-end analysis, expert reviews, teaching experiments, and reflections to the local instructional sequences has been conducted in developing, evaluating and implementing the RME prototypical materials for teaching multiplication and division of multi-digit numbers in Indonesian primary schools. This cyclic process (called 'developmental research') steers the development of each version of the RME prototype. This process will be elaborated in more detail throughout the eight chapters of this book. It includes the rationale of this study, the research design, and its results as well. The next section presents the rationale of this study.

1.2 RATIONALE

The learning process of mathematics is the event that should be taken into account in order to analyze the effectiveness of the instruction (Koehler & Grouws, 1992). According to Plomp and Brummelhuis (1998) the learning process takes place as a result of the interaction among four aspects: teacher, pupil, content, and infrastructures. It is also a result of structural conditions derived from the learning infrastructure and the personal characteristics of the actors involved, and their interaction in the process. Each actor plays their role depends on the activities (traditional - teachers' centered or RME - pupils' centered) arranged by the teachers (Simons & Zuylen, 1995). In RME learning arrangement, the pupils should take more responsibility for their learning and actively engage in the learning process. Guided by the teacher, the pupils learn, discuss, and formulate the informal and formal mathematics forms in order to understand mathematics concepts and procedures. In contrast, in a 'traditional' learning process, the teacher takes control to each activity. The teacher extensively directs, explains, and gives questions in the context of whole-group instruction followed by pupils working on paper-and-pencil assignments.

The traditional learning process described above has been practiced in the Indonesian classrooms for years (elaborated in section 1.2.1). It is an integral part of the Indonesian mathematics education, which is trying to find an effective teaching method that conforms with the Indonesian condition and culture. Started in the 1975 curriculum in which the modern mathematics became the dominant materials to be learned by the pupils. Then the 1984 curriculum revision took place emphasizing on the set theory in teaching mathematics. Next and nowadays, the 1994 curriculum focuses on understanding arithmetic. The Pupils' Active Learning (Cara Belajar Siswa Aktif - CBSA) was introduced and emphasized in the instruction process. This method has been demanded to meet the teaching objectives, but in practice it was practically displeased because of the shortage of teaching aids and the lack of teachers' pedagogical competence (Suyono, 1996).

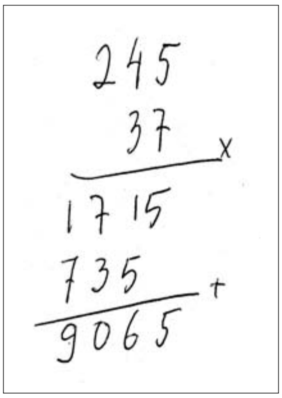
Considering the circumstances above, the problems of finding a proper mathematics teaching method that suits the Indonesian culture still remains. There are several questions emerging in relation to this study, such as; does the Indonesian mathematics education really need improvement? Is there any promising teaching approach that suits the Indonesian condition and culture? Why and how is this approach significant in improving the mathematics learning achievement of the pupils? The next section describes how those questions originated.

1.2.1 An example of Indonesian mathematics education

The goals in teaching arithmetic in the 1994 curriculum are to develop the ability to count, to enhance pupils' mathematics content knowledge in order to be used in doing mathematics and in their advance study, and to structure pupil's attitude to be critical, honest, disciplined, efficient, and effective (Depdikbud, 1993). These goals drive the teachers to apply the mechanistic approach, practicing symbols and emphasizing on application of algorithm (Treffers, 1987). For instance, the short algorithm procedure is taught in learning multiplication and division (elaborated in section 2.5).

The teaching of multiplication and division using this model does not suit the goals of teaching mathematics mentioned above. This model focuses on memorizing abilities rather than on the pupils' understanding. Whenever the pupils apply this procedure and have difficulties (for instance, they forgot steps) they do not have anything to fall back on. Then they develop a corrupt procedure (see figure 2.3 and 2.5 in section 2.5).

From observation conducted in schools it was concluded that there are misconceptions of the pupils in doing the procedures after learning the standard algorithm (short model) in the classroom (Armanto, 2000). Some teachers argue that by teaching the pupils in this mechanistic approach, the pupils can understand and apply algorithms easily to solve other problems. This argument is hardly true because in answering a multiplication contextual problem given by the researcher, only one third (1/3) of pupils can solve the problem correctly. Meanwhile another two-third (2/3) of pupils has difficulties to the idea of multiplication algorithm. The following conversation gives an indication of this misconception.



The interviews:

Observer: *Did you find the answer?*
Pupil: *Yes, sir. This is my calculation.*
Observer: *Why did you calculate like this?*
Pupil: *Because the teacher taught me to do that.*
Observer: *Why did you multiply 7 x 5 in the first place and not 3 x 5? (The observer points the numbers of the multipliers).*
Pupil: *Because 7 is the last number of 37 and the teacher showed me to multiply it first.*
Observer: *The multiplication of 3 x 5 = 15. Why did you put 5 (from 735) under the second 1 of 1715?*
Pupil: *I don't know. The teacher did that. I think I should follow her. And it works. I got the right solution.*

Figure 1.1
The pupils' valid procedure

From the conversation above it can be seen that the pupil did not understand the meaning of the multiplication algorithm. They memorized the teachers' way of applying the algorithm. The problems occurred whenever they applied the algorithm to answer another problem. Not only did they have no concepts to fall back on but also they did not realize the connection between the numbers and the algorithm they applied when they solve a contextual problem (see some Figure 2.2 - 2.5 in section 2.5).

In learning division a similar misconception also occurred (Armanto, 2000). The following story describes what happened in the classroom. The researcher introduced a contextual problem to the pupils as follow and ask them to find the solution:

> *36 performances of Wayang Kulit (Shadow Puppets) were conducted in a RRI Auditorium. The tickets sold are 7416 tickets. How many tickets are sold for each performance in average?*

The researcher found several mistakes made by the pupils. They used a mechanistic way of solving problem, the way they were taught by the teacher. The researcher asked several pupils to write their solutions on the blackboard. After five pupils wrote their solution, the researcher asked the class how many pupils got the same solution as each solution on the blackboard. The answers were surprising. Only two pupils got the right answer (206). Most pupils got 26 as the solution. Several pupils got 1106, 215 and 116.

Those two examples describe the pupils' weaknesses after learning multiplication and division in a mechanistic way. Other weaknesses of Indonesian mathematics education and its reasons will be illustrated in the next section, including a discussion of the need of improvement.

1.2.2 The need of improvement

The weaknesses in mathematics teaching in primary schools in Indonesia have been studied by many experts (Jailani, 1990 and Haji, 1994). These weaknesses, including unable to comprehend mathematical concepts and to construct and solve a mathematical forms from a story problem, make mathematics more difficult to learn and to understand and the pupils become afraid of mathematics. The results of National Examinations (EBTANAS) showed that mathematics was continuously the lowest score the pupils obtained compared to other subjects (Depdikbud, 1997). The TIMSS results (1999) showed that the Indonesian pupils score were in the 33rd level from 37 countries involved in the TIMSS evaluation.

There are many reasons for this low achievement in learning mathematics, one of which is the low quality of teachers' competence of mathematics (BPPN, 1996). According to the educational statistics 1995/1996, 89% (1,049,468) of primary school teachers were not qualified as teachers because they had no minimum prerequisite to be teachers in primary schools (Depdikbud, 1997). It was found out that the primary school teachers have mastered only 57% of the concepts, facts and procedures of mathematics (Depdikbud, 1997).

Another weakness of teachers is due to teachers' competence in the pedagogical aspects. Suyono (1996) found that teachers: (1) have low ability in using variety of teaching methods, (2) teach the basic skills only for answering the tests (teaching for tests), and (3) teach using conventional methods without considering the logical thinking, critical and creativity aspects of the subject matter. The 3rd aspect is related to the teachers' knowledge of students' learning cognition.

These facts show that the teachers conventionally teach mathematics with practicing mathematics symbols and emphasizing on giving information and application of mathematics algorithms (mechanistic algorithmic mathematics education, Treffers, 1987). The teachers taught ready-made mathematics rather than doing mathematics (Freudenthal, 1973). De Feiter and Van Den Akker (1995) state that this conventional method as over-dependence on lecture method ('chalk and talk'), passive nature of learners, only correct answers accepted and acted upon, lack of learner questioning, and whole-class activities of note-taking.

The picture of the classroom above has been the main characteristic of Indonesian elementary schools for years and still is nowadays (Armanto, 2000). Considering its consequence and drawbacks, it can be concluded that improving the teachers' competence of teaching becomes apparent as the most essential improvement to be accomplished to increase pupils' mathematics understanding in Indonesia.

1.3 THEORETICAL ROOTS

In learning mathematics in the mechanistic approach, many pupils will leave the classroom with a collection of well-practiced procedures and formulas but with only a hazy grasp of their meaning (see section 2.5). In another words, the pupils necessitate a better teaching model that gives opportunities to learn and understand mathematics they are learning. From research and other experiences in a variety of countries, it is known that a contingent approach of teaching can provide the desired result (Kilpatrick & Silver, 2000). This model suggests that the teacher orchestrates the discourse and sets up a situation and then responds to what the pupils are saying by building on their observations, seeking clarification, and challenging them to explain and justify. The goal is to help pupils to develop their own and one another's understanding. The contingent model is a core part of the RME approach.

Based on projects and studies in a number of countries (the Netherlands, Australia, the United Kingdom, Germany, Denmark, Japan, Malaysia, South Africa, and USA, see De Lange, 1996) it is known that the RME theory is a promising direction to improve and enhance pupils' understanding in mathematics. It is believed (see section 3.5) that the RME approach would be an effective approach to encounter the pupils' low performance problems in mathematics education in Indonesia that are caused by a number of factors, such as insufficiency of the teachers' mathematics knowledge and pedagogical approach, and the cultural aspects of the teaching and learning activity in the classroom (see further in Chapter 2).

The RME theory was build upon the Freudenthal argument that mathematics is a human activity, an activity of mathematizing whether subject matter from everyday-life reality or mathematical matters. Besides mathematizing the problems which are real to pupils, there also has to be a room for the mathematization of concepts, notations and problem-solving procedures. As a human activity, mathematics should be reinvented by the pupils, in whom they convert a contextual problem into a mathematical problem (horizontal mathematization) and later on they structure the mathematical problem on different mathematical levels (vertical mathematization). This is called progressive mathematization (Treffers, 1987), a reinventing process of mathematical insights, knowledge, and procedures (Gravemeijer, 1997). The emphasis of this process is on allowing the pupils to regard the knowledge they acquire as their own, private knowledge; an understanding of which they themselves are responsible for. This theory will be elaborated in section 3.3.

The RME approach suggests facilitating the pupils' learning process by exploring the contextual problems in which the mathematics is embedded. Opportunity to do mathematics using their own understanding helps the pupils restructuring their own knowledge. Guidance from teacher supports the pupils reinventing informal and formal mathematics forms (concepts and procedures). These are the essential anchors of developing the pupils' learning attitude. Implementing this approach in Indonesia would be a promising direction toward the improvement of teachers' competence and students' understanding of mathematics.

In this study learning multiplication and division in Grade 4 of Indonesian primary schools begins with introducing a contextual problem to encourage the pupils to use their understanding of repeated addition and repeated subtraction. This is the beginning of the learning trajectory towards the multiplication and division algorithm. In order to facilitate this RME teaching approach (elaborated in section 3.4) the study develops exemplary materials by considering the RME theory (from now on the RME materials are called 'RME prototype') and the Indonesian contexts. These materials guide the teacher as well as the researcher to apply the RME approach effectively in the classroom.

1.4 AIMS OF THIS STUDY

From the description above it is realized that Indonesian mathematics education needs to be improved especially in the instructional materials and the teachers' competencies (mathematics content, pupils' learning cognition and pedagogical aspects). In order to improve their competence, the teachers should develop a new vision of mathematics instruction, including a deep understanding of goals of the changes: from teaching to learning, from teacher-centered to pupil-centered, and from ready-made-mathematics to problem solving. This new vision requires teachers to build awareness of the mathematics content, to translate and practice the knowledge, and to reflect deeply on teaching and learning (Loucks-Horsley, 1998). For instance, the need of the pupil to be presented with situations in which they can understand and explore many mathematics activities assumes a considerable change in the conception of the nature of teaching. These tasks of teaching are complex in practice and it needs a specialized knowledge of the teachers in order to provide opportunity for pupils to explore their own and others' ideas individually and collectively.

These roles in the new vision drive teachers to be more professional in managing their classroom. It needs an alternative program, a sustainable reform agenda, to develop teachers' competence in the future. Loucks-Horsley, et al., (1996) suggests several possible alternatives of developing teachers to a new role of teaching mathematics. One of which is to conduct a n*ew formal curriculum implementation*. It is suggested to construct, learn, use, and refine a particular set of instructional materials in the classroom. Feiter and Van den Akker (1995) add that providing teachers with instructional materials and practicing a form of teaching approach are two alternatives that can be an effective alternative to improve the teachers' competencies. This study provides the RME exemplary materials with which the teachers can practice the RME teaching model under the guidance of the researcher. This study formulized characteristics of an RME prototype to teach multiplication and division of multi-digit numbers in Indonesian primary schools. The study was lead by the following research question:

> *What are the characteristics of the RME prototype for teaching multiplication and division of multi-digit numbers to Indonesian primary school pupils?*

The characteristics of the RME prototype are seen in two different aspects: local instructional sequences and quality aspects of the prototype. In the matter of intervention of teaching and learning mathematics, the characteristics refer to the explicit formulation of local instructional activities that is made up of three components: (1) learning goals for pupils; (2) planned instructional materials; and (3) a conjectured learning sequence (Gravemeijer & Cobb, 2001). Meanwhile, the quality aspects are defined as the degree to which the RME prototype:

- is in agreement with the state-of-the-art of the RME theory (validity);
- is usable and easy for the Indonesian teachers and pupils (practicality);
- can be used as intended by teachers and pupils (implementability);
- improves the pupils' performance in learning multiplication and division of multi-digit numbers (effectiveness).

These characteristics are elaborated in section 4.2.3.

Validity of the RME prototype refers to the internal agreement of the prototype toward the RME theory. Two aspects of validity were analyzed in this study: content and construct validity. Content validity of the RME prototype refers to the presence of the state-of-the-art knowledge of the Indonesian mathematics education circumstances and the RME theory. Construct validity related to the consistent link of the components in the RME prototypical materials. *Practicality* of the RME prototype referred to the initial satisfaction of the target groups (pupils and teachers) toward the materials and the teaching model suggested in the RME. *Implementability* of the RME prototype refers the proper teaching organization established by the teacher in teaching multiplication and division in Indonesian setting. It is analyzed from three aspects: the use of contextual problems, the interactive teaching model, and the establishment of mathematical norms in the classroom. The proper implementability would be reached after the teachers had an intensive coaching and discussion with the researcher before the learning process begins. These three quality aspects lead the study to the first sub-research question:

> *To what extent is the RME prototype valid, practical, and implementable for teaching multiplication and division of multi-digit numbers in Indonesian primary schools?*

The *effectiveness* of the RME prototype refers to the expected learning progress, understanding, and performance of the pupils in learning multiplication and division of multi-digit numbers. This lead to the second sub-research question:

> *To what extent is the RME prototype effective for teaching multiplication and division of multi-digit numbers in Indonesian primary schools?*

Those sub-research questions steer the study to choose the fitting research method and to develop reliable appraisals to be involved in measuring and analyzing the variables (elaborated in section 4.2). The research questions also reflect on the materials' characteristics to obtain quality aspects and the development process of the prototypical materials.

1.5 RESEARCH DESIGN

To address the research question and its sub-research questions mentioned above, a developmental research approach was chosen to analyze the development and improvement of the prototypical product. In the field of curriculum, it is a formative research design (Van den Akker, 1999 and Van den Akker & Plomp, 1996) and a type 1 developmental research study (Richey & Nelson, 1996), in which the research activities were conducted, the products were analyzed during a cyclic developmental process, from exploratory phase through (formative and summative) evaluation phase. The design and activities will be elaborated in section 4.2.1.

In mathematics didactics, the developmental research aims at developing an instructional sequence for specific topic where the researcher constructs prototypical instructional activities in an iterative process of designing-testing and redesigning-retesting. It is a 'theory-guided bricolage' (Gravemeijer, 1994), of which its core is in the cyclic process of thought and teaching experiments (Freudenthal, 1991). Like a handyman, the researcher can make use of all the domain specific knowledge concerning mathematics education: classroom experience, textbooks, exemplary instructional activities, relevant research, and educational psychology. The activities begin with a preliminary design of the provisional instructional activities, followed by a cyclic teaching experiments, and end up with a retrospective analysis. The design and research activities of this process are elaborated in section 4.2.2.

Based on those types of developmental research mentioned above, this study developed the RME prototypical materials in a cyclic process of front-end analysis, expert reviews, teaching experiments, and reflection to the local instructional sequences. These cyclic processes lead the study to build a conjectured local instructional theory of teaching multiplication and division of multi-digit numbers in Indonesian primary schools. The research design of this study is illustrated as follows.

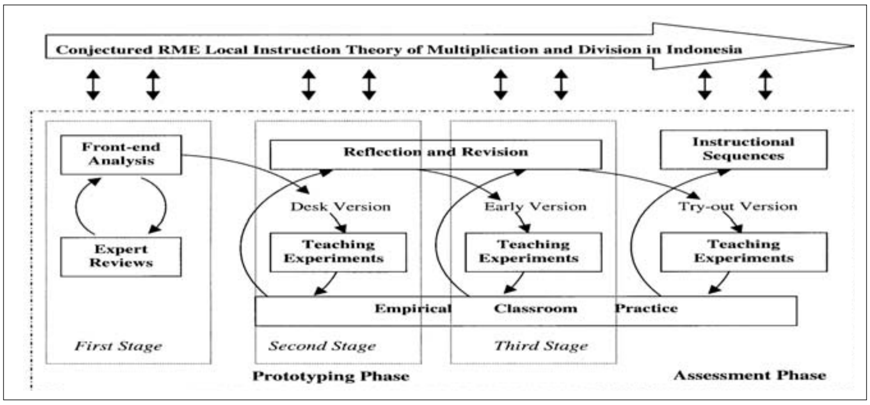


Figure 1.2
The cyclic process of developmental research

During the front-end analysis the researcher analyzed the present condition of the Indonesian mathematics education and the prospective RME approach to improve the condition. Conducting the expert reviews two RME experts and an Indonesian mathematics education expert analyzed the prototypical materials being developed in several walkthrough sessions. Then, two cyclic processes of teaching experiments were carried out in which reflections and revisions of the instructional sequences took place consecutively. Next, the last teaching experiments were conducted to analyze the effect of the instructional sequences to pupils' performances. After all, this cyclic process of front-end analysis, expert reviews, teaching experiments, and reflections shaped the local instructional sequences toward a local instructional theory of teaching multiplication and division of multi-digit numbers for Indonesian primary schools. And in the other way the local instructional theory guided the cyclic process of the research activities. It is a reflexive relationship of the local instruction theory and the cyclic research activities in this study.

In line with the cyclic research activities mentioned above, this study was conducted in two consecutive phases. *Firstly*, ***prototyping phase*** focused on analyzing validity, practicality, and implementability of the RME prototype. To address those aspects, this phase was held in three stages. *The first stage* focused on validity of the RME prototype. It referred to the presence of the state-of-the-art knowledge of the Indonesian circumstances and the RME theory (content validity) and the consistent link of the components in the prototypical materials (construct validity). Held on a cyclic process of front-end analysis and expert reviews, the first stage created a desk version of the RME prototypical materials. The process involved two RME experts and an Indonesian mathematics education expert. This stage and its results are illustrated in chapter 5.

The second stage of the prototyping phase focused mainly on practicality of the RME prototypical materials. Practicality referred to whether the RME prototype was usable and easy to the teachers and pupils. Using several sub-sequences of the RME desk version the teaching experiments took place in Yogyakarta on September – November 1999. Two primary schools were chosen purposively as sample (SD Puren and SD Kanisius) considering teachers' willingness to apply the RME approach and their competence (experienced). Three teachers and the researcher conducted the learning process using some sub-sequences of the RME materials. It aimed at having sense and insights of conducting the actual RME learning activities. Reflecting on these teaching experiments, revision of the instructional sequences was made. This reflection and revision activities involved the Indonesian and the RME experts. Analyzing the actual pupils' learning trajectory and teachers' impressions toward the RME teaching approach, the stage produced an early version of the RME prototype. This stage and its results will be elaborated in chapter 6.

The third stage of the prototyping phase focused primarily on the implementability of the RME prototype. Implementability referred to whether the teachers used the RME prototypical materials as intended. As an effect of the implementation, the initial effectiveness of the RME materials was also analyzed. It referred to whether the pupils improved on the intended level of understanding. The third stage began at August 2000 in Medan (SDN 101746 and 101748 Klumpang) and at October - November 2000 in Yogyakarta (SD Rejodadi and SD Sonosewu II). The stage developed and revealed a try-out version of the RME prototype. This stage will be elaborated in chapter 7.

Secondly, ***assessment phase*** analyzed the implementability and the effectiveness of the RME prototype in teaching multiplication and division in Indonesian primary schools (elaborated in section 4.2.3). In this phase the non-equivalent pretest-posttest control group design was chosen (Krathwohl, 1998) because the schools were not formed by random assignment. Eight schools were chosen as the experimental group and another eight schools were chosen as the control groups. They were chosen purposively. The potential lack of comparability was taken into considerations in this phase (see section 4.4). The assessment phase was conducted in Medan on August – December 2001. The phase and its results are elaborated in chapter 8.

Each phase of this study utilized various instruments and different people to collect the data to judge the validity, practicality, implementability and effectiveness of the RME prototype. The important concern was to make sure that triangulation (confirming data from different sources, confirming observations from different observers, and confirming information from different data collection methods) would be possible. The triangulation had to be applied to enhance reliability and internal validity of the findings. It relates to the significance of various individuals participated in the study and variety methods of data collection selected and applied such as interviews, observations, logbooks, tests, and quizzes (the overview of instruments are elaborated in section 4.5).

1.6 STRUCTURE OF CHAPTERS

To address the development and implementation of the RME prototype in Indonesian settings, this study first analyzed the Indonesian mathematics education contexts (Chapter 2). Then the RME will be described (Chapter 3). It analyses why RME is a promising direction to improve Indonesian pupils' understanding of mathematics. It also describes the factors determining teaching and learning mathematics and the RME method of teaching multiplication and division. It ends up with a hypothetical local instructional theory of multiplication and division. The next chapter (Chapter 4) is about the research design. The developmental research in curriculum field and in mathematics didactic will be discussed as well as the research design of this study and the appraisals used. The next four chapters (Chapter 5, 6, 7, and 8) explain the results of this study concerning the quality aspects of the RME prototype. Then, Chapter 9 illustrates the conjectured local instructional theory of multiplication and division of multi-digit numbers. It relates to the intended pupils' learning trajectory. The last chapter interprets the conclusion, which includes the summary, reflections, and recommendations of this study.

CHAPTER 2
MATHEMATICS EDUCATION IN INDONESIAN PRIMARY SCHOOLS

A concise picture of this study has been illustrated in the previous chapter, including the Indonesian mathematics education as the settings of research. This chapter explores the settings in more detail focusing on the teaching and learning of mathematics in Indonesian primary schools. The goals of learning mathematics in 1994 curriculum are to prepare the students to use and apply their mathematics understanding and thinking in solving problems in their life and in learning other different knowledge. These goals influence teachers to conduct the learning process mechanistically focusing on memorizing facts and procedures of mathematics. It effects on the pupils' confusions and weaknesses. One of which is in learning multiplication and division of multi-digit numbers. This chapter illustrates variety of weaknesses found by considering its background (the curriculum, the goal of teaching, and the teacher education).

2.1 INTRODUCTION

The declaration of the Nine-year Basic Education in 1995 had an intense influence in the mathematics curriculum in Indonesia. It began by implementing the 1994 curriculum to all Indonesian schools from primary schools until senior high schools. It substitutes the learning time, the contents and the way of teaching them. This chapter describes the 1994 mathematics curriculum in primary schools (section 2.2), the teachers and teacher education (section 2.3), the goals of learning mathematics in Indonesia (section 2.4), the weaknesses in teaching and learning multiplication and division (section 2.5), and the direction of improvement in the future (section 2.6).

2.2 THE 1994 MATHEMATICS CURRICULUM

In the last three decades, there have been three changes in Indonesian mathematics curriculum the primary schools. *Firstly*, the implementation of the 1975 mathematics curriculum. The curriculum focused on teaching and learning arithmetic, where the pupils learned and studied basic arithmetic skills (addition, subtraction, multiplication and division). The teachers emphasized the operations using conventional teaching model, teaching how to do calculation, give an example and practice the calculation. The pupils imitate and do the calculation on their own. The teachers did not illustrate any means of the mathematics concepts for the benefit of students' understanding.

Secondly, the 1984 curriculum emphasized the use of set theory in teaching and learning mathematics in the classroom. The aim was to enhance the curiosity of the pupils to investigate mathematics concepts. This new direction was wanted by many mathematics educators because it expected to meet the goals. But it had been stirred up most criticism from the parents and society (Depdikbud, 1997). The curriculum itself was inspired by many movements all over the world such as the modern mathematics and the idea of teaching for understanding.

Designed by the ministry of Education and Indonesian mathematics education experts, the teachers were supported by a prescribed teaching guide, a prescription of what and how to teach for a certain week. For practical teaching process, the teachers obliged to create the lesson plan for each content. They constructed the instructional objectives, the methods and teaching aids used (in details), activities for pupils and teachers, and the test items to be applied in a definite time. In facilitating this activity, most teachers acquired a training session in their region (PKG or MGMP). Though they spent much time on writing the plan they hardly applied it in the classroom because it was only for the sake of administration. The daily practical lesson plan was actually "in the heads of the teachers" (Marsh & Willis, 1995).

Thirdly, the 1994 curriculum had significance changes in many perspectives compared to the 1984 curriculum. The difference is on the interval time of the teaching and learning system. In the 1994 curriculum, the trimester system orientation has been applied in altering the semester system (six months of learning). In the 2002/2003 of school year the semester system will be applied again (see www.pdk.go.id, the latest news, 6 March 2002). For primary schools, the 1994 curriculum focused on the teaching and learning arithmetic and the set theory was not a priority anymore. Like in the 1975 curriculum, the pupils had to learn the arithmetic aspects. A new aspect was that the pupils' understanding of mathematics became the key element to be taken into account in the teaching-learning process.

In implementing the 1994 curriculum, the teachers were not well prepared how to teach for understanding, how to do the learning process, how to work smoothly, how to turn for assistance, how to do consistently with the plan, and how to evaluate the effect on the pupils. They had minimum competencies and resources to make up a particular topic, were not able to spend the long period of time needed to prepare the instructional materials, and were not financially adequate to provide instructional aids. This low competence influenced teachers to teach the lower level of thinking by using paper-and-pencil strategy combine with the concepts-operations-example-drilling approach (Suyono, 1996). The teachers and its education are described in the following section.

2.3 THE TEACHERS AND TEACHER EDUCATION

According to the schooling statistics 1995/1996, there are more than one million primary school teachers in Indonesia (see Table 2.1. below). It is 0.5% of the 200 million Indonesian citizens. They have been spread all over Indonesia to be the self-content teachers; i.e. teaching all subject matters to the pupils in a classroom. In order to be the primary school teachers, they have to graduate from SPG (Sekolah Pendidikan Guru or School of teacher education), a senior high school where the pupils learn to be teachers in primary school.

Table 2.1
Numbers of teachers in Indonesian schools

School category	**Numbers of teachers**
Primary Schools (SD)	1,179,177
Junior High Schools (SLTP)	413,910
Senior High Schools (SLTA)	331,946

Source: Indonesia Educational statistics in brief 1995/1996, Depdikbud, 1996.

There were several weaknesses during their learning in the SPG. *Firstly*, pupils entering the SPG were coming from the lowest achievement pupils. Since the low salary of teachers in the primary school, the motivation of the pupils in attending the SPG was low and only the low ranking pupils applied for the SPG. *Secondly*, the lack of teaching aids influenced the teaching and learning process and the pupils' preparation to be teachers was not effectively accomplished. These weaknesses produced poor quality teachers (see chapter 1.2.1 and table 2.2 below). Cipra (1992) also mentioned that since the pupils were not provided with enough mathematical learning experiences, they go to classroom to teach as they were taught.

It was in 1995 after the declaration of the nine years basic education that the policy in recruiting teachers in the Primary schools was changed. The teachers have graduate from a university (FKIP or Faculty of Education) or an institute of teacher training (IKIP - Since 1998, those 10 IKIPs have been changed to universities). A PGSD (Teacher Training for Primary School) program in a university prepares pupils to be primary school teachers. The Indonesian government subsidized this teacher training education program and there is a National examination entrance for this program. This program is delivered to all pupils who graduated from senior high school pupils.

There are two aims of this program. *Firstly*, the program is to increase the quality of the primary School teachers. According to the schooling statistics 1995/1996 (Depdikbud, 1996), 89% of the primary school teachers were not capable to teach subject matters (see Table 2.2. below). It is assumed that by giving the pupils more opportunity to study in the field of education in the university the prospective teachers could enhance their knowledge of subject matters and pedagogy.

Table 2.2
Numbers of unqualified teachers in Indonesian schools

School category	Numbers of teachers	Percentage
Primary Schools (SD)	1,049,468	89%
Junior High Schools (SLTP)	235,929	57%
Senior High Schools (SLTA)	86,306	26%

Source: The schooling statistics 1995/1996, Depdikbud, 1996.

Secondly, the program also gives opportunity to the primary school teachers to develop their competencies in teaching subject matter. For this reason a D2 (a two-year university program) program is prepared for the teachers in the universities. The government believed that this program would be an alternative solution for improving teachers' qualification mentioned in Table 2.3 below. Most teachers (79.81%) graduated from the senior high school (SPG). It is very dispirited description of the Indonesian Primary school teachers when it is compared to the primary school teachers in United States where 52% of the teachers graduated from the university and 42% of the teachers graduated from Master Program (Digest of Education Statistics, 1997).

Table 2.3
Percentage of the highest degree earned by the Indonesian teachers

School category	Percentage	Graduate from
Primary Schools (SD)	0,06%	SD (Primary schools)
	6,33%	SLTP (Junior high schools)
	79,81%	SLTA (Senior high schools)
	3,47%	PGLSP (Senior high Schools)
	6,45%	BA (Diploma program, D2 or D3)
	3,87%	Drs (University or institute)

Source: The schooling statistics 1995/1996, Depdikbud, 1996.

The teacher recruitment policy of Indonesian Education Ministry has been changed in order to restructure teacher education in Indonesia since the declaration of the nine years basic education in 1995. The declaration influenced the abolishment of the SPG as a school for teacher education. To be a teacher in primary schools one should graduate from a university. This policy is based on the teacher's pre-requisites developed by Leinhardt (1988) and Carpenter & Fenneme (1992). They suggested that the teachers need a certain competence in doing mathematics themselves, a positive attitude towards mathematical activity, an understanding of the ways pupils think, and a specific didactical knowledge. Learning more about the mathematics contents and the pedagogical knowledge in a university would be an alternative to develop teachers' competencies in order to pursue the main goal of teaching mathematics. The goals are illustrated in the following section.

2.4 THE GOALS OF LEARNING MATHEMATICS IN INDONESIA

In primary schools, the mathematics curriculum aims at preparing the pupils to use and apply their mathematics knowledge and mathematical way of thinking in solving problems in their life and in learning other different knowledge

(Depdikbud, 1993). It means that the pupils should develop their counting ability, enhance their mathematics content knowledge, and structure their attitude to be critique, honest, disciplined, efficient, and effective.

In order to accomplish the goals of the 1994 curriculum, the conventional teaching and learning process (see section 1.3) needs to be changed to another approach where the teachers challenge the pupils with well-selected mathematical problems and a classroom culture that encourages and facilitates learning. Becker and Shelter (1996) believed that learning in this environment, pupils will improve their learning activities: learning actively, learning individually, learning cooperatively, and learning in strands and contexts.

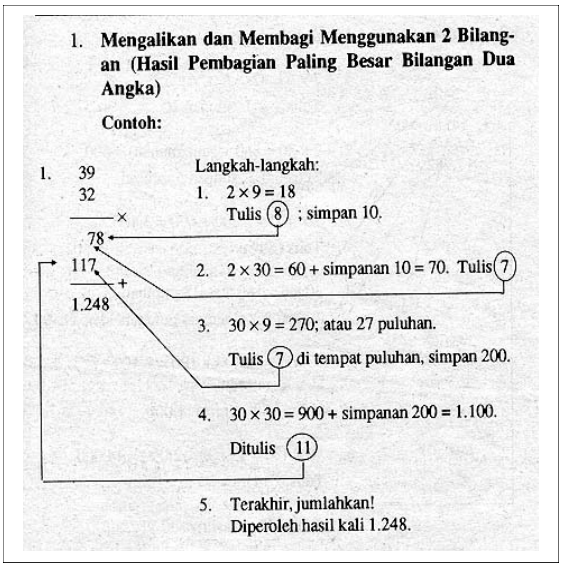
In order to encourage the students to be self-active and responsible for their own learning, the learning process should take place in meaningful contexts, in which the inter-relatedness of mathematics and its connections with the real world exists. The teachers as facilitators accomplish the learning activities by selecting mathematical tasks to engage pupils' interests and intellect, providing opportunities to deepen their understanding of the mathematics, orchestrating classroom discourse in ways that promote the investigation and growth of mathematical ideas (Romberg, 1998).

2.5 THE WEAKNESSES IN TEACHING AND LEARNING MULTIPLICATION AND DIVISION

To achieve the goals illustrated above, the learning process of mathematics was conducted by practicing the chalk and talk model or the concepts-operations-example-drilling approach (Suyono, 1996). The teachers explain the mathematics operation and procedures, give some examples, and ask the pupils to do the other similar problems. This model of teaching is called the mechanistic way of teaching (Treffers, 1987). The teachers teach mathematics with practicing mathematics symbols and emphasizing on giving information and application of mathematics algorithms.

Applying this model of teaching in the classroom has had a discouraging effect in pupils' achievement. A study from Semiawan (Akbar, 1998), the results from EBTANAS - National Examination - in several years (1994 - 2001, Depdikbud, 1997), TIMSS (1997) proved this. It means that many pupils do not possess the knowledge, skills, beliefs, and motivation that are needed to efficiently solve mathematics problems and to effectively employ in their daily life.

In learning multiplication and division of multi-digit numbers for instance, the teachers use teachers' guide from the books published by the ministry of National Education (i.e. "MARI BERHITUNG", Moesono & Sujono, 1994) or published by a private publisher such as Erlangga, Yudistira, Surabaya (i.e. "MATEMATIKA/ BERHITUNG", 1997). The books explain the algorithm way of teaching multiplication and division of multi-digit numbers as follows.

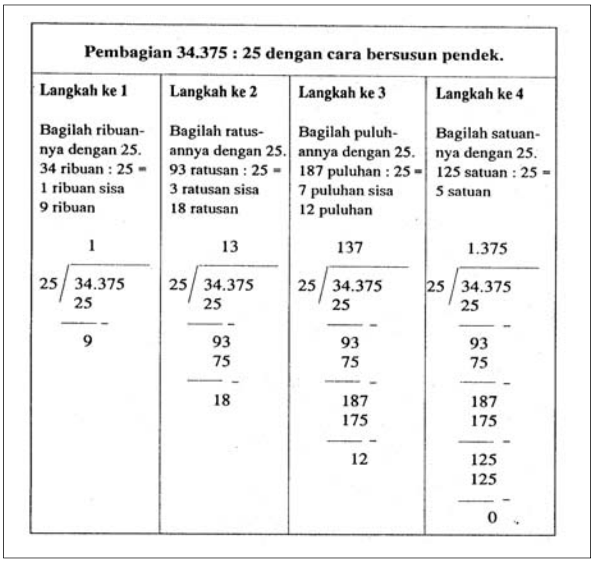


1. **Mengalikan dan Membagi Menggunakan 2 Bilangan (Hasil Pembagian Paling Besar Bilangan Dua Angka)**

Contoh:

1. 39
32 ×
78
117 +
1.248

Langkah-langkah:
1. 2 × 9 = 18
Tulis 8 ; simpan 10.
2. 2 × 30 = 60 + simpanan 10 = 70. Tulis 7
3. 30 × 9 = 270; atau 27 puluhan.
Tulis 7 di tempat puluhan, simpan 200.
4. 30 × 30 = 900 + simpanan 200 = 1.100.
Ditulis 11
5. Terakhir, jumlahkan!
Diperoleh hasil kali 1.248.



Pembagian 34.375 : 25 dengan cara bersusun pendek.

Langkah ke 1	Langkah ke 2	Langkah ke 3	Langkah ke 4
Bagilah ribuannya dengan 25. 34 ribuan : 25 = 1 ribuan sisa 9 ribuan	Bagilah ratusannya dengan 25. 93 ratusan : 25 = 3 ratusan sisa 18 ratusan	Bagilah puluhannya dengan 25. 187 puluhan : 25 = 7 puluhan sisa 12 puluhan	Bagilah satuannya dengan 25. 125 satuan : 25 = 5 satuan
1 25 / 34.375 25 —— - 9	13 25 / 34.375 25 —— - 93 75 —— - 18	137 25 / 34.375 25 —— - 93 75 —— - 187 175 —— - 12	1.375 25 / 34.375 25 —— - 93 75 —— - 187 175 —— - 125 125 —— - 0

Figure 2.1
The algorithmic model of teaching multiplication and division

The mechanistic learning begins with teachers' explanation of each step of operations of multiplying and dividing multi-digit numbers and then the pupils imitate the step of operations as the teachers did. The starting point is in the formal level of the world of symbols where the instruction becomes the presentation and drill of rules and regulations or the algorithmic mathematics education (Treffers, 1991). It is a description of teaching the algorithm mechanistically by using place value, using mental algorithm, and using standard (column) algorithm. The pupils learn from the teachers' explanation and imitate the strategy to solve another problem.

By observing and analyzing the pupils' log several weaknesses were found such as multiplying 1-digit numbers, incorrect adding, corrupt operation procedure, unjust mixed procedure, incorrect place value, and guessing multiplication (Armanto, 2000). This observation was conducted in a classroom with 42 pupils in Yogyakarta during the first stage of the prototyping phase. The analysis showed that most pupils (60% out of 42 pupils) had a lack of memorizing multiplication facts. Some pupils applied the repeated addition, while others utilized their hands in adding numbers. Figure 2.2 followed illustrates the mistakes.

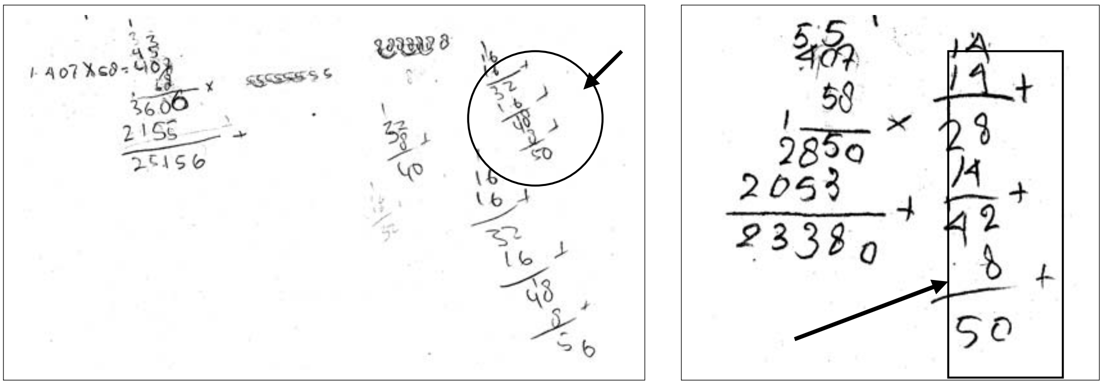


Figure 2.2
Incorrect multiplication and repeated addition (*pointed by the author)*

Both incorrect examples showed that the pupils incorrectly doing the multiplication of 7 x 8 by adding 8 seven times (the right picture) and by adding 7 eight times (the left picture). They got 50 rather than 56.

Other mistakes occurred because of the mechanistic way of learning the multiplication. The pupils memorized the operation procedures and tried to apply them in answering problem. Using their own understanding the pupils applied a corrupt operation procedure. Figure 2.3 below illustrates the mistakes.

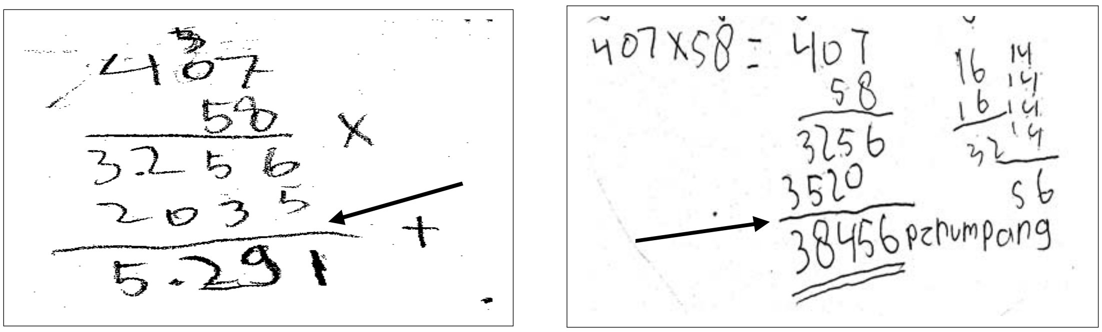


Figure 2.3
The corrupt multiplication procedure (*pointed by the author)*

In solving the division problems there were some mistakes identified such as incorrect multiplying numbers, incorrect adding numbers, a corrupt division procedure. Analyzing the pupils' log, it was found out that many pupils utilize the multiple addition in multiplying 1-digit and 2-digit numbers. It made the incorrect adding numbers. Figure 2.4 below illustrates the mistakes.

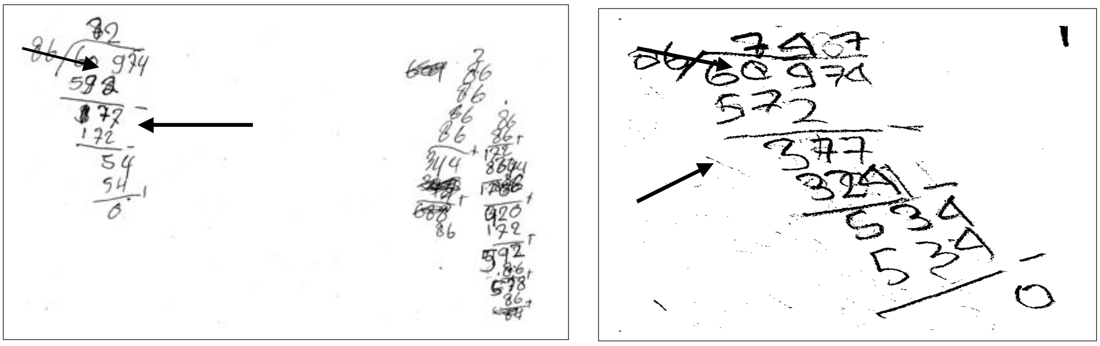


Figure 2.4
Incorrect addition and multiplication (*pointed by the author)*

The pupils also made mistakes in multiplying 1-digit numbers to 2-digit numbers (in the picture 7 x 86 ≠ 572 but 602). The right picture in Figure 2.4 above illustrates the mistakes.

Another mistake occurred when the pupils applied the standard division algorithm that they learnt mechanistically in the classroom. The pupils memorized the steps but incorrectly applied the procedure. The following figure illustrates the mistakes.

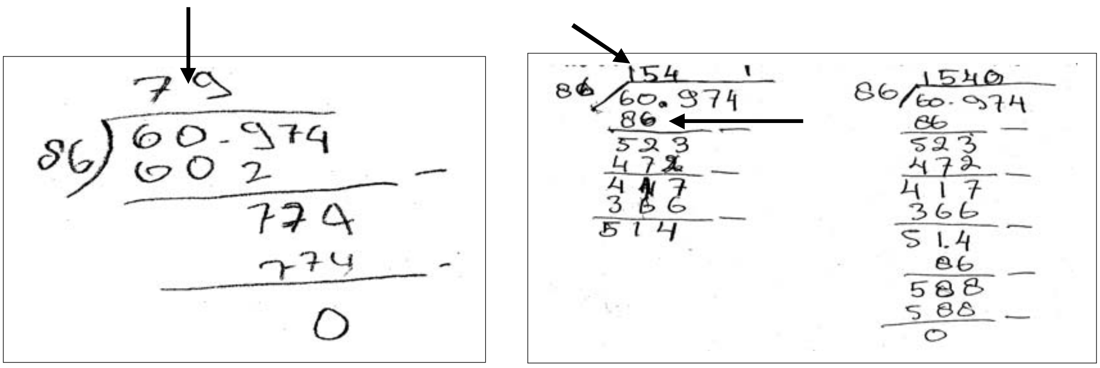


Figure 2.5
The corrupt division procedure (*pointed by the author)*

The mistakes showed that pupils had difficulties in learning multiplication and division mechanistically. The repeated addition was the main choice to get the product, even though in some reasons (carelessness, for instance) they made incorrect addition. Other weaknesses referred to applying procedures. Pupils depend on the explanation of the teacher. They did not understand other strategies because the teacher only taught the standard algorithm. These were the drawbacks that made pupils structure a "buggy procedure" in solving problems (Carroll & Porter, 1998). Pupils did not comprehend the means of the multiplication and division algorithm.

2.6 DIRECTION OF IMPROVEMENT IN THE FUTURE

The weaknesses mentioned above apparently show the necessity to improve the curriculum. The learning techniques for solving closely defined problems whose terms include a few key words giving the pupils the clue as to which mechanical process to apply should disappear. Meanwhile presenting pupils with a situation in which they can understand and explore many mathematics activities assumes a considerable change in the nature of teaching. The variety of questions in which mathematics occurs makes it possible to provide an exploratory attitude to the pupils in order to offer them a wide variety of techniques in solving mathematics problems.

Regarding these facts, the need of innovation in implementing mathematics curriculum for Indonesian schools becomes more important. Firstly, there has to be found a representative approach in teaching mathematics in the way that all pupils can understand and master the facts, concepts, and procedures in order to continue their study and solve problems of their everyday life. Secondly, there should be found a proper alternative program solution in order to improve the teachers' competences to teach mathematics in the classroom. Finding solutions for innovating mathematics teaching in Indonesian is the most important thing to be conducted to improve teachers' role in the classroom and to give pupils chances to do mathematics activities in favor of improving their own skill to answer the problems.

One kind of change that is beneficially pursued is what is called "realistic mathematics education (RME)": mathematics education that is compatible with the idea of mathematics as a human activity (Freudenthal, 1983). In this philosophy, the mental activity of the learner is at the center. Mathematics is an activity of doing and reinventing mathematics (mathematizing subject matter). Analyzing and reflecting own mathematical activity is the main key principle of reinventing mathematics. The RME theory is illustrated more in the next chapter.

CHAPTER 3
REALISTIC MATHEMATICS EDUCATION (RME)

The Indonesian mathematics education has been described in the previous chapter, including its domain drawbacks (the teachers' understanding of the mathematics concepts, of the pedagogical aspects, and the pupils' learning cognition) that need to be improved. One of the promising strategies is the Realistic Mathematics Education (RME) approach. Rooted in Freudenthals' interpretation of mathematics as a human activity, the RME approach activates pupils to reinvent mathematics forms by encountering contextual problems. It also prepares the teachers to learn more about the didactical phenomenology and the pupils' learning trajectory. This chapter illustrates the teaching and learning mathematics in a general view and gives its focus on RME theory and its instruction model. An example of teaching multiplication and division realistically is also summarized concisely.

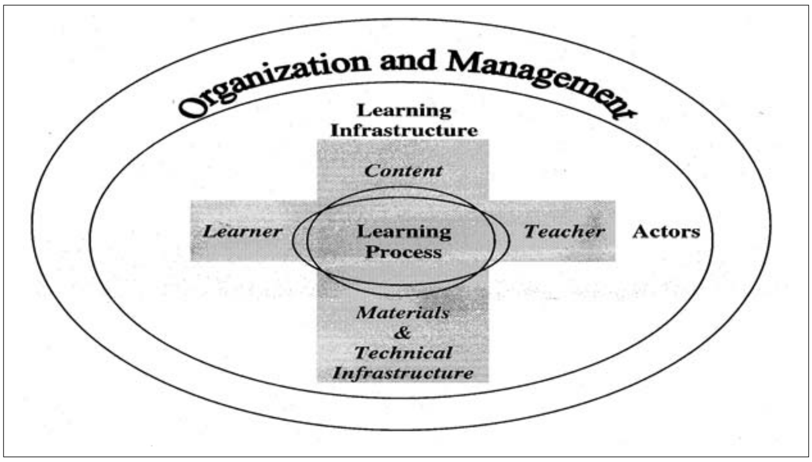
3.1 INTRODUCTION

As discussed in the previous chapter, the instructional materials and the teachers' competence should be improved in order to remediate weaknesses in the Indonesian pupils' mathematical understanding. It will be argued in this chapter that the RME approach would be a promising model to develop teachers' competencies. The following section (3.2) discusses in general terms of the mathematics teaching and learning. Next the principles of the RME approach are elaborated in section 3.3. Then the concepts of section 3.2 and 3.3 are combined in section 3.4 that illustrates RME local instructional approach for teaching multiplication and division (section 3.4). Then a conjectured learning example of teaching multiplication and division using RME approach is illustrated in section 3.5.

3.2 THE TEACHING AND LEARNING MATHEMATICS

In having a significant picture of classroom activity this study examines two dimensions of learning process proposed in Figure 3.1 below by Plomp and

Brummelhuis (1998). The horizontal dimension represents the relation between the actors in the learning process: the teacher and the learner. The vertical dimension represents the learning infrastructure, consisting of the content (and the goals), and of teaching and learning materials and assessment procedures. The outside circle represents the school organization and management that provides the context or environment of the arrangement of the learning process. The learning process takes place as a result of the interaction among the four forces: teacher, leaner, content, and materials. It is also a result of structural conditions derived from the learning infrastructure, the beliefs, and the personal characteristics of the actors involved, and their interaction in the process.



Source: Plomp and Brummelhuis, 1998.
Figure 3.1
The wheel of learning process

The role of actors (pupils and teachers) is characterisized by activities conducted in the learning process (Simons & Zuylen, 1995). There are three main categories of activities: preparation, executing instruction, and regulation. The preparatory activities include cognitive and affective aspects. The cognitive aspects contain the activities of choosing and defining sub-goals, the learning strategies, and mobilizing prerequisite knowledge. While the affective aspects consist of challenging pupils and focusing their attention. The executing instruction comprises several activities such as absorbing knowledge, practicing skills, reflecting and formulating conclusions, relating to what is being learned, and getting an overview. In regulatory activities, the focus is also on cognitive and affective aspects. The cognitive aspects comprise testing, monitoring, and reflecting on the learning process and progress, evaluating and taking recovery actions. The affective aspects include to maintaining motivation, generating feedback, and doing self-assessment.

In the "traditional" arrangement of the learning process (described in chapter 2), the teachers take control over each activity. In contrast, the RME approach suggests that the pupils are supposed to take responsibility for their own learning and actively engage in interactive discussion in the classroom. Guided by the teacher, the pupils reinvent informal and formal mathematics models in a process of mathematizing contextual problems. The pupils actively execute the instruction in order to understand the mathematics concepts and procedures.

In the learning process of mathematics, however, Koehler and Grouws (1992) mention that two factors involve interactively in the classroom processes, i.e. pupils' behavior and teachers' behavior (as like in Figure 3.1 above). In the classroom processes, pupils' own behaviors influence the outcomes of their learning process (achievement and attitudes) and the pupils' characteristics give some involvement in their behaviors. Pupils' confidence in learning mathematics, their belief of its usefulness, and their feelings of discovering mathematics are some components that influence pupils' behavior in the classroom.

The teachers' behavior depends on three characteristics, one of which is the teachers' knowledge. The components of the teachers' knowledge are the knowledge of mathematics, of learners' cognition in mathematics, and of pedagogical aspects (Fennema & Franke, 1992). Teachers' knowledge of mathematics includes teacher competence on the concepts, procedures, and problem-solving process within the domains of mathematics. It also includes the concepts underlying the procedures, the inter-relatedness of these concepts and the use of the concepts and procedures in solving problems. Pedagogical knowledge includes teachers' knowledge of teaching procedures such as effective strategies planning, classroom routines, behavior management techniques, classroom organizational procedures, and motivational techniques. And knowledge of learning cognition includes competence of how pupils acquire the knowledge of the mathematics content being addressed, as well as understanding the process the pupils will use and the difficulties and successes that are likely to occur.

Also influencing teachers' behaviors are the teachers' attitudes and beliefs about teaching and mathematics content. Ernest (1988) mentions that the research literature on mathematics teacher's beliefs indicates that teachers' approaches to mathematics teaching depend fundamentally on their systems of beliefs, in particular on their conceptions of the nature and meaning of mathematics and on their mental models of learning and teaching mathematics. For example, teachers who believe that pupils learn by explicit examples and repetition or by extensive practice, and who see their role as dispenser information, would behave differently in the classroom than teachers who believe pupils learn by reinvention and who see their role as the facilitator for the pupils. These later teachers might ask more open-ended questions, engage in more problem posing and are less tied to the textbook.

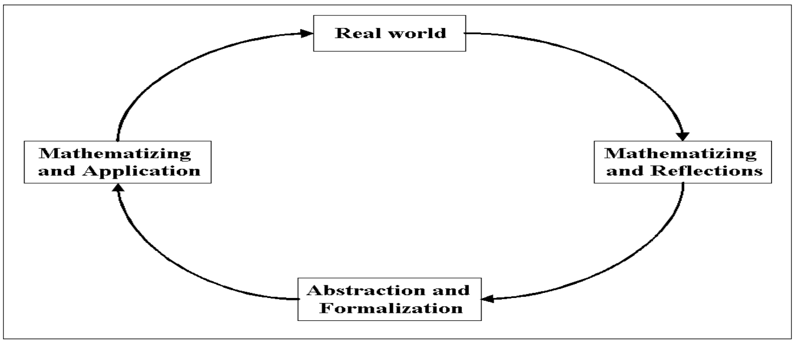
The CGI (Cognitively Guided Instruction) model (Fennema, Carpenter, & Peterson, 1989) suggests that the classroom process is based on "teacher decisions which are presumed to be based on their own knowledge and beliefs as well as their assessment of pupils' knowledge through their observation of pupils' behaviors". This classroom instruction model considers three tenets, i.e. that instruction must be based on what each pupil knows, it must take into consideration how pupils' mathematical ideas develop naturally, and pupils must mentally active as they learn mathematics. In implementing these tenets, the teacher might provide pupils time to solve problems and ask them to explain how they encounter the problem to find the solution. The emphasis of this activity is on the process of mathematizing the problem, rather than on the answer.

From the RME viewpoint, the goal of instruction is not to develop the pedagogical strategies to help pupils receive mathematical knowledge, but rather to structure, monitor, and adjust activities for pupils to engage in (Gravemeijer, 1997 and Koehler & Grouws, 1992). Cobb, et al. (1991) explain that 'mathematical learning is not a process of internalizing carefully packaged knowledge but is instead a matter of reorganizing activity, where activity is interpreted broadly to include conceptual activity or thought". This is the teaching-by-negotiation model where teacher guides and facilitates the pupils' construction of knowledge. The teacher and pupils work as co-partnership of each other building positive social interaction. They can verbalize their thinking, explain or justify their solutions, and also ask for clarification to the teacher.

3.3 REALISTIC MATHEMATICS EDUCATION (RME)

The development of the RME (Realistic mathematics education) and its underlying educational theory evolved after thirty years of developmental research in teaching and learning mathematics in the Netherlands and is rooted in Freudenthal's interpretation of mathematics as a human activity (De Lange, 1994; Freudenthal, 1973; and Gravemeijer, 1994). According to Freudenthal, mathematics must be connected to reality, stay close to children and be relevant to society in order to be of human value. This point of view involves regarding mathematics not as subject matter but, rather, as a human activity. He argued that human beings have to learn mathematics not as a closed system, but rather as an activity, the process of mathematizing reality and mathematizing mathematics (Goffree, 1993).

Freudenthal (1971, 1973) also argued that as a mathematizing activity, mathematics can best be learned by doing. The mathematics should never be presented to pupils as a ready-made product and that pupil should reinvent mathematics. They should be treated as active participants in educational process, in which they themselves develop all sorts of mathematical tools and insights (Heuvel-Panhuizen, 1996). De Lange (1987) called this process as "conceptual mathematization". The process can be illustrated as follows:



Source: De Lange, 1987.

Figure 3.2

The conceptual mathematization process

The real world situation is first explored intuitively by the pupils, for the purpose of mathematizing it. This process intends to organize, structure the problem, to identify the mathematical aspects of the problems. By reflecting on their own mathematical activities they discover regularities and relations. This exploration leads to development of reinvention of mathematics concepts. The interaction among pupils and teachers and with social environment, the pupils can formulize and abstractize the mathematical concepts and their conceptual mathematization grows. These activities lead them to mathematizing different problems. This increases the reinforcement of the concepts after being structured and improves the readjustment of the perceived real world (De Lange, 1994). Note that mathematics itself will become part of the real world that gets mathematized.

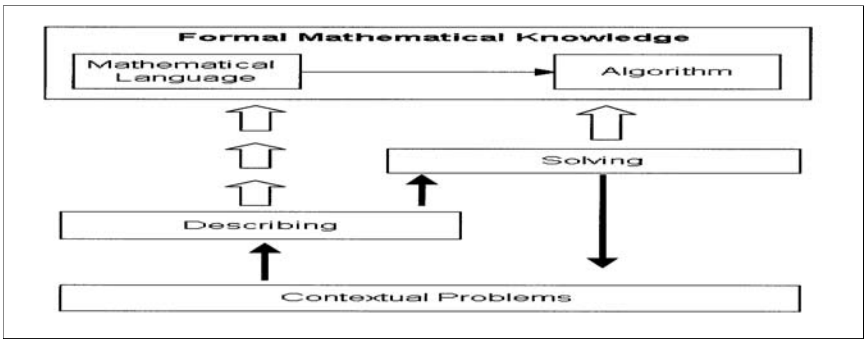
3.3.1 The principles of RME

The first principle of RME is called "guided reinvention and progressive mathematization" (Gravemeijer, 1994). The pupils should be given the opportunity to experience a process of reconstructing or reinventing mathematical ideas and concepts through encountering many varieties of contextual problems. This principle assumed that knowledge cannot be instructed (transmitted) by the teacher, but it can only be constructed by the learners.

The reinvention of mathematical forms begins when pupils use their everyday language (informal description) to structure contextual problems into formal or informal mathematical forms. In learning multiplication for instance, they develop repeated addition whenever they solve the "Tiles" problems (see section 9.4.1 item a) and the "Playing cards" problem (see section 9.3 item 1). The pupils use their former knowledge to create a type of strategy in order to structure the contextual problems into mathematical ideas. When they use the mathematical tools (adding numbers, for example) to calculate the numbers and find the solution, they develop mathematical concepts and procedures in their understanding. These activities grow incrementally as many times as they encounter the contextual problems.

In RME, the progressive mathematization can be distinguished into two components: horizontal and vertical mathematization (Treffers & Goffree, 1985). In horizontal mathematization, one can identify that the contextual problems should be transferred into mathematical stated problem in order to make it more understandable. Via schematizing, formulating, and visualizing one tries to discover regularities and relations and to transfer them to specific mathematics formulation in a general contexts. Starting with solving common problems in common situations, horizontal mathematization produces models (genuine or informal schemes and notations of the mathematical models) (Treffers, 1991). In doing horizontal mathematization the pupils, guided by the teacher, identify specific mathematics in a general context, schematize, formulate and visualize a problem in different ways, discover relations and regularities, and transfer the real word problems to a mathematical problem and model (De Lange, 1996).

On the other hand, vertical mathematization is a process of a variety of reorganizations the mathematical activity itself. Finding shortcuts, discovering connections between concepts and strategies, and then applying these discoveries are implicit in vertical mathematization. For instance, the formal mathematics forms are treated with mathematical tools, such as operations, concepts, and procedures. By representing the relation of the mathematical formula and using mathematical regularities, the model is treated constructively by which the solution is found. At last the learners can reformulate and generalize the problem by confronting the solution to real world context, condition, and problem. Guided by the teacher the pupils represent a relation in a formula, prove regularities, refine and adjust models, use different models, combine and integrate models, formulate a new mathematical concept, and generalize them (De Lange, 1987).
Freudenthal (1991) describes that the horizontal mathematization leads from the world of life to the world of symbols, while vertical mathematization means moving within the world of symbols. The symbols themselves are shaped, reshaped, and manipulated mechanically and comprehensively. He emphasized, however, that the differences between these two worlds are far from clear cut. The two forms of mathematization were equal value and both activities could take place on all levels of mathematical activity. Gravemeijer (1994) illustrated the activities of the horizontal and vertical mathematization as in the following Figure 3.3.



Source: Gravemeijer, 1994.
Figure 3.3
The reinvention model

Gravemeijer (1994) described that over time the informal notations and solution procedures develop into standard notations and fixed algorithms. The pupils also should get accustomed to using formulas in communicating, to all kinds of graphic or tabular representation and learn to use mathematical models and judge their relevance (De Lange, 1996). The essential importance of the reinventing process is the fact, in a case of multiplication and division, that the pupils are confronted with problem situations in which they produce gradually and cleverly the repeated addition and subtraction algorithms themselves (Dekker, Ter Heege, & Treffers, 1982; Gravemeijer, 1994; and Treffers, 1991). And in contrast, conventionally (mechanistic algorithmic instruction process) the pupils' own activities are often eliminated with all resulting consequences.

The second principle of RME relates to the idea of didactical phenomenology (Freudenthal, 1983). The didactical phenomenology means that the contextual problem and situation chosen to introduce the mathematics topic should be in favor of two purposes, i.e. to reveal the kind of applications that have to be anticipated in instruction and to consider their suitability as an impact for a process of reinvention and progressive mathematization. It is also to fulfill four functions (Treffers and Goffree, 1985):

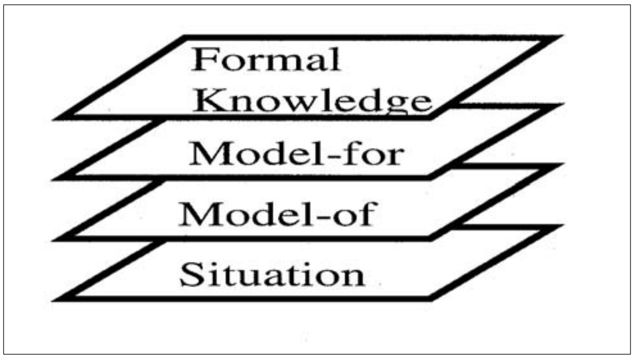
- concept formation (to allow pupils natural and motivating access to mathematics),
- model formation (to supply a firm basis for learning the formal operations, procedures, notations, and rules in conjunction to other models as the support for thinking),
- applicability (to utilize reality as a source and domain of applications),
- practice (to exercise the specific abilities of the pupils in applied situations).

In order to concretize the exploration of mathematical ideas, Freudenthal (1983) describes the didactical phenomenology process by the following approach:

- starting from those phenomena that beg to be organized in order to create the opportunity for the learner to construe these means of organizing.
- In order to teach groups, rather than starting from the group concept, looking around for phenomena might compel the learner to constitute the mental object that is being mathematized by the group concept.

This Freudenthal's prescription using reality as a source for mathematization, together with the macro-structure according to the three Van Hiele levels, becomes the first framework of instruction theory.

The third principle of RME is found in the role of which the emerged model plays in bridging the gap between informal knowledge and formal mathematics (Gravemeijer, 1994). In RME, models are presented and developed by the pupils themselves. They enhance the models by using their former models and knowledge about mathematics. At first the model is used as the model of situation they encounter in the problem that is familiar with them. By the process of formalizing and generalizing, the model is developed and used as a model for mathematical reasoning until they acknowledge of the formal understanding of mathematics (mathematical language, symbols, and algorithms). The following Figure 3.4 illustrates the emerged model of RME.



Source: Gravemeijer, 1994.
Figure 3.4
The emerged model of RME

The Freudhental's philosophy can be considered as a global theory (Gravemeijer, 1997) because it claims to be applicable to all mathematics topics. It has been elaborated in many prototypes for specific topics that represent *local theories* (e.g. local instruction theories on fractions, addition and subtraction, written algorithms, matrices, differentiating, and exponential functions). In another words, global theory can be concreted and reconstructed in analyzing local theories. It serves as a design for new formal curriculum of RME domain specific (local) instructional model. The local instruction model will be elaborated in the next section.

3.3.2 The RME tenets of teaching

Treffers (1987) traced and reconstructed five tenets of progressive mathematization as a representation of development guidelines of a domain specific instruction theory for RME.

The use of contextual problems: In RME, contextual problems play its role as a meaningful starting point from which the intended mathematics can emerge rather than as applications at the end of the learning sequence.

Bridging by vertical instruments: The intention is given on formatting mathematics concepts and models of the problem situations that arise from problem-solving activities. It can help to bridge the gap between the intuitive phenomenological level and the level of subject-matter systematics (the level of mathematics as a formal system).

Student contribution: The constructive element is visible in the large contribution to the course coming from the student's own constructions.

Interactivity: Explicit negotiation, intervention, discussion, cooperation, and evaluation are essential elements in a constructive learning process in which the students' informal methods are used as a lever to attain the formal ones.

Intertwining: The holistic approach, incorporates application, implies that learning strands can not be dealt with as separate entities, instead an intertwining of learning strands is exploited in the problem solving activities.

The five tenets were characterized by an integrated relation of the five pairs of learning and teaching principles by Treffers (1991).

Firstly, constructing and concreting. He convinced that learning is a constructive and long-term activity stimulated by concreteness. It contradicts with the idea of learning by absorbing knowledge that is presented or transmitted. The constructive characteristic is seen as pupils develop their own strategy (for instance, constructing repeated addition for multiplication problem) for themselves. For this reason the teaching must involve contextual problems that can be realized by pupils (thus 'realistic' does not necessarily mean 'real life').

Secondly, levels and models. The second learning principle is the learning of a mathematical concept or skill is a process which is often stretched out over the long term and which moves at various levels of abstraction. These level characteristics can be seen in strategies developed by pupils (see Figure 3.6 and 3.7 in section 3.4

below). The first strategy in division for instance (see the left picture of Figure 3.7) refers to the division situation of the buses; the arithmetic is in the informal context-bound level. Later the applicability of division procedure is broadened and the calculations are understood completely within the formal number system. To raise from the context-bound level to formal arithmetic pupils must have tools or materials to bridge the gap between the concrete and abstract level. Materials, such as situation model, schemes, diagrams, and symbols serve this purpose.

Thirdly, reflection and special assignments. The third principle of learning is; learning mathematics is promoted through reflection on own thought process of others. Finding and interpreting the remainder in the Feijenoord problem is an example of reflection. For facilitating this reflection in the teaching process, pupils must have opportunity and stimulus at the essential junctions in the course, to reflect on learning strands that have already been encountered and to anticipate on what lies ahead. Important assignments that should be given to pupils consider the following aspects: (a) the magnitude of the numbers with which pupils dare to work, (b) the level of schematizing at which they calculate, (c) possible systematic errors, and (d) the application of the types of division.

Fourthly, social context and interaction. The fourth learning principle is that learning occurs in a society and is directed and stimulated by that socio-cultural-context. In learning division the discussion takes place from the very beginning of understanding contextual problem. Pupils' question such as "Is this a multiplication or division problem?" for instance attracts other pupils to start exchanging ideas. The rule of deploying ten busses; a quick and understood strategy that is followed by their peers become an interactive discussion among pupils and teachers. Further the curtailment of strategies also motivate pupils to understand the procedures as well as the mathematical tools being used.

Fifthly, structuring and interweaving. The last learning principle is that learning mathematics does not consist of absorbing a collection of unrelated knowledge and skill elements, but is the construction of knowledge and skills to a structured entity. For instance, division concepts and mental objects can be connected with three other basic operations via the bus items; i.e. repeated addition, repeated subtraction, or supplementary multiplication (in multi-digit numbers it is the multiplication of 1-

digit numbers). Later division becomes more of an independent operation with a structure of its own. But the connection with other operations continues to exist. Doing arithmetic sums, mental arithmetic, long and short arithmetic procedures, and applications in division establish a structured entity. For the teaching principle, the learning strands must, where possible, be intertwined with each other. The pure arithmetic and its applications must intertwine each other from the very beginning. Consequently, reality is both the source and the application area of mathematical concepts and structures, hence the term realistic mathematics education emerges (Treffers, 1991).

3.4 TEACHING MULTIPLICATION AND DIVISION REALISTICALLY: A CONJECTURED LOCAL INSTRUCTIONAL THEORY

Based on the principles and the five tenets of RME instructional theory mentioned above, this study proposes a RME local teaching model for multiplication and division of multi-digit numbers in Indonesian primary schools as follow:

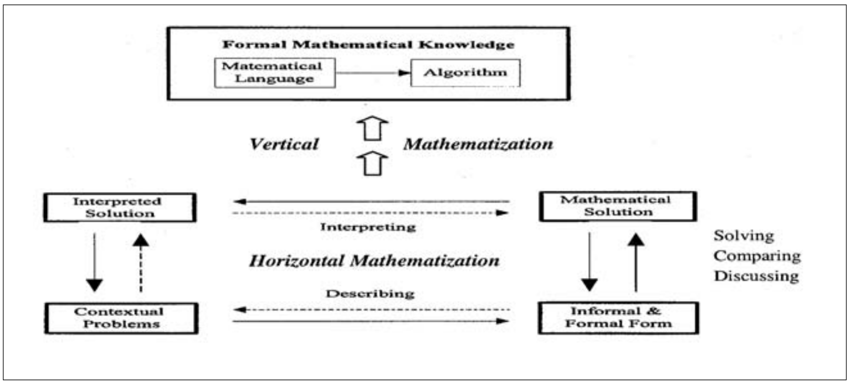


Figure 3.5
The local RME model of teaching multiplication and division

The learning process begins with encountering a contextual problem where mathematical concepts of multiplication and division embedded in the problems. For instance, the "Tiles" problem (see Appendix A) is the contextual problem to be solved in the first day of learning multiplication. The problem is provided for motivating pupils to use their own daily life language for creating many models of repeated addition, such as counting them all, repeated addition of 5 numbers, 10 numbers, or 14 numbers (14 tiles in each row). Using these strategies the pupils apply mathematical tools to find the solution. And then the mathematical solution becomes the interpreted solution when the pupils realize its rationale by comparing it to the context of the problem.

Having several models of repeated addition pupils then discuss the strategies and its mathematical tools being used to generalize the comparison among the strategies and to justify which model is the most understandable, more effective and efficient to solve problems. These discussions will improve pupils' understanding. They can use and apply the strategy they are comfortable with to solve another problems.

In these activities, the teachers have to play their roles by concentrating on pupils' understanding of the problem, and their use of mathematical tools in the strategy they apply. Question such as: "Is this a multiplication or division problem?" (Armanto, 2001) was always asked by the Indonesian pupils when they deal with a contextual problem because of their dependency to the teachers' order. Rather than giving answer "Yes" or "No" answer, the RME didactical aspects suggest that the teachers can ask pupils a leading question or give hints (for instance, drawing picture illustrating the problems). It encourages pupils to form their own mathematical model (informal or formal).

In learning multiplication of numbers Treffers (1991) suggests that the mental arithmetic must be developed first. It is done by operating the numbers which retain their own value. Meanwhile the column arithmetic is done with the individual digit. In conjunction with both arithmetic model (mental and column algorithm), it is essential that pupils have mastered the zero rule (Gravemeijer, 1994 and Treffers, 1991). This illustrate that the learning process of multiplication departs from a context problem to build a mental arithmetic in the first place and it ends at column arithmetic.

Considering this manner and the result from the preliminary study (Armanto, 2000), the multiplication learning starts by understanding the addition process. The learning process begin with encountering context problems for building pupils understanding of repeated addition. From the class discussion it was found out that the repeated addition of ten numbers became the first option to be applied in multiplying numbers. Then the learning continues with the reinvention of the multiplication by 10 and next by multiplication by tens. The learning ends at formulizing the column algorithm or the standard multiplication algorithm.

For instance, the following contextual problem is justified as a good example for starting the teaching process of multiplication. The problem motivates pupils to create various models of repeated addition.

A. Skillful mason

Pak Budi is a skillful mason. He is asked to build a wall that needs 204 bricks in each layer. The wall contains 52 layers. How many bricks does the wall need?

The following Figure 3.6 illustrates the repeated addition strategies the pupils develop when they encounter this problem.

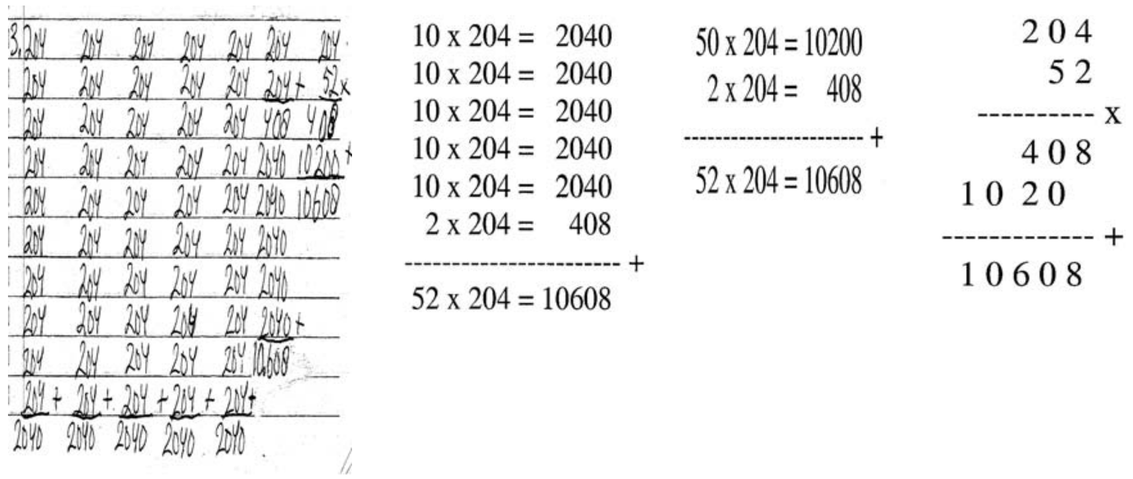


Figure 3.6
Various strategies of repeated addition and standard multiplication algorithm

The understanding of division begins with the understanding of distribution process. This process actually does not require explanation (Gravemeijer, 1994) because pupils have had their own informal and/or formal knowledge of sharing things with their peers. By letting them to share things with others they do mathematics, division as the inverse of multiplication. According to Romberg (1996), three aspects (model, language, and symbol) are involved in these activities. Modeling the distribution process of 36 marbles to 3 pupils, using mathematical language of 36 divided by 3, and writing mathematical symbol of the division process, i.e. 36 ÷ 3 = are the processes of understanding the division.

Gravemeijer (1994) found out that the pupils develop all kinds of informal procedures, such as dividing on a geometrical basis, distributing one by one, grouping of triads, and using multiplication facts. These solutions reflect the pupils' implicit understanding of division concepts as repeated subtraction, fair sharing or distribution, and as the inverse of multiplication. Repeated subtraction in division can be conceived as the counterpart of repeated addition in multiplication and also can be applied to solve ratio division and distribution division type of problems. Gravemejier (1994), however, argues that ratio-division contexts are more likely to be interpreted as repeated subtraction, than distribution-division contexts.

In learning and teaching long division on big numbers, the RME suggests using contextual problems as the starting point. The long division processes can be illustrated with the problem situation of busing the Feijenoord supporters (Gravemeijer, 1994). The problem is as follows:

> *"1296 supporters want to visit the away soccer game of Feijenoord. The treasurer learns that one bus can carry 38 passengers and that a reduction will be given for every ten buses. How many buses are needed?"*

By giving explanation the teacher can ask how to fill the bus with supporters or giving clues and hints about the ideas of filling the bus by drawing some buses. The pupils develop a model of the situation: filling buses as a model of repeated subtraction. After they understand the context, using their informal everyday language and formal language of mathematics, they create the formal mathematical model of the context. When they try to solve the problem, they come up with ideas of solution as the repeated addition of 38's, or stepwise multiplication of 38's, or by guessing number and multiply them with the 38's, or stepwise multiplication starting with 10, 20, 30, etc. After several solutions created, the teacher stimulates the pupils learning process by giving them opportunity to compare their solutions. Obviously, most found the first jump to 10 x 38 is a nice short cut to fill the supporters into the busses. And then the pupils can repeated the multiplication process in order to find solution of the problem. The pupils repeatedly use addition, multiplication and subtraction to solve the problem. Figure 3.7 illustrates the strategies pupils might have in solving the Feijenoord problem.

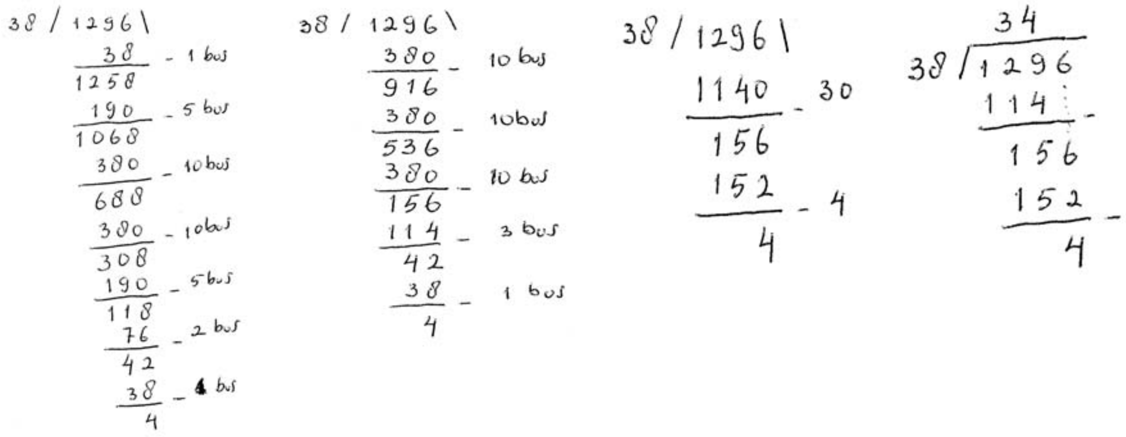


Figure 3.7
Various strategies of repeated subtraction and standard division algorithm

Whenever the pupils try to find the dividend as closely as possible, the pupils try to combine the dividend all together by adding multiples of the divisors to make it easy to understand by using tens or hundreds. It is easy to understand the procedure since the repeated division gives pupils opportunity to compare dividends, to add or decrease them, to multiply and subtract the result, and concentrate on what is left after several subtractions. A curtailed procedure is a resemblance of the standard procedure of division algorithm.

In order to develop and implement the RME learning process of multiplication and division several aspects have to be taken into account. First, a chosen research design to be employed aiming at developing RME instructional materials and implementing the RME approach in teaching multiplication and division of multi-digit numbers in Indonesian primary schools. The materials have to be in accordance with (1) the RME theory; (2) the Indonesian culture and conditions; (3) the 1994 curriculum; and (4) the teachers' and pupils' familiarity. Second, the appraisals to be used to analyze the representatives of the RME materials in Indonesian contexts. It includes analyzing particular measurable variables of effective implementation of the RME exemplary materials in the classroom (see the evaluation level of Kirkpatricks, 1987). These two aspects will be elaborated in the following chapter, the research design.

CHAPTER 4
RESEARCH DESIGN

The previous chapters describe the main aspects of this study, concerning Indonesian mathematics education that needs to be improved and the RME theory as a promising approach to be implemented in Indonesia. This chapter defines the research question and elaborates the research design of this study. It introduces the developmental research as the research approach used in this study and illustrates its potential cyclic process in developing, implementing, and evaluating the prototypical materials. In line with the developmental approach, this study follows a cyclic process of front-end analysis, expert reviews, teaching experiments, and reflections to the local instructional sequences. The cyclic process was carried out in two different phases: prototyping and assessment phase. Designed in three stages, the prototyping phase focused on developing, implementing and revising the RME prototypical products. The assessment phase aimed at evaluating the effectiveness of the RME prototype. In each phase a variety of data collection methods, various data resources (teachers, pupils, and experts) and observers were utilized to realize triangulation in order to have a study with quality.

4.1 INTRODUCTION

This study focused on characteristics of the prototypical RME formal curriculum (the RME prototype). It referred to the local instructional sequences and quality aspects of the RME prototypical products for teaching multiplication and division of multi-digit numbers in Indonesian primary schools. This study referred the research question as follows:

What are the characteristics of an RME prototype for teaching multiplication and division of multi-digit numbers in Indonesian primary schools?

To address this question a developmental research approach (Freudenthal, 1991; Gravemeijer, 1996; Richey & Nelson, 1996; Van den Akker, 1999; and Van den

Akker & Plomp, 1993) was chosen as the most suitable approach to investigate the development, implementation, and improvement of a prototypical product. The design will be explained in section 4.2. After the general term of developmental research is defined, two sections followed will elaborate the approach in two different fields: curriculum (section 4.2.1) and mathematics didactics (section 4.2.2). Next section (4.2.3) explains how the insights from the two perspectives are combined in developmental research approach used in this study. It will elaborate the research question, the aims, the phase and its cyclic process, and the main knowledge of conducting the approach. Then section 4.3 and 4.4 elaborate each phase of this study (prototyping and assessment phase), illustrating the intentional aims and its sub-research question, participants, research activities, data collection methods, and data analysis. The last section (4.5) presents an overview of the instruments and data analysis that were used in this study.

4.2 DEVELOPMENTAL RESEARCH

The idea that curriculum development should be guided by research is widely accepted by the educational research community (Richey & Nelson, 1996; Van den Akker, 1999; and Walker & Bresler, 1993). It is believed that curriculum development should be guided by something more informed than tradition and popular opinion. Meanwhile, the 'traditional' research approaches (e.g. experiments, surveys, correlation analysis), with their focus on descriptive knowledge, hardly provide prescriptions with useful solutions for a variety of design and development problems in the field of education (Van den Akker, 1999). After all, the call for a more inquiring approach to development has been issued from nearly every major figure in the history of curriculum studies.

The experts mentioned above believed that the developmental research has much to gain from a revival of the traditional research that casts research in a more central role in curriculum development. They said that developmental research is disciplined inquiry conducted in the context of development of a product or program for the purpose of improving either the products being developed or developers' capabilities to create better things in its kind in the future situations. It aims at supporting the development of prototypical products (including empirical evidence of effectiveness) and generating methodological directions for the design and evaluation of such products (Van den Akker & Plomp, 1993). In this approach, the scientific contribution (knowledge growth) is seen as equally important as the practical contribution (product improvement).

The idea of developmental research have been represented in many different fields, such as curriculum (Kessels, 1993; Keursten, 1994; McKenney, 2001; Nieveen, 1997; Thijs, 1999; Van den Berg, 1996; Visser, 1998; and Voogt, 1993), media and technology (Flagg, 1990), learning and instruction (Brown, 1992; Collins, 1992; Greeno, Collins, & Resnick, 1996). In the area of teacher education, the proposal of Elliot (1991) and Hoolingsworth (1997) made the developmental research well established. In mathematics didactics, the developmental research is based on the idea of an interactive, cyclic process of development and research in which theoretical ideas of the designer (theory bricolage) lead the development of products that are tested in classroom settings, producing theoretically and empirically founded products, learning activities, and local instructional theories (Freudenthal, 1991; Gravemeijer, 1994 and Simon, 1995).

4.2.1 Developmental research in curriculum

Developmental research is a research approach that analyzes new procedures, techniques, and tools based upon a methodological analysis of specific case. Richey and Nelson (1996) characterize the developmental research into "type I" and "type II" approach. Type 1 refers to the research studies in which the product development process is described and analyzed and the final product is evaluated. Here, the roles of designer and researcher coincide within a specific developmental context that occurs throughout the entire developmental cycle. Type 2 relates to the research studies, directed towards a general analysis of the design, development or evaluation as a whole, or towards any particular component of the research. In this type the researcher are not involved in the developmental process, but in studying the process being practiced by others in order to come to conclusions concerning design principles in general nature.

Van den Akker (1999) calls the developmental research Type I (Richey & Nelson, 1996) a formative research, in which the research activities are conducted during a cyclic developmental process of specific intervention, from exploratory phase through (formative and summative) evaluation phase, and aiming at optimizing the quality of the intervention. He argues that the main knowledge to be gained is in the form of design principles (substantive, procedural/methodological, and theoretical/empirical) to support the development of the RME materials. The cyclic developmental process is articulated in the following heuristic sentence: "If you want to design intervention X (for the purpose/function Y in context Z), then you are best advised to give that intervention the characteristics A, B, and C (substantive emphasis), and to do that via procedures K, L, and M (procedural emphasis), because of arguments P, Q, and R (theoretical/empirical emphasis)."

This sentence implies three main aspects: substantive, procedural, and theoretical/ empirical emphasis. The substantive emphasis refers to three quality criteria of the products being developed: validity, practicality, and effectiveness (Nieveen, 1999). These criteria will be developed in section 4.3. The procedural emphasis relates to the developmental activities (Nieveen, 1997, 1999; Richey & Nelson, 1996; and Van den Akker, 1999). These activities are as follow:

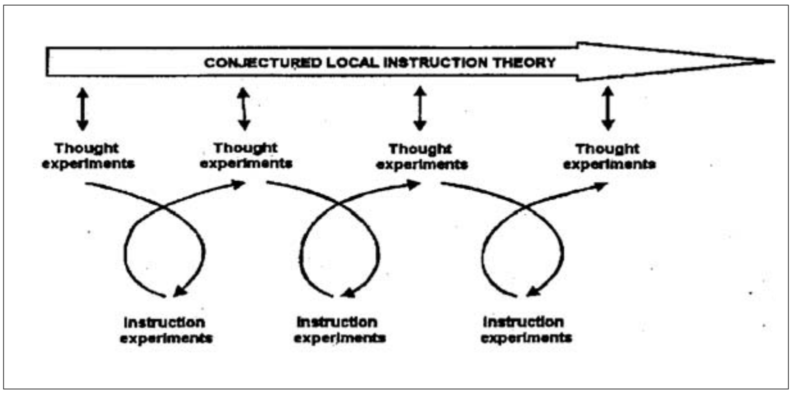
- A front-end analysis to describe the starting situation (context, available theory, and research results).
- A formative analysis to develop, evaluate, and revise the materials.
- A summative analysis to judge whether the prototype was effective enough to improve pupils' performance.

4.2.2 Developmental research in mathematics didactic

In mathematics didactics the developmental research activities are seen as a cumulative cyclic process of thought and instruction experiments resulting in empirically tested instructional sequences, called local theories (Freudenthal, 1991 and Gravemeijer, 1994. Unlike curriculum developmental research (see section 4.2.1 above) where principally directed to general curricula at all level (micro and macro), the developmental research in mathematics didactics focuses at micro level of how to teach mathematics topic according to RME approach. However, it is also theory-oriented that accumulates knowledge in a long-term research process.

The process is very similar to that of the mathematical teaching cycles (Simon, 1995) that serve the development of local instructional theory. It is a process of developing prototypical materials for a specific topic where the researcher constructs a provisional set of instructional activities, that are worked out in an iterative process of (re) designing and testing (Gravemeijer, 1999). The cyclic process aims at designing and testing a conjectured local instruction theory on how to teach specific subject. The activities begin with a preliminary design of the prototypical instructional activities, followed by a teaching experiment and end up with a retrospective analysis.

The core element of developmental research is on the classroom teaching experiments in which the local instructional theories and prototypical instructional sequences are developed. In the course of the teaching experiments, the researcher develops sequences of instructional activities that embody conjectures of pupils' learning route. The development is based on designing and testing instructional activities in daily basis. During the teaching experiments the researcher also carries out anticipatory thought experiments, in which he/she foresees both how the proposed instructional activities might be realized during the interaction in the classroom and what pupils might learn as they engage in the activities. These provide useful information to guide the revision of the instructional activities for the next instructional activity. Finally a well-considered and empirically-based instructional sequence is construed by reconstructing the sequence retrospectively. When this process of teaching experiment and revision is repeated a number of time, the rationale of the instructional sequence can be refined until it acquires the status of a local instructional theory (Gravemeijer, 1994). In fact, there is a reflexive relation between the thought and teaching experiments and the local instruction theory. At one hand the conjectured local instructional theory guides the thought and teaching experiments, and at the other hand, the micro instruction experiments shape the local instructional theory. This relationship is illustrated in the following figure.



Source: Gravemeijer, 1999.

Figure 4.1

The cumulative cyclic process

4.2.3 Developmental research in this study

a. Aim and insights

This study was aimed at developing and implementing an RME prototype for teaching multiplication and division of multi-digit numbers in Indonesian primary schools. According to Nieveen (1999), a prototype is a product that is designed before the final product will be constructed and fully implemented in practice. Smith (1991, p.42) defines a prototype as a "preliminary version or a model of all or a part of a system before full commitment is made to develop it." In this study, the term prototype means a product that is being designed during the development and implementation process to be used as exemplary materials by others. The RME prototype that is to be developed represents a RME formal curriculum for multiplication and division of multi-digit numbers. It was constructed in a process of developmental research.

There were two types of knowledge that were implied in this developmental research study. *Firstly*, it concerned the characteristics of the prototypical RME local instructional theory for teaching multiplication and division of multi-digit numbers. In the matter of the intervention for teaching and learning mathematics, the characteristics refer to the explicit formulation of local instructional activities that is made up of three components: (1) learning goals for pupils; (2) planned instructional materials; and (3) a conjectured learning sequence (Gravemeijer & Cobb, 2001).
Secondly, during the developmental process of the local instructional theory, it evolves the design principles (substantive, procedural/methodological, and theoretical/ empirical) to support the development of the materials. Substantive emphasis refers to characteristics of the instructional materials. In this study it referred to the quality aspects of the RME prototypical materials being developed (elaborated in item b of this section). Procedural/methodological emphasis associates with the cyclic process in conducting the research activities of this study (elaborated in item c of this section). Theoretical/empirical emphasis relates to the rationale of the intervention; the theoretical background of this study (see Chapter 2).

b. Characteristics of the RME prototype

Characteristics of the RME prototype referred to the quality aspects of the instructional materials. It can be related to a typology of curriculum representations:

ideal, formal, perceived, operational, experiential, and attained curriculum (see Goodlad, Klein & Tye, 1979; adapted by Van den Akker, 1988, 1990), resulting in a framework with three quality criteria: validity, practicality, and effectiveness (Nieveen, 1999). For a good understanding of these curriculum representations, first the definitions of representations are illustrated as follows:

- *Ideal curriculum*: reflects the original assumptions, visions and intentions that are laid down in a curriculum document.
- *Formal curriculum*: reflects the concrete curriculum documents such as pupil materials and teacher guides. In some studies the term "intended curriculum" is used, which refers to a combination of the ideal and formal curriculum (Nieveen, 1999).
- *Perceived curriculum*: represents the curriculum as interpreted by its users (teachers)
- *Operational curriculum*: reflects the actual instructional process as it was realized (also often referred to as curriculum-in-action or the enacted curriculum).
- *Experiential curriculum*: reflects the curriculum as the pupils experience it.
- *Attained curriculum*: represents the learning results of the pupils.

Nieveen (1999) mentioned that validity was defined as whether the components of the materials were developed based on the state-of-the-art knowledge (content validity) and all components were consistently linked to each other (construct validity). Meanwhile, practicality referred to whether the materials were usable and easy to teachers and pupils. There should be a consistency between the intended (ideal and formal) curriculum and perceived curriculum (the curriculum as interpreted by the teachers) and the intended and operational curriculum (the actual instructional process or the curriculum-in-action). In addition, the effectiveness was related to whether the pupils appreciate the learning program (representing the intended and the experiential curriculum) and the desired learning takes place (referred to the intended and the attained curriculum).

In this study, characteristics of the RME prototype were operationalized in four quality aspects: validity, practicality, implementability, and effectiveness. As an addition to the Nieveen's quality aspects, the implementability aspect was introduced. It was defined as whether the RME prototype can be applied as intended in the classroom. Proper implementation means that the teachers' operationalization of the subjects and the RME approach they perceive and apply in the instruction experiments are in accordance with the intentions of the curriculum designer (intended curriculum). It was represented by the actual curriculum in action or the operational curriculum. Meanwhile, during the instruction experiments the pupils engage in the actual learning process of the RME approach. These situations represent the curriculum that pupils experience it (experiential curriculum). From this illustration it can be summarized that in the teaching experiments two curriculum representations are interacting each other to meet the intended curriculum: (1) the teachers' operational curriculum and (2) the pupils' experiential curriculum. The consistency of the intended, operational, and experiential curriculum represented the implementability of the RME prototype. The links between the four quality aspects and the curriculum representations in this study are summarized in the Table 4.1.

Table 4.1
The links of quality aspects and curriculum representations

	Quality aspects			
	Validity	*Practicality*	*Implementability*	*Effectiveness*
Representation	Intended (Formal) curriculum Representing: - RME theory - Indonesian conditions	Perceived curriculum is useful	Consistency between: intended and operational intended and experiential	Consistency between intended and attained curriculum

Source: Nieveen, 1999.

c. Procedural activities

This study was aimed at constructing, revising and evaluating a provisional set of RME instructional activities. It was conducted in an iterative process of (re)designing and testing a conjectured local instruction sequences for teaching multiplication and division of multi-digit numbers. The cyclic activities consisted of the front-end analysis, expert reviews, teaching experiments, and reflections to the local instructional sequences (see Figure 4.2).

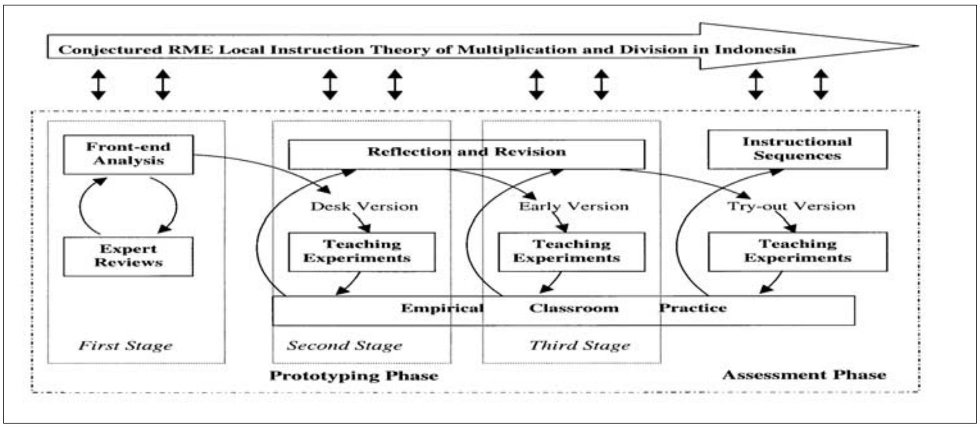


Figure 4.2
The cyclic process of this study

This study developed and implemented the instructional sequences in two cyclic phases: ***prototyping*** and ***assessment phase***. ***The prototyping phase*** (section 4.3) was conducted in three stages. In *the first stage* (section 4.3.1) the activities started at a cyclic process of front-end analysis and experts reviews. The front-end analysis was conducted to analyze the current situation of the Indonesian mathematics education concerning its drawbacks on the curriculum, the teaching and learning activities, pupils' performances, and teachers' competences. It also analyzed the appropriateness of the RME theory to improve the situation. During this analysis the Indonesian and RME experts were involved to review the feasibility of applying the RME approach in Indonesian circumstances. It included analyzing whether the RME materials (developed by the researcher) representing the Indonesian circumstances and the RME theory (content validity) and whether the components of the materials linked to each other (construct validity). *The first stage* produced a desk version of the RME prototype.

Then, *the second stage* (section 4.3.2) emphasized mainly on the practicality of the prototypical materials. It began with conducting the teaching experiments using the desk version. It was a core activity of this study in which the researcher developed sequences of instructional activities that conjectures the pupils' learning trajectory. The researcher envisioned both how the proposed instructional activities might be realized during the interaction, and what pupils' might learn as they engage in them. During the teaching experiments the researcher, as well as other three Indonesian primary school teachers, tried-out several sub-sequences of hypothetical learning trajectory in the classroom. It aimed at grasping a sense and experience of being in the RME actual learning activities. This also helped the teachers to analyze whether the RME approach usable and easy to be applied in Indonesian primary schools. Reflecting on the empirical classroom practice and the experts' suggestions, the researcher revised the hypothetical learning sequences and restructured an early version of the RME prototype.

Next, the teaching experiments was carried out in *the third stage* (section 4.3.3) of the prototyping phase in which the whole sequences of the proposed learning sequences were taught. It was assumed that analyzing the pupils' actual learning process when the instructional activities are conducted in the classroom can guide the revision of its activities. During the teaching experiments the researcher and other four primary school teachers applied and evaluated the whole sequences of the learning trajectory. The Indonesian and RME experts were also involved in revising the instructional materials considering pupils' portfolios, teachers' logbook, and experts' reviews. *The third stage* produced a try-out version of the prototypical instructional materials.

The assessment phase (section 4.4) focused on evaluating whether the RME prototypical materials (the try-out version) were used as intended and effective to improve pupils' performances. During the teaching experiments the whole instructional sequences were also examined and refined until it acquired the status of a prototypical instructional sequence. Observing and evaluating teachers' applying the prototypical materials, the researcher conducted a semi-summative analysis in the procedural activities that ended up with a judge of whether the prototypical instructional sequences improve pupils' performances. This phase produced the ornate version of the RME prototype, representing the conjectured local instructional theory of the multiplication and division of multi-digit numbers.

After all, the phases of this study and its emphasis can be summarized in Table 4.2.

Table 4.2
The phases and its emphasis in this study

Phase/Stage	**Quality aspects**			
	Validity	*Practicality*	*Implementability*	*Effectiveness*
Prototyping				
▪ First stage	√[a]	√[b]	-	-
▪ Second stage	√[b]	√[a]	-	-
▪ Third stage	√[b]	√[b]	√[a]	√[b]
Assessment	-	-	√[a]	√[a]

Note: √[a] Major emphasis;
√[b] Minor emphasis.

The following section illustrates the cyclic process of this study. It describes how the research was conducted, included the aims to meet, the research question, activities, the participants, and the appraisals used.

4.3 PROTOTYPING PHASE

The prototyping phase was held in cyclic process of front-end analysis and expert reviews, teaching experiments and its reflections (see Figure 4.1 above). It was aimed at analyzing the quality of the prototypical products being developed: characteristics of the RME prototypical materials. The phase was lead by the following sub-research question:

> *To what extent was the RME prototype valid, practical, and implementable for teaching multiplication and division of multi-digit numbers in Indonesian primary schools?*

The phase was conducted in three consecutive stages: the first, the second, and the third stage. Each stage is illustrated in the following section.

4.3.1 The first stage of prototyping phase

The first stage of the prototyping phase was conducted in a cyclic process of front-end analysis and expert reviews. The front-end analysis and expert reviews were subjected to analyze current situations of Indonesian mathematics education (drawbacks and strength) and to discus the RME theory as a promising approach to be implemented in Indonesian schools (see section 1.2 and 1.3 and section 2.5 and 3.3). These activities led the study to produce a desk version of the RME prototype. This version was reviewed by the Indonesian mathematics education and the RME experts for its content and construct validity. This stage was led by the following question:

> *To what extent was the RME prototype valid for teaching multiplication and division of multi-digit numbers in Indonesian primary schools?*

Valid materials were defined as the degree to which the materials were in agreement with the existence of state-of-the-art knowledge of the Indonesian mathematics education and the RME theory (content validity) and the consistent link of the components in the materials (construct validity). In this phase the agreement was analyzed from the initial perceptions of the experts toward whether the RME theory and the Indonesian contexts were embedded in the materials. It comprised the RME philosophical and didactical tenets as well as the local Indonesian contexts.

The Indonesian mathematics education expert and the RME experts reviewed the internal quality of the RME prototype. For this reason, walkthrough evaluation sessions (Nieveen, 1999) with the RME and Indonesian experts were held intensively. These experts analyzed different aspects of the RME prototype. The Indonesian experts analyzed whether the materials comprised the Indonesian contexts (curriculum, content, and contexts) that the teachers and pupils were familiar with. And the RME experts analyzed whether the RME theory was embedded in the materials. Details of the questions addressed during interviews in the walkthrough sessions which summarized in section 4.5. The data from the interviews were transcribed for analysis and the decision of improving the materials was made subsequently. The results of this first stage of the prototyping phase are presented in chapter 5.

4.3.2 The second stage of prototyping phase

The second stage of the prototyping phase focused mainly on the practicality of the RME prototypical materials. It was conducted in cyclic teaching experiments in three Indonesian primary schools. During the teaching experiments some subsequences of the instructional activities were applied in order to allow the researcher and the teachers to grasp a sense of conducting the RME learning activities. The second stage was led by the following question:

> *To what extent was the RME prototype practical in teaching multiplication and division of multi-digit numbers in Indonesian primary schools?*

Practical materials defined as the degree to which the materials are usable and easy to the target groups (the Indonesian pupils and teachers). It was analyzed from the teachers' initial perceptions towards the materials, the RME instruction approach, and the learning climate. In this stage these perceptions were collected after the teachers grasped the idea of conducting some subsequences of the RME learning trajectory in the classroom. Various data collection methods, such as teachers' logbook, pupils' portfolios, and interviews were employed. Reasons and details of these appraisals are illustrated in section 4.4. Based on these perceptions and reflection to the actual classroom practice the researcher revised, restructured, and improved the content, the format, and the organization of the RME instructional activities.

The teaching experiments were conducted in two primary schools in Yogyakarta, Indonesia: SD Puren and SD Kanisius on September – December 1999 (see the participants in Table 4.1 below). The schools were chosen using the purposive (pragmatic) sampling based on several considerations: (1) the chosen school principals and teachers were accepting voluntarily to apply the RME approach in their classrooms and (2) the chosen teachers were characterized as competence and experienced teachers. These teachers with these qualifications had admirable experiences of teaching and learning the subjects in conventional didactical approach. Their expertise was needed to analyze and judge whether the RME approach is usable and easy to the Indonesian teachers and pupils.

The researcher conducted the teaching experiments in a class of SD Puren and two teachers of SD Kanisius taught the other two classes using the RME approach. Before the teaching experiments began, the teachers and the researcher discussed the materials and its learning subsequences that were going to be taught in the classrooms. These activities were conducted before and after the teaching activities. From these discussions the researcher and the teachers analyzed whether the materials were useable and whether the instructional sequences went along as intended. Being a teacher, conducting research, and building the prototype gave the researcher three viewpoints that enhanced its constructive research attitude: "teachers' sense" of the actual learning environment in RME approach; "researchers' viewpoint" of conducting the study; and "developers' role" in developing the RME materials.

Table 4.3
The participants in the second stage of prototyping phase

Schools	**Type**	**n**[a]	**Teacher type**	**Treatment**
SD Puren, Pringwulung	State	38	Experienced	TC[b]
SD Kanisius, Demangan	Private	43	Experienced	OC[c]
		42	Experienced	

Note: [a]Numbers of pupils; [b]The taught class (by the researcher); [c]The observed class (by the researcher).

The teaching experiments exposed several results (see Armanto, 2000). These results became the basic foundation for building and developing an early version of the RME prototype (elaborated in chapter 6). This early version was applied and revised in the third stage of the prototyping phase, which is described in the following section.

4.3.3 The third stage of prototyping phase

The third stage of prototyping phase aimed at applying, evaluating, and revising the early version of the RME prototype for teaching multiplication and division of multi-digit numbers in Indonesia. The phase focused primarily on implementability of the RME prototype. As the results of the implementability, the initial effectiveness of the RME prototype was also analyzed. This stage was led by the following question:

> *To what extent was the teachers' implementation of the RME prototype and the pupils' performances in multiplication and division of multi-digit numbers in Indonesian Primary schools?*

The teachers' implementation of the RME prototype was defined as the degree to which the teachers used and applied the instructional materials in the classroom in a proper activity. It referred to whether the teachers could apply the RME approach as intended. It meant that teachers organized teaching experiments using the RME instruction activities. In RME didactical aspects, the proper implementability was identified whenever the teachers actively organized the learning process by (1) introducing the contextual problems; (2) conducting an interactive teaching approach; and (3) establishing socio-mathematical norms in the classroom (Verschaffel & De Corte, 1997; and Cobb & Gravemeijer, 2001). The norms that are expected to occur during the learning activities refer to the expected ways of explaining and acting in whole-class discussions that are specific to mathematics.

The pupils' performances in the RME learning activities referred to the initial effectiveness of the RME prototype. It was defined as the degree to which the pupils perform on the expected level of understanding. It assessed the pupils' learning progress, understanding, and achievement. Pupils' learning progress was analyzed from their valid use of the strategies in daily basis. Pupils' understanding was examined from their level of solution stage (see Table 4.7 in section 4.5.2) in solving the daily and weekly quiz. Meanwhile, pupils' achievement referred to pupils' correct answers in applying the strategies being learnt in solving the post-test problems.

During the teaching experiments the data on the implementability and the effectiveness were collected using several data collection methods and various people: teachers' logbooks, pupils' portfolios, in-depth interviews, teaching profile checklist, daily and weekly quizzes, pre-test and post-test. The appraisals and the data resources assumed the presence of triangulation of data, methods, and observers in order to have a data of good quality to draw a conclusion. These appraisals will be elaborated in section 4.4.

The third stage of prototyping phase began in July 2000 until December 2000. It took place in 4 primary schools, 2 schools in Medan (SD 1 and SD 3 Klumpang) and 2 schools in Jogyakarta (SD Rejodadi and SD Sonosewu II). The schools were chosen purposively based on whether (1) the principals and teachers accepted voluntarily to apply the RME approach in their classrooms and (2) the chosen teachers had various competencies: novice (teaching the subjects less than 2 years), moderate (teaching the subjects between 2-5 years), and experienced teachers (teaching the subjects more than 5 years). These teachers represented a balance from point of view of experience.

Table 4.4

The subjects of the third stage of the prototyping phase

Schools	Type	n[a]	Teacher types	Treatment
SDN 101748 Klumpang	State	16	Novice	TC[b]
SDN Sonosewu II	State	33	Moderate	
SDN 101746 Klumpang	State	20	Experienced	OC[c]
SDN Rejodadi	State	25	Moderate	

Note: [a]Numbers of pupils; [b]The taught class (by the researcher); [c]The observed class (by the researcher).

Four classes of pupils aged 10-12 years participated in the teaching experiments: two taught classes (TC) and two observed classes (OC) (see Table 4.3 above). In the TC classes, the teaching experiment was conducted by the researcher during the hours allocated for the subject. For each subject the lesson consisted of four units of about 2 x 40 minutes each, spread over a period of a week. In the OC classes, the teachers themselves conducted the teaching process using the RME prototype.

The teaching experiments were held in two separate periods, i.e. from 31 July - 12 August 2000 (in the first tri-mester) in Medan and 30 October - 12 November 2000 (in the second tri-mester) in Yogyakarta. During the teaching experiment, each class (TC1, TC2, OC1, and OC2) was given the pre-test, post-test, daily quiz, and weekly quiz (available in the teachers' guide). It measured the pupils learning progress and performance on the multiplication and division of multi-digit numbers. All classes were observed and all pupils' portfolios were analyzed.

This third stage resulted in a try-out version of the RME prototype (Armanto, 2001). This try-out version was used and applied in the assessment phase. The phase is described in the following section.

4.4 ASSESSMENT PHASE

The assessment phase focused on analyzing whether the teachers used the RME prototype as intended (implementability) and whether the RME prototype improved the pupils' performances (effectiveness). The phase was conducted and the data were analyzed to answer the following sub-research question:

To what extent was the RME prototype implementable and effective to teach multiplication and division of multi-digit numbers in Indonesian Primary schools?

The implementability referred to the degree to which the RME prototype can be taught properly (in the meaning that teachers organize the learning process as intended). The proper learning processes were related to the teachers' teaching profile in the RME prototype approach. The proper learning instruction was verified if the teachers started the learning process by introducing the contextual problems, conducting an interactive teaching approach, and establishing the socio-mathematical climate in the classroom.

The effectiveness was defined as the degree to which the RME prototype improved the pupils' cognitive performance (or coherence of the intended – attained curriculum). It dealt with the pupils' learning progress, their understanding, and their level of achievement. These cognitive performances established whenever pupils solved the contextual problems of the quizzes and tests correctly, performed the valid solutions correctly, and progressed toward the expected level of understanding.

This assessment phase employed several instruments: logbooks, portfolios, in-depth interviews, teaching profile, quizzes, and tests. It also utilized various individuals (teachers, pupils, experts, and university pupils) to review and gather the data. The instruments and the people involved in collecting the data as well as the sources of the data assumed the existent of the data triangulation to determine the quality of conclusions. The instruments were elaborated in section 4.4.

In the assessment phase, the teaching experiments were conducted in a non-equivalence pretest-posttest control group design (Krathwohl, 1998) because the schools were not formed by random assignment. It was a quasi-teaching experiment where teachers from the experiment groups (EG) taught the subjects using the RME prototype and teachers from control groups (CG) taught the subjects conventionally. And the researcher did not set the learning situation in which many variables involved are being restrained, but the researcher role was to analyze the circumstances of whether the learning activities and the teaching experiments took place.

The phase began by purposively choosing 8 primary schools in Hamparan Perak, Medan as the experiment groups (EG) and 8 schools as the control groups (CG). Although both groups were purposively sampled, yet it is assumed that both groups represent typical Indonesian schools for the following several reasons: (1) the principals and teachers willingness of applying RME approach; (2) diverse teachers with different competences; (3) the closest school location area; and (4) time consumed in conducting the research.

Even though the control groups had several classes from the private schools (see Table 4.5 below), for several reasons it could be assumed that both groups were equivalent. The reasons were related to the fact that the mathematics curriculum employed in Indonesia was centralized, and each school utilized the same curriculum and similar approach in teaching the subjects. The last reason was concerning the fact that in both groups' pupils had no significant differences in their performance in the pre-test (see section 8.2.2 item c.2). The following table summarizes the subjects of the assessment phase.

Table 4.5
The subjects of the assessment phase

Schools	**Type**	**n**[a]	**Teacher types**	**Treatment**
SDN 101746 Klumpang	State	23	Experienced	
SDN 101747 Klumpang	State	12	Moderate	EG[b]
SDN 101748 Klumpang	State	26	Novice	
SDN 101749 Klumpang	State	31	Novice	(291 pupils)
SDN 101750 Klambir Lima	State	37	Experienced	
SDN 101751 Klambir Lima	State	45	Moderate	
SDN 101752 Klambir Lima	State	52	Novice	
SDN 106153 Klambir Lima	State	65	Experienced	
SD PAB 15 Klambir Lima	Private	43	Experienced	
SD PAB 26 Tanjung Gusta	Private	57	Moderate	CG[c]
SD PAB 1 Klumpang	Private	53	Novice	
SD PAB 14 Klambir Lima	Private	14	Novice	(310 pupils)
SDN 065854 Tanjung Gusta	State	35	Experienced	
SDN 107395 Klumpang	State	37	Moderate	
SDN 101750 Klambir Lima	State	44	Experienced	
SDN 105283 Klambir Lima	State	27	Moderate	

Note: [a]Numbers of pupils; [b]The experiment group; [c]The control group.

Considering the need of having a good internal validity of the study, 3 university students were involved in the study as the observers. They were taught intensively about the RME theory and were trained about the teaching profile checklist and the research being conducted. It took place in July 2000 in the State University of Medan.

Before the learning activities began in the classroom, the intended curriculum had to be comprehended by the teachers (Fullan, 1984). For this reason the study created a staff development activity that consisted of: (1) a day of RME training, and (2) guided application of RME approach. These processes introduced the RME approach and the whole research process to the teachers. The staff development activities were as follow:

1. *A day of explanations and small group discussion among teachers, observer, and principals.*
   The discussions were aimed at enhancing teachers' knowledge and skill in teaching multiplication and division of multi-digit numbers using the RME prototype. The researcher discussed the use of the RME materials, the main aspects of the RME teaching, the role of teachers, the role of interactive discussion, and the use of the contextual problems in the learning process. All participants acted as learners in learning the subjects. The discussion was devised to have a clear understanding of (1) the RME learning process and the materials; (2) the conjectured pupils learning progress; (3) the pupils' difficulties; (4) the use of the contextual problems; (5) the interactive teaching methods; and (6) the establishment of the socio-mathematical climate. During the small group discussions the pupils took the pre-test.

After the small discussion the teaching experiments began. The teachers from the experiment group (EG) conducted the teaching and learning activities in their classroom using the RME instructional materials.

2.a. The learning process using the RME materials

8 EG teachers taught the subjects and the observers observed the learning process by filling the teaching profile checklist. The researcher acted as an observer, a facilitator, and a guide for the teacher in conducting the learning process. The pupils learnt the subjects, had homework and solved the daily and weekly quiz (see Appendix A). The data from the quiz represented the pupils' learning progress in the subjects (see section 8.2.2 item a).

During the learning sessions of reflection was also done to examine how the proposed instructional activities were conducted and what the pupils might learn.

2.b. A small group reflection at the end of the day

The reflection sessions were aimed at discussing the classroom activities every day and to find out the agreement of the learning process with the teaching profile of the teachers. It was also to have a conjectured plan for the next learning activities. This activity was reiterated everyday during the teaching experiments among teachers, observers, and researcher.

The assessment phase produced the ornate version, the future RME prototype. The results of this phase are elaborated in Chapter 8. The next section describes an overview of the instruments utilized in each phase of this study.

4.5 THE OVERVIEW OF THE INSTRUMENTS AND THE DATA ANALYSIS

As mentioned in the previous sections a variety of data collection methods (instruments) were utilized in this study. The use of all instruments was aimed at justifying the "goodness" of the study. Utilizing various sources (experts, teachers, and pupils), different observers, and variety of appraisals determined the objectivity of the study. This triangulation (data, observer, and methods) assured quality of the data and provided the quality control of this study. The overview of the instruments is summarized in the following Table 4.6.

Table 4.6
The overview of the data collection method in each phase

Phase/stage quality aspect	Questio n-naire	Log-book	Inter-view	Port-folio	Check-list	Quiz	Test
Prototyping							
The first stage							
Validity			√	√			
The second stage							
Practicality	√	√	√				
The third stage							
Implementability	√	√	√				
Initial effectiveness			√	√		√	√
Assessment							
Implementability	√	√	√		√		
Effectiveness			√	√		√	√

The following segments illustrate the instruments that guided the data collection activities and its analysis.

4.5.1 The instruments

Two different *questionnaires* utilized in this study: pupils' characteristics and teachers' characteristics. The pupil questionnaire contained the structured items about the pupils' backgrounds (e.g. age, gender, time consumed to learn, and opinions towards mathematics and the learning process). The teacher questionnaire items asked about teachers' background such as age, diploma, and experiences as teachers teaching the subjects. The teachers were also asked to give their opinion towards mathematics and its teaching process. The items were adopted from TIMSS questionnaire (1999). These questionnaires were given to all participants and the data were collected to illustrate reasons behind the teachers' opinion towards the RME learning climate occurred in the classroom (see section 7.3.2).

The *teacher logbook* was kept by the teachers during the two weeks instruction process. Each week the teachers gave their opinion addressing the following issues:

- General impression of the learning process (usefulness, easiness, and aims met);
- Performance of the lesson (instruction activities);
- Pupils' participation in learning (active, independent, and interestedness);
- The RME materials (content, lay-out, language usage, and level of exercise).

The teachers described their opinion about the subject being investigated by indicating the extent to which they agreed with several position statements on a 5-point Likerts' format in the logbook and were asked to give written explanations. The *logbook* was developed based on the idea that the less time the teachers needed to complete the logbook the more likely they would use it.

Interviews were structurally held with diverse individuals in different phase. In the prototyping phase *unstructured interviews* with Indonesian and RME experts were held in walkthrough sessions aiming at analyzing the content and construct validity of the RME materials. It addressed several issues:

- content appropriateness and relevance;
- context applicability and familiarity;
- time sufficiency; and
- materials usability and easiness.

The *interviews with experts* were carried out based on the idea that the more issues discussed with the experts, the more improvement the materials could get and the more valid the materials would be.

During this prototyping phase the *unstructured interviews* were also conducted with teachers and pupils concerning the practicality of the RME approach. It addressed the functional issues of the RME approach concerning the teachers' answers and reasons given in the logbook. It was an effort to build trustworthiness between the researcher and the teachers and to have a data of good quality.

In the third stage of prototyping phase and the assessment phase *interviews* with teachers and pupils were held for different intentions. The teachers were interviewed before and after the instruction process to cross-examine their answers and reasons given in the logbook. Meanwhile the pupils were interviewed during the learning process aiming at analyzing their learning progress and understanding towards the subjects. The scheme of interviews with pupils was developed of the existing scheme from Verschaffel and De Corte (1997):

- How do you do that? ("That" indicates the strategies the pupils use to solve problems)
- How do you know that is right?
- How do you find that?
- What do you think of the learning activities?
- What do you think of the contextual problems?
- Why do you think that is right?
- Why do you do that?

Interviews were conducted with pupils from different levels of ability. These interviews depended on their answers to the problems and on the group discussions.

The pupils' *portfolios* contained the compilation of pupils' workbook comprising the strategies they used when solving contextual problems. The *portfolios* served the study to (1) review pupils understanding; (2) find out the actual solution procedures; and (3) analyze difficulties and weaknesses. The portfolios also gave insight of the actual learning trajectory the pupils routed in learning the subjects.
The *teaching profile checklist* displayed the teaching activities conducted by the teachers. The items were utilized 3 aspects of teaching: introduction to the lesson (preparation activities), body of lesson (instruction activities), and the conclusion. The general impression of the whole instruction process was also analyzed. Introduction activities contained 5 items relating to the way teacher introducing the contextual problems, delivering questions and hints, responding to pupils' idea, and encouraging and drawing conclusions. These items analyzed the teachers' use of the contextual problems. The instruction activities consisted of 10 items referring to the way the teachers conducting interactive teaching approach. They comprised whether teacher allowing pupils to choose own approach, to explore problems individually or in groups, and the way teacher focusing pupils' attention, interacting with pupils, and encouraging them to discuss and draw own conclusions. The regulation activities contained 6 items relating to the establishment of socio-mathematical norms in the classroom. The items analyzed to the way the teacher

asking pupils to report, encouraging them to comment and compare strategies, delivering questions, and drawing conclusions. The 5 items of the general impression concluded the learning activities occurred in the classroom (how teacher acknowledged and discussed pupils' idea summarized pupils' answers and whether the classroom atmosphere encouraged pupils' asking and discussing the strategies). The data analysis of this checklist is illustrated in the next section 4.4.2.

The items developed in the *teaching profile checklist* were adopted from Thijs (2000) because of several reasons. *Firstly*, the checklist had been tried out and used in several African schools and produced a valid and reliable data. *Secondly*, several well-known experts in the field of curriculum instruction were involved in judging the goodness of checklist. *Thirdly*, it was developed based on the idea of the need of having a good judgement of teachers' capability in conducting learning activities in RME approach. The other reason was that the less time needed to complete the checklist the more likely the learning activities were observed. Last but not least, RME experts also analyzed the items of the checklist resulting on some revisions on the items and the options.

Two different *quizzes* were developed in this study. The *daily quiz* contained 3 contextual items in each subject and the *weekly quiz* consisted of 2 contextual items for each subject (multiplication and division) that fit with aims of RME approach. It aimed at finding pupils learning progress and understanding (including their difficulties and weaknesses) after participating in the learning climate of the RME approach. It compiles the pupils' reinvented procedures of multiplication and division algorithms. The level of difficulty and its experientially real of the item contexts and the numbers involved were judged by the RME experts. The data collected from these quizzes were analyzed using the degree of progress toward a valid solution (Table 4.8) from Malone, et al. (1989).

The items in the quizzes were pilot-tested in the formative phase (see Appendix C item 1C and 1D). It showed that the reliability of the daily quiz items was considered acceptable: α = 0.5575 for multiplication items and 0.8050 for division items. The items were in the middle level of difficulty (Kehoe, 1995). Meanwhile for the weekly quiz it was found out that α = 0.3808 for the multiplication and 0.7468 for the division. The initial multiplication items were not acceptable then they were modified by the researcher, judged by the RME experts, and used in the assessment phase. The data analysis can be found in the appendixes.

There were two tests developed in this study: pre-test and post-test. These tests consisted of 10 items in each subject (multiplication and division): 6 items of contextual problems and 4 items of conventional problems. The items were structured based on the time consumed and available for conducting the tests and the RME experts' agreement to the structure (amount, contexts, and numbers involvement) of items used in the tests. The items were aimed at: (1) examining the intuitive (context-bound) level of the pupils' understanding (pre-test) and (2) analyzing pupils' performance after engaging in RME approach (post-test). It included the pupils' learning progress, understanding, and achievement in solving contextual problems as well as conventional problems. These elements referred to the consistency of the intended and attained curriculum (effectiveness). The data from these tests were analyzed using the categorization of pupils' achievement (Table 4.9) and the degree of progress toward a valid solution (Table 4.8) from Malone, et al. (1989).

Data in the formative phase (see Appendix C item 1.A and 1B) showed that the reliability scale was considered acceptable: α = 0.7999 for the pre-test and 0.8573 for the post-test. Considering Kehoe (1995) good item criteria, the items were in the middle level of difficulty. It can be concluded that the items had a good quality for assessing the pupils' performance in learning the subjects. The analysis can be found in the appendixes.

4.5.2 Data analysis

The data collected using the instruments mentioned above were analyzed in different quantitative and qualitative strategies.

The teachers' and pupils' background *questionnaires* provided quantitative data that were analyzed by computing descriptive statistics, including means, frequencies, and percentages. Then these results were summarized qualitatively concerning the level of the scale given in each item (most items were scaled on a 5-point Likert format).

The teachers' *logbook* informed quantitative and qualitative data about the teachers' impression towards the instructional process and the materials. Frequencies of the quantitative data were tabulated from numbers of teachers that were in favor with each option of each item. The cumulative percentages of this tabulation were analyzed qualitatively referring each issue in the logbook.

The *interviews* conducted with the experts, teachers, and pupils mainly resulted in qualitative data. Based on the written transcription and notes made during the interviews, the data were summarized and analyzed with techniques of memoing (Miles & Huberman, 1994). The memos created by the researcher were based on the comments, questions, and suggestions given by the participants. It was intended to build a conceptual link of data from several aspects of the study: participants, methodological, and substantive. For instance, questions such as "Do you think this context is real for pupils?" and "What do you think of these numbers involved in the contexts" were transcribed to improve the contextual problems in the materials.

The pupils' portfolios informed several aspects: the pupils' learning progress, their understanding, and their reinvented procedures. Using the degree of progress toward a valid solution (Table 4.8) from Malone, et al. (1989), the portfolios were analyzed from pupils' written strategies they developed in solving problems. It mainly informed the qualitative data of pupils' learning performance.

The *teaching profile checklist* addressed four key implementability issues (introduction, instruction, and regulation activities; see its conceptual characteristics in section 4.5.1). For each issue, 5 - 10 items were constructed illustrating what activities the teacher should conduct and what activity pupils would be done. For each item, frequencies were tabulated from the amount of times the observer chose the option (the items were scaled on a 5-point Likert format). And then the mean score and its standard deviation were calculated. These descriptive statistics illustrated the variation of teachers' capability in conducting the RME learning process and they also eliminated observers' disparity in judging the activities being conducted by the teacher. After all, the cumulative average of the items was calculated referring to the teachers' level of performances in carrying out each implementability issue. Then the average scores were categorized as follow:

Table 4.7
The teachers' level of conducting the RME learning activities

Score	Level	Interpretation Values
1	Very poor	The activities were conducted very poorly
2	Poor	The activities were conducted poorly
3	Fair	The activities were conducted fairly
4	Good	The activities were conducted well
5	Very good	The activities were conducted as intended

Both *quizzes* (daily and weekly quiz) gave quantitative and qualitative data. The quantitative data were analyzed by the degree of progress toward a valid solution (Table 4.8) from Malone, et al. (1989). It was coming from the pupils' written procedures in solving contextual problems after engaging the learning process. It informed the pupils' learning performance (progress, understanding, and achievement) in daily and weekly basis. Qualitative data were examined from the reinvented strategies (procedures) the pupils applied in solving the problems. It informed the valid solution strategies the pupils understand most.

The data from the tests were analyzed in several ways. *First,* for the pre-test, the pupils' intuitive mathematical forms were analyzed qualitatively to find its common general pattern (cf. Miles & Huberman, 1994). For the post-test the general pattern was examined from the valid solution the pupils applied. *Second*, to score each item of both tests, the study utilized the scoring scale from Malone, et al. (1989, see Table 4.8 below). The scale compared the solutions of the pupils in five stages (Noncommencement, Approach, Substance, Result, and Completion).

Noncommencement stage was characterized by the pupils were inability to begin solving the problem (their handwork were meaningless). These pupils got a score of 0. The approach stage was characterized by the pupils approaching the problem with a meaningful work, but an early impasse was reached (the pupils got score of 1).

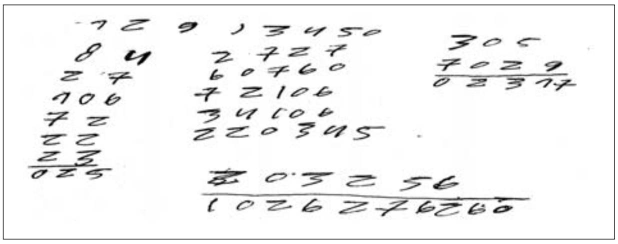
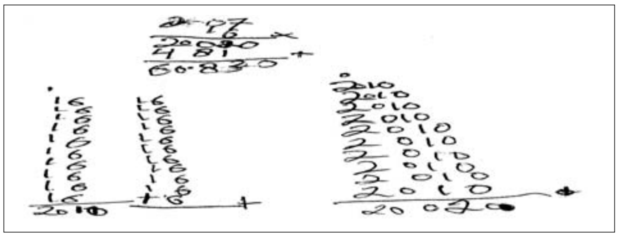
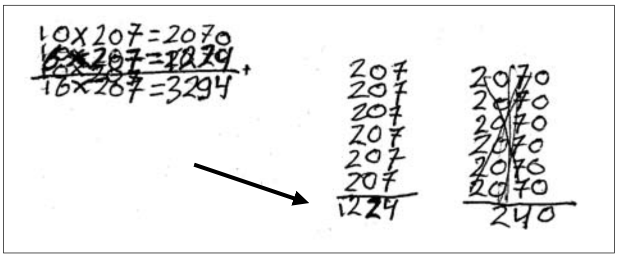
The substance stage was characterized by the pupils proceeding toward a rational solution but major errors obstructed the correct solution process (the pupils were given a score of 2). In this study the major errors referred to the mistakes pupils had when calculating numbers using multiplication of 1-digit numbers or adding numbers consecutively. This calculation process was conducted by the pupils before they find the solutions.

When the pupils nearly solved the problem; but minor errors produce an invalid final solution, then they were in *the result stage*. They got score of 3. In this study the minor errors are related to the mistakes pupils made in calculating the final results for having a correct solution.

The completion stage was characterized by the pupils having the valid solution with an appropriate method. They got a score of 4. The Malones' stage and its examples of pupils' works were illustrates in Table 4.8.

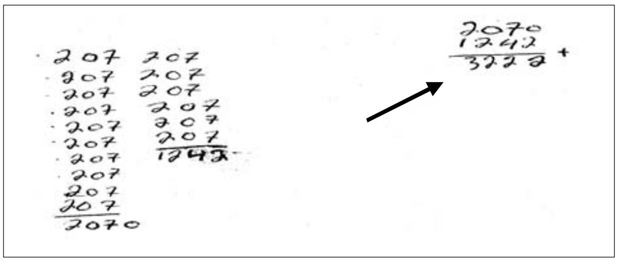
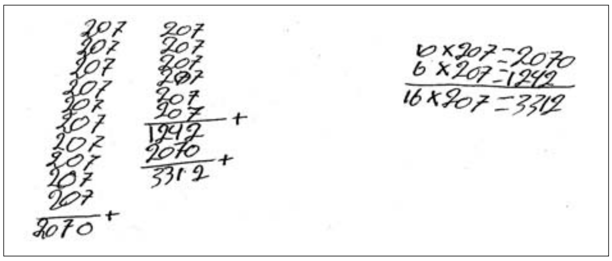
Table 4.8

The degree of progress toward a valid solution

Score	Solution stage	Example
0	*Noncommecement* The pupil is unable to begin the problem or hands in work that meaningless.	
1	*Approach* The pupil approaches the problem with meaningful work, indicating some understanding of the problem, but an early impasse is reached.	 Early impasse*: Calculating 207x6 in standard algorithm*
2	*Substance* Sufficient detail demonstrates that the pupil has proceeded toward a rational solution, but major errors obstruct the correct solution process.	 Major error*: Calculating 6x207 by repeated addition*

To be continued

Table 4.8 *(Continued)*

3	*Result* The problem is very nearly solved; minor errors produce an invalid final solution.	 Minor error*: Calculating the final results*
4	*Completion* An appropriate method is applied to yield a valid solution.	

Source: Malone, et al., 1989.

Third, the data from both tests (pre-test and post-test) were analyzed descriptively to find its mean and standard deviation. The mean score determined the pupils' level of problem solving achievement. Then the data were categorized in Table 4.9:

Table 4.9
The level of pupils' achievement

Interval score	**Level**	**Interpretation Values**
00 - 13	Low	Low achievement
14 - 27	Mediocre	Mediocre achievement
28 - 40	High	High achievement

Next, pupils' level achievement in the tests was distinguished based on the type of the problems they solved (contextual and conventional). It was categorized in the Table 4.10.

Table 4.10
Level of pupils' achievement in contextual and conventional problems

Contextual problems			**Conventional problems**		
Score	*Level*	*Interpretation values*	*Score*	*Level*	*Interpretation values*
00 - 08	Low	Low achievement	00 – 05	Low	Low achievement
09 - 16	Mediocre	Mediocre achievement	07 – 11	Mediocre	Mediocre achievement
17 - 24	High	High achievement	12 - 16	High	High achievement

Fourth, in comparing the effect of the experimental program, this study utilized the independent t-test and the analysis of variance (ANOVA). The analysis was performed based on the type of groups (EG and CG), the type of items (contextual and conventional), and type of tests (pre-test and post-test) as the dependent variables, and with the pupils' performance score as the independent variables. The main significant effect and its interaction among the variables would be analyzed using the Tukey HSD test (using the .05 significance level).

4.6 CONCLUDING SUMMARY

During the cyclic process of developmental research, sensitivity of the design was an essential aspect to be taken into account. Sensitivity refers to "the likelihood that an effect, if present, will be detected (Lipsey, 1990). Consideration of sensitivity is based upon the distinction between the "true", but unknown, state of affairs in a population and the state that is observed by the researcher in the teaching experiments. A design is sensitive to the extent that the researchers' conclusion reflects the "true" state (Lawson, 1997). If any difference between the true and experimental state is considered as error or variability then anything that reduces this error will increase a design's sensitivity. Factors of errors such as subject heterogeneity (choosing teachers, schools, and pupils as samples), experimental variability (control of experimental arrangements and procedures), measurement error (the appropriateness of appraisals), and statistical power (the use of statistical tests) were considered to be important sensitivity aspects.

In this study those error factors were carefully encountered. However, in some circumstances, many features could not be removed; they remained although it was possible to reduce its affect. In some cases, the teachers' eagerness of controlling the class by giving orders made them answered the pupils' questions by telling the correct solution right away. Meanwhile the RME theory suggests the norms should be established in the discussion process. Choosing the experiment group and the control groups (teachers, schools, and pupils) purposively and comparing the data decreased the sensitivity of the design. After all, having a homogeneous pupils' achievement in both groups, using various data collection methods, utilizing variety of sources (experts, teachers, pupils, and observers), analyzing data with a carefully chosen methods would reduce some unforced errors. It assured the quality of data, the conclusions, and the study as well.

CHAPTER 5
FIRST STAGE OF PROTOTYPING PHASE: STRUCTURING AN RME DESK VERSION

The previous chapters illustrate the backgrounds of this study concerning the improvement needed in Indonesian mathematics education, the promising RME approach, and the chosen developmental research designs. This chapter describes the development process of the early version of the RME prototype in the first stage of the prototyping phase. Focusing on validity (content and construct) of the RME prototype, the first stage of the prototyping phase constructed and revised the RME exemplary materials for teaching multiplication and division of multi-digit numbers in Indonesia. The experts (Indonesian mathematics education and RME experts) assured validity of the desk version.

5.1 RESEARCH DESIGN

The first stage of prototyping phase was aimed at (1) analyzing the current situations of the Indonesian mathematics education and discussing the promising RME approach and its possibility to be applied in Indonesian schools and (2) developing, evaluating, and revising the RME instructional materials. This phase ended up with a desk version of the RME prototype.

This stage was conducted in a cyclic process of the front-end analysis and expert reviews (see Figure 4.2 in section 4.2.3). The front-end analysis examined the Indonesian situations, discussed the prospective use of RME approach, and ended at constructing a desk version of the RME materials. The initial intention was on analyzing (content and construct) validity of the instructional materials (the state-of-the-art knowledge of the Indonesian circumstances and the RME theory). Two RME experts and an Indonesian mathematics education expert were involved in reviewing and justifying the validity of the RME prototypical materials. The experts analyzed the materials in several walking through sessions in which the researcher held interviews to get their initial agreement of the content. The activities were conducted from January until June 1999 producing the desk version of the RME prototype.

After all, this stage was led by answering the following question:

> *To what extent was the RME prototype valid in teaching multiplication and division of multi-digit numbers in Indonesian primary schools?*

Validity of the RME prototype referred to whether the components of the RME materials were developed based on the state-of-the-art knowledge (content validity) and all components were consistently linked to each other (construct validity). To evaluate the content and construct validity the researcher interviewed an Indonesian mathematics education expert and two RME experts in walkthrough sessions. The Indonesian expert was needed to analyze whether the materials represent the Indonesian circumstances (mathematics curriculum and contexts). The RME experts examined the materials from one page to another to judge whether the RME theory was embedded in the materials. They also judged the consistency of the components linked in the materials. These activities were held in cyclic process of front-end analysis and expert reviews. After all, the validity of the RME prototype was obtained whenever the experts were satisfied with the presence of the content and construct validity of the RME prototype. The following section illustrates the results of the first stage of the prototyping phase.

5.2 RESULTS

5.2.1 The content validity of the RME prototype

To assure the existence of the content validity in the RME prototype the phase took several considerations into account. *Firstly*, the materials had to suit the Indonesian education circumstances and culture. This means that (1) the contents were subjected to the 1994 mathematics curriculum in primary schools and (2) the contextual problems involved were addressed to the familiarity of the teachers and pupils. *Secondly*, the arithmetic contents were chosen by considering the teachers' competencies toward the contents. The more the teachers' understanding the contents the less difficulty they had in implementing the learning process. Thus, this study was mainly dealt with the teachers' competences on RME didactical approach and pupils' leaning cognition. *Thirdly*, the materials developed was structurally represented the RME theory and its instruction approach.

The first and second reason led this phase to choose multiplication and division of multi-digit numbers as the contents to be developed. These subjects became the core parts of the mathematics curriculum in primary schools and considerably important to develop pupils' ability to manipulate numbers. And it was observed that teachers had good competency in solving problems of these subjects. The contents were taught in the second trimester of Grade 4 in the primary schools (see section 5.2.3 below). In RME these contents were already developed (see Treffers, 1991). Treffers distinguished mental and column arithmetic procedures in solving multiplication problems. Meanwhile, Gravemeijer (1994) found varieties of pupils' strategies in solving division problems. The need of finding a learning trajectory with appropriate contextual problems to facilitate the learning process for Indonesian primary schools still remained.

As mentioned before this first stage of prototyping phase was conducted in a cyclic front-end analysis and expert reviews. During this process the researcher analyzed current situation of Indonesian mathematics education (see chapter 2). It was found out that the teachers taught the subjects mechanistically (see section 2.5). It influenced the pupils' confusions and drawbacks and also promoted pupils' dependent learning attitude (see section 1.2.1 and 2.5). Several reasons were found out, the main weakness was on the instructional materials being used in teaching the subject and the teachers' competencies (in the subject contents, in the didactical approach, and in the pupils' learning cognition; see section 1.2.2 and 2.3).

These conditions guided the researcher towards a discussion of what kind of approach to be applied to improve pupils' understanding as well as teachers' competencies. Considering several studies and projects developed in many countries, it was believed that the RME theory was the promising approach to be applied in Indonesia. Then the thought discussions went on to how to create the materials, what kind of research to be conducted, what kind of appraisals, methods, and individuals are involved, and how to apply the RME approach considering the teachers' and pupils' backgrounds.

As mentioned in section 1.4, Feiter and Van den Akker (1995) and Loucks-Horsley, et al., (1996) suggested that providing teachers' materials and guidance and practicing a form of teaching approach would be an essential alternative to improve the teachers' competencies. Taking this suggestion into account the researcher developed a desk version of the RME instructional materials. The materials consisted of several items: a teachers' guide, a pupil book, and a daily basis of instructional activities. The structure of the subjects created in the desk version materials (e.g. the goals, the learning activities (route), the contexts involved in the problems, and guidance of conducting instruction process) took into account the 1994 mathematics curriculum in Indonesian primary schools (see section 5.2.3 below). Developing these instructional desk version materials helped the researcher develop the initial requisite experiences towards the new understanding of the RME approach.

To help the researcher to construct the materials, expert reviews were utilized. An Indonesian mathematics education expert and two RME experts were utilized to analyze the content and construct validity of the materials (the consistence of the construct validity will be elaborated in the next section). The Indonesian mathematics education expert analyzed the validity of the materials concerning whether the materials represented the Indonesian circumstances, including the curriculum to be taught and the contexts the teachers and pupils were familiar with. The RME experts examined whether the RME theory and its instructional principles were embedded in the materials and whether the components of the materials were linked to each other.

The expert reviews were conducted in several one-to-one formative walkthrough sessions, in which the researcher interviewed the experts while they were analyzing one page to another the existence of the content validity of the materials (the questions addressed can be seen in section 4.5). All comments and reactions were transcribed independently considering its relevant improvement to the materials (organization, contents, contexts, and numbers involved). Based on interpretation of these sessions, revised decisions were generated to improve the materials.

In these activities the researcher developed and reevaluated various contextual problems in each day of learning multiplication and division (see Appendix A). Six items were constructed for each day of teaching, three items aimed at giving opportunity for the pupils to understand the contents by mathematizing the problems and develop the procedures using their own learning process. The other three items served as the homework and enrichments. The items were developed and evaluated formatively considering its functions (Treffers & Goffree, 1985): model formation; concept formation; practice; applicability. For instance, the first three items in the first day of learning multiplication were "Tiles", "To the zoo", and "Skillful mason" (see Appendix A). Encountering the "Tiles" problem, the pupils were given a picture of a 14x14 square and asked to find the number of tiles to build this square. To solve this problem, it was found out that some pupils build up several models of repeated addition, such as counting them all, addition of 5 numbers, addition of 14 numbers, and addition by ten numbers (see examples in Figure 9.14 in section 9.4.1 item a). It is a *model formation*, in which pupils constructed variety of models representing the "tiles" problem into mathematical forms. The "Tiles" problem gave access and motivated pupils to create their own model using their own understanding. Whenever the pupils use the mathematical tools for finding out the solution by calculating the numbers repeatedly, then it can be said that the pupils were formatted their mathematical concepts and procedures. This calculating numbers activity supplied a firm basis for formalizing operations, notations, and rules (*concept formation*). Having many various strategies (repeated addition) and discussing them with teachers and other peers would guide them toward the understanding of the use of repeated addition; the different models of solving problems; and the mathematical tools being used in the calculation process. It was also found that pupils and teacher determined which strategy was the most understandable, and more effective and efficient to solve the problem. Then they could *practice* the strategy they were comfortable with to solve the other two problems (see some solutions in Figure 9.15 in section 9.4.1 item a). Solving many contextual problems that were taken from everyday life situation was intended to lead pupils towards their understanding of the relationship of the daily problems and the mathematics subjects; meaning that mathematics was *applicable* to solve many daily life problem situations.

During this stage the researcher also developed the appraisals used in the second stage of prototyping phase, such as quizzes (daily and weekly) and tests (pre-test and post-test). These appraisals were used to judge pupils' learning performances: learning progress, understanding, and achievement. These three items for each subject (multiplication and division) in daily quiz aimed at finding the pupils' learning progress in day-to-day basis. Two items in weekly quiz presumed to analyze pupils' learning achievement at the end of the learning process (eight hours per week). 10 items in the pre-test and the post-test spreading in two different types (contextual and conventional) were aimed at finding the pupils' achievement before and after the learning activities using the RME prototype. All items were judged in terms of its validity and its level of difficulty by the experts. Its reliability and the actual level of difficulty were analyzed after conducting the third stage of the prototyping phase (illustrated in section 4.5.1).

5.2.2 The construct validity of the RME prototype

The consistence link of the components in the RME prototype was analyzed by the Indonesian and RME experts. In several walkthrough sessions they were interviewed to examine the organization of the materials, including the learning trajectory structured, the content lay-out constructed, the contexts used, the numbers involved, and the time available. The experts examined the problems and the inappropriateness of the materials in connection with the instructional process in the classroom. During the sessions, the experts' comments and the problems found were transcribed and interpreted: based on those results the materials were improved. These activities were iteratively conducted until the experts satisfied with the consistence link of the components in the materials.

The consistence link of the materials was analyzed based on whether the RME tenets were accessible in the materials. *The use of contextual problems* was the first tenet to be taken into account. In RME, contextual problems play its role as a meaningful starting point from which the intended mathematics can emerge rather than as applications at the end of the learning process. Considering this aspect the researcher constructed the contextual problems involved in the learning process based on its familiarity with the pupils, the level of difficulty, the reasonability of numbers included, and the length of the sentences. The experts always involved in judging the appropriateness of all contextual problems used in the study.

The contexts and the numbers involved in the contextual problems were the most important aspects examined by the experts. They suggested the contexts or the numbers that were beyond imagination, irrational and uncommon that had to be eradicated. For instance, a contextual problem created in the desk version such as "*Budi has 5 chickens. He feeds them everyday. <u>Each chicken lays 2 eggs a day</u>. How many eggs does he get in two weeks?*" was beyond imagination. But a contextual problem such as

"*Pak Budi is a skillful mason. He is asked to build a huge wall that needs 204 bricks in each layer. The wall contains 52 layers. How many bricks does the wall need?*" was realistic, rational and imaginable. Streefland (1990) mentioned that the main important thing was that pupils could realize the problem, thus 'realistic' does not necessarily mean 'real life'. During the session, the researcher and the experts examined each contextual problem and its numbers involved and the decision was made to modify or to except the problems. Other contextual problems created in the RME prototype can be seen in Teacher Guide and Pupil Book (see appendix A).

Bridging by vertical instruments was considered essential for the materials. The intention was given to the opportunity for pupils to format their own mathematical concepts and models of the contextual problems. It helps them to bridge the gap between their intuitive informal levels to the level of mathematics as a formal system. The experts emphasized this tenet by giving its attention to the contextual problems utilized the learning paths in each day. For instance, using the "Tiles" problem (see section 9.2) was reasonable to start learning path in the first day of learning multiplication for pupils (elaborated more in section 9.4). It was because pupils still had difficulties in developing their own informal models on their own. They were used to get orders from the teacher to copy the teachers' mathematical models. The "Tiles" problems helped them to shift from "having orders" to that of "doing by themselves". The problems could be easily solved by counting the tiles at once, or by rows, or by column, or by using other addition strategies. It led the pupils to invent the repeated addition strategies of multiplication 14 x 14. The experts label these repeated addition strategies as vertical instrument for building pupils understanding of multiplication (quotation from interviews with Gravemeijer, 2002). These learning activities were parts of the whole learning multiplication sequences (see pupils' learning trajectory of multiplication in section 9.4).

The pupils' reinvented strategies to solve the problems were seen as constructive elements in the learning process (elaborated more in section 9.4). The experts emphasized the usefulness of these pupils' contributions (own productions) to understand their learning progress. As an example, in this study the pre-test items were provided to visualize pupils' initial reinvented strategies in solving contextual and conventional problems of multiplication and division. It was found out that pupils used repeated addition to solve not only the contextual problems such as

"Playing cards" problem (see Figure 9.1 in section 9.3) but also the conventional problems such as multiplying 86 x 37 and 608 x 45 (see Figure 9.11 in section 9.3). These conditions appeared as an initial element of starting teaching multiplication (see also Gravemeijer, 1994 and Treffers, 1991).

Interactivity in the learning process was another tenet that is considered important by the experts. The explicit negotiation, intervention, discussion, and evaluation were the essential elements in a constructive learning process developed in the RME prototype. The experts influenced the urgency of utilizing guidance questions and hints and making them available for teachers in the materials in order to facilitate interactivity in the learning process. They determined that the Indonesian pupils' attitude in learning and teachers' beliefs in teaching would be essential aspects to be put into consideration. This study prepared such guidance in the Planning Instruction (the first page of each section in the Teacher's Guide)

In conjunction with the teaching process another essential aspect due to the availability of sufficient time for learning the subjects. The experts determined that 8-hour each for teaching multiplication and division was not sufficient to cover the whole activities in the RME approach. It was because of several reasons: (1) the pupils' difficulties in learning the contents; (2) their familiarity towards different approach; (3) teachers' knowledge and skills in didactical aspects of the RME approach; and (4) the time allocated to similar sequences in Dutch textbooks. Considering the obligation of teaching the contents in harmony with the 1994 mathematics curriculum, the experts suggested to having intensive discussion with teachers before, during, and after the learning process. This suggestion had been employed in each instruction experiment conducted in each phase of this study. It was found that these discussion activities gave a significant improvement towards the teachers' performance in conducting the RME approach in the classroom.

5.2.3 Concluding summary

In this first stage of the prototyping phase the cyclic process of front-end analysis and expert reviews had been conducted in analyzing the (content and construct) validity of the RME prototype. During the front-end analysis the present condition of Indonesian mathematics education was analyzed (see chapter 2). And then the RME theory and its approach in teaching mathematics were evaluated to see its prospective use in improving Indonesian condition. It was found out that the experts (Indonesian and RME experts) were satisfied with the existence of the state-of-the-art knowledge (content validity) of the Indonesian circumstances and the RME theory that embedded in the RME materials. The interviews held during the walkthrough session proved this result. The experts were also satisfied with the consistence link of the components (construct validity) of the materials, including organization (aims, structure, and the learning route), mathematical contents (multiplication and division concepts and strategies), contexts used, numbers involved, and appraisals. The RME prototypical materials were developed based on the RME tenets and its consistent link had been judged satisfactory by the experts. However, time available for teaching multiplication and division of multi-digit numbers was judged insufficient. The following section illustrates the structure of the RME prototype.

5.3 DESIGNING THE RME PROTOTYPICAL MATERIAL

This study developed the RME prototype materials to teach multiplication and division of multi-digit numbers based on several aspects: (1) the 1994 mathematics education curriculum for Indonesian primary schools; (2) the RME theory and its instructional principles; (3) the hypothetical RME learning trajectory of learning multiplication and division; and (4) the empirical learning trajectory as a result from the preliminary phase. Each aspect gave its contribution and influenced the development of the RME prototype.

On the 1994 mathematics curriculum for primary schools in Indonesia, the multiplication and division of multi-digit numbers were taught in the second trimester. The contents, the objectives, and the time available in teaching multiplication and division of multi-digit numbers were illustrated in the following table.

Table 5.1
The content, the aims, and the hours to teach the content

Contents in the 1994 curriculum	Aims	Hours
Unit 4: Numbers (Part 3) *3.1. Numbers and its symbols* Introduction to numbers between 50.000 – 100.000	*The pupils are able to comprehend the whole numbers to 100,000*	2
3.2. Multiplication Multiplying within tens continuously (10 x 10 x 10 x 10 x 10 = 100.000) Multiplying between tens (30 x 20 x 40 = 24.000) Multiplying between 2-digit numbers Multiplying 2-digit numbers with 3-digit numbers	*The pupils are able to:* *multiply between 2-digit numbers* *multiply 2-digit numbers by 3-digit numbers*	8
3.3. Division Dividing 4-digit numbers with 1- or 2-digit numbers Dividing 5-digit numbers with 1- or 2-digit numbers	*The pupils are able to:* *divide 4-digit numbers with 2-digit numbers* *divide 5-digit numbers with 2-digit numbers*	8
3.4. Multiplication and division (mixed) Finding solution by employing more than two basic computations Finding solution of story problems	*The pupils are able to find solution by employing more than two basic computations*	6

During the first stage of prototyping phase, the researcher developed sub-objectives of the contents and its descriptions based on RME approach. Because the hours of teaching could not be changed (that was 8 hours for each subject), the researcher distributed the time equally to each sub-objectives. It was indeed a weak point of the RME prototype because experts and teachers mentioned that the time available was not enough to cover all learning activities. It was because the teachers' adjustment took much more time than expected (see section 5.2.1 and 5.2.2 above). Discussions and guidance given during the learning process was not sufficient to improve teachers' competence in applying the RME approach. On the other hand, the teachers and the principal did not agree to prolong the teaching experiments because other mathematics subjects should be taught as well in the second trimester. Considering this fact, the researcher decided to distribute the 16 hours (á 40-minute) equally for each session of teaching. The following table illustrates the contents, objectives, and the hours distributed in teaching the subjects in the RME approach.

Table 5.2

The contents, objectives and the hours in RME approach

Contents in the RME prototype	**Objectives and sub-objectives**	**Hours**
Multiplication	*The pupils can understand, explore, and justify the conventional algorithm for multiplication in terms of repeated addition and the decimal numbers*	
▪ Repeated additions of ten numbers	▪ *Pupils can use repeated additions of ten numbers*	*2*
▪ Multiplication by 10	▪ *Pupils can use multiplication by 10*	*2*
▪ Multiplication by multiples of ten	▪ *Pupils can use multiplication by multiples of ten*	*2*
▪ Standard multiplication algorithm	▪ *Pupils can use standard multiplication algorithm*	*2*
Division	*The pupils can understand, explore, and justify the conventional algorithm for division in terms of repeated subtraction and the decimal numbers*	
▪ Unstructured repeated subtraction	▪ *Pupils can use the unstructured repeated subtraction*	2
▪ Limited structured repeated subtraction	▪ *Pupils can use the limited unstructured repeated subtraction*	2
▪ Structured repeated subtraction	▪ *Pupils can use the structured repeated subtraction*	2
▪ Standard division algorithm	▪ *Pupils can use the standard division algorithm*	2

It was expected that the researcher would have to reconstruct the objectives and its sub-objectives based on the pupils' problem solving procedures after the teaching experiments conducted in the second stage of prototyping phase. It was predicted that most pupils applied varieties of strategies to solve multiplication problems. They developed the repeated addition (with various strategies, for instance doubling, adding 5-numbers, and adding 10-numbers together), multiplication by 10, and multiplication by multiples of ten; mental procedure, standard multiplication algorithm, and try-and-error strategy, or combining multiplication by multiples of ten and standard algorithm (see Armanto, 2000).

Considering these strategies the researcher believed that there was a path of learning multiplication of multi-digit numbers. From Gravemeijer (1994) it was found out that repeated addition was the strategy the pupils used in the first place and repeated addition of ten numbers was the effective strategy to solve the multiplication problem. From these notions it was believed that the repeated addition of ten numbers would be the first strategy the pupils should develop. This would be facilitated by offering a structured contextual problem and having an interactive discussion in the classroom. The teachers should deliver questions, hints and encouragement to discuss the strategies to assure that pupils found the repeated addition of 10-number was effective for solving multiplication problems (see section 9.4.1 item a).

From there, pupils can be stimulated to be aware of the need of more effective strategy. A contextual problem that could be solved with a long repeated addition of 10 numbers, such as 125 for 46 times would be an interesting discussion to find another prospective, easier, less time consumed, reliable, and understandable strategy. It was the multiplication by 10. The teacher could facilitate this reinvention by strategically representing the amount of numbers in each column (that is 10) and the number being added in the column (the multiplied number), for instance 10 x 125 for four times (see section 9.4.1 item b). Then by realizing that the multiplication by 10 could be changed into multiplication by multiples of ten, it made the learning process understandable (see section 9.4.1 item c). Next and the last learning activities were on reinventing multiplication algorithm. Having learnt the multiplication by multiples of ten would guide the learning process of the multiplication algorithm. The teacher could facilitate the learning by comparing both strategies or by writing the multiplication by multiples of ten downward (see section 9.4.1 item d).

And then the researcher restructured the learning route for teaching each session of the instructional process. Because of the tendency of pupils' dependency on the teachers' orders and their weaknesses in manipulating numbers, the RME experts and the researcher agreed upon the need of preliminary games in order to increase the pupils' skills in numbers, attitude and motivation. Table 5.3 illustrates the multiplication learning route.

Table 5.3
The learning route of teaching multiplication

Objective:	After engaging in this multiplication section the pupils can understand, explore, and justify the conventional algorithm for multiplication in terms of repeated addition and the decimal numbers		
Day 1 *Hours: 2 x 40 minutes*	***Day 2*** *Hours: 2 x 40 minutes*	***Day 3*** *Hours: 2 x 40 minutes*	***Day 4*** *Hours: 2 x 40 minutes*
***Sub-objective*:** Pupils can reinvent repeated additions	***Sub-objective:*** Pupils can reinvent multiplication by 10	***Sub-objective:*** Pupils can reinvent multiplication by multiples of ten	***Sub-objective:*** Pupils can reinvent standard algorithm
Learning route: *Preliminary game* Multiplication facts Addition of multi-digit numbers	***Learning route:*** *Preliminary game* Multiplication facts Multiplication by 10 (tens)	***Learning route:*** *Preliminary game* Multiplication facts Multiplication by 10 and by multiples of ten	***Learning route:*** *Preliminary game* Multiplication facts Multiplication by 10 and multiples of ten
Solving problems ▪ Tiles ▪ To the zoo ▪ Skilful mason	*Solving problems* ▪ Potatoes ▪ Books ▪ The teacher	*Solving problems* ▪ Using waters	*Solving problems* ▪ A Fan ▪ A jumping frog ▪ Plane and car

Table 5.3 above illustrates that teaching multiplication was conducted in 4 days (2 x 40 minutes each) and in each day there was a sub-objective to be achieved. This sub-objective was reached by conducting the preliminary games and solving contextual problems. In each preliminary game there were 1-2 games provided. These games were conducted for two reasons: (1) practicing and recalling multiplication facts; (2) generalizing multiplication by 10 and by multiples of ten; and (3) attracting pupils' attention and motivation to learn the content of the subject matters (see the introduction activities in section 7.2.1 item a and 8.2.1 item a). This game took place for 5 -10 minutes.

After the preliminary games, the teaching and learning multiplication began. In each day the pupils solved and encountered contextual problems that led the pupils to develop, increase, and grow their understanding towards the multiplication procedures. They learnt how to use their former knowledge and develop informal and formal mathematics forms to solve the problems. The problems involved in each day were judged by the RME experts and the Indonesian experts.

For learning division pupils developed several procedures in solving contextual problems: repeated subtraction with table of multiplication by 10; repeated subtraction with table of multiplication by unit, 10, and 100; try-and-error, guess-multiply-check, standard algorithm, structured and unstructured repeated subtraction, and combination of two strategies. However, most pupils applied the unstructured repeated subtraction rather than the structured or the standard division algorithm. They argued that even though the unstructured was a long way strategy, but easily understood and they could find the right answer as well. The idea behind this statement was that they had difficulty in multiplying numbers and it was easy to multiply numbers that they were familiar with.

Considering those facts the researcher constructed the objectives into several sub-objectives, as shown in Table 5.2. The development of the division learning route was taken place (see the following table). In all these processes the RME and the Indonesian mathematics education experts were involved.

Table 5.4
The learning route of teaching division

Objective:	After engaging in this division section the pupils can understand, explore, and justify the conventional algorithm for division in terms of repeated subtraction and the decimal numbers		
Day 1 *Hours: 2 × 40 minutes*	***Day 2*** *Hours: 2 × 40 minutes*	***Day 3*** *Hours: 2 × 40 minutes*	***Day 4*** *Hours: 2 × 40 minutes*
***Sub-objective*:** Pupils can use the unstructured repeated subtraction	***Sub-objective:*** Pupils can use the limited structured repeated subtraction	***Sub-objective:*** Pupils can use the structured repeated subtraction	***Sub-objective:*** Pupils can use the standard division algorithm
Learning route: *Preliminary game* Multiplication facts Multiplication by 10 and 100	***Learning route:*** *Preliminary game* Multiplication facts Multiplication by 10 and 100	***Learning route:*** *Preliminary game* Multiplication facts Multiplication by 10, 100, and 1000	***Learning route:*** *Preliminary game* Multiplication facts Multiplication by 10, 100, and 1000
Solving problems ▪ Lebaran day ▪ Pupils in line ▪ Reading a book	*Solving problems* ▪ Chicken farm ▪ Jumping on the rope ▪ Graduation	*Solving problems* ▪ Using waters	*Solving problems* ▪ The zoo ▪ Stack of paper ▪ Kangaroo's jump

Using Table 5.3 and 5.4 above the researcher developed the hypothetical learning trajectory (HLT) of each subject. It is a conjectured learning path in which the pupils might learn the subjects. In HLT, the pupils' mental activities are illustrated; including what are the expected strategies they use, which expected thinking process occurs, what are they about to think and to see, and how are they expected to reason.

In constructing the HLT for teaching multiplication for instance, the "Tiles" problem was expected to motivate pupils to use their counting ability or repeated addition strategies. The strategies might be doubling, triples numbers, five numbers in a row, or repeated addition of ten numbers. Discussing those strategies would lead the pupils toward the mathematical tools that were used in each strategy that produce the same answers. Reasoning and understanding are the key element to be built in this discussion. They can use one of the strategies to answer the next two problems: "To the zoo" and "Skillful mason". On the other situation, whenever pupils have other sophisticated strategies, such as multiplication by 10 or the standard algorithm, the discussions become more interactive and effective. Asking pupils these questions: "How do you know that these strategies work?" or "Why do these strategies produce the same result?" will stimulate pupils' thinking as well as the discussions. What attitude was expected that the pupils could reason many different strategies they employed. It leads them toward the understanding of strategies; in terms of making sense of the contextual problems, the solution procedures, and the relationship between the strategies and the mathematical tools being used.

Similar approach was approved for building the HLT of division. The "Lebaran day" problem asked pupils to reuse repeated subtraction strategy. The problem was about 1400 people going back to Surabaya from Jakarta using train with 86 people in each wagon (see Appendix A). It could be represented by subtracting the number of people that could be brought by several wagons. For instance, 5 wagons could carry 5 x 86 = 430 people. Teachers had to realize that pupils could also use repeated addition of 86 for 5 times to find the result. These calculations required pupils' ability of multiplication by 1-digit numbers or repeated addition. The pupils also needed a skill of subtracting multi-digit numbers because they were about to subtract 1400 - 430. The cyclic process of multiplying (or adding) and subtracting numbers was the main mental activities of pupils in carrying out the repeated subtraction strategies. Different alternatives of numbers used by the pupils made the repeated subtraction strategy varied. Some pupils might use 10 and 5 wagons. Other might use five wagons consecutively. Whatever numbers the pupils used, they would end up with the same result. The differences were only in how many times they conduct the calculations until they find the solution. Having these strategies will guide pupils toward the discussions of how to do the repeated subtraction more efficient. In other cases pupils may use the structured repeated addition (using tens and unit at one time) or the standard division algorithm. In this situations the discussions became more interactive. Asking the same question mention above, pupils would pay their attention to give reasons that led them toward understanding the strategies. Having comprehending the understandable strategy the pupils would use it to solve other contextual problems.

Those HLTs were constructed in daily basis of learning involving many contextual problems that represent the multiplication and division of multi-digit numbers. The structure of the contextual problems in the RME approach and the learning trajectory of multiplication and division will be elaborated in section 9.2 and 9.4 respectively. The learning routes mentioned above were structured comprehensively in the RME prototype. Several subsequences of this version was used and applied in the second stage of the prototyping phase (illustrated in the next chapter).

CHAPTER 6
SECOND STAGE OF THE PROTOTYPING PHASE: ASSEMBLING AN RME EARLY VERSION

The previous chapter illustrates the development process of the desk version focusing on its (content and construct) validity in the first stage of the prototyping phase. This chapter describes briefly the research design and the results of the second stage of the prototyping phase. Focusing on the practicality of the RME prototypical materials, this stage used and revised the RME exemplary materials for teaching multiplication and division of multi-digit numbers in Indonesia. Using the desk RME version, the researcher and three experienced teachers applied several sub-sequences of the learning activities for teaching multiplication and division of multi-digit numbers. It was aimed at having sense and insights of conducting the instruction activities using the RME approach. This stage ended up with restructuring an early version of the RME prototype.

6.1 RESEARCH DESIGN

The second stage of prototyping phase was aimed at analyzing the practicality of the RME prototype in Indonesian primary schools. Practicality was defined as the degree to which the RME prototypical materials were usable and easy to Indonesian teachers and pupils. The stage was led by answering the following sub-research question:

To what extent was the RME prototype practical for teaching multiplication and division of multi-digit numbers in Indonesian primary schools?

The practicality of the RME prototype was justified by three teachers from two schools in Jogyakarta, Indonesia that were chosen purposively. It was assumed that trying out a new approach needed experienced teachers to apply the approach. They are the essential elements for the effective implementation because the relevance of the approach, its clarity, complexity, and practicality of the approach (see characteristics of the change from Fullan, 1984) are several aspects to be taken into account. These experienced teachers judged the practicality of the RME prototypical materials.

The teachers chosen were classified as experienced teachers (more than 5 years of teaching the subject) and they voluntarily accepted to conduct the RME instruction approach (see Table 6.1 below). It was assumed that having these two purposive elements helped teachers to apply the RME approach, encountering the complexity of the implementation process and analyzing whether the instructional sequences were easy and usable for the Indonesian teachers and pupils. The following figure illustrates the teachers' background.

Table 6.1
Teachers' backgrounds

Items	**SD Puren**	**SD Kanisius A**	**SD Kanisius B**
Age	40-49	30-39	30-39
Gender	Female	Female	Male
Education	BA or eq.	BA or eq.	BA or eq.
Experience in teaching	13	5	11

The teachers as well as the researcher analyzed the RME desk version (structured in the first stage of the prototyping phase, see chapter 5) and used several sub-sequences of the version in the classroom. The aim was to get a sense and experiences of conducting the RME approach. Being in the instruction activities in the RME approach would give an insight for teachers to analyze and judge the usable and ease of the RME approach in Indonesian primary schools.

The pupils of Grade 4 age 10 - 11 years old were involved. They were 123 pupils, 56 boys and 67 girls (see Table 6.2 below). Their backgrounds varied in the achievement and the economics status (from teachers' excerpt during the interview).

Table 6.2
Pupils involved in the teaching experiments

Items	**SD Puren**	**SD Kanisius A**	**SD Kanisius B**
Numbers of pupils	*38 pupils (17 boys and 21 girls)*	*43 pupils (20 boys and 23 girls)*	*42 pupils (19 boys and 23 girls)*
Ownership of:			
▪ calculator	20%	95%	90%
▪ table	68%	95%	95%
▪ room	56%	95%	95%
▪ computer	5%	80%	60%

During the teaching experiments the teachers completed the *logbook* twice (once a week for 2 weeks), illustrated their agreement towards several issues: the RME instruction process, the prototypical materials (The teacher guide and Pupils book), and the pupils' engagement in learning process. The logbooks were used as initial information to *interview* teachers in order to realize reasons of the answers given. These interviews were transcribed and coded and the data were used to revise the instructional materials. The *interviews* were also conducted with pupils from different ability. It informed their impression of the learning conditions. Pupils' *portfolios* were also collected to analyze their actual learning trajectory that representing their informal and formal mathematics strategies applied in learning the subjects.

There were two aspects analyzed during the teaching experiments: how the proposed instructional sequences used and what the pupils might learn during the learning process. These aspects were also cross-examined with transcription data from the teachers' logbooks, the pupils' portfolios, and the experts' reviews. All problems, comments, and reactions were analyzed and the materials were improved. These activities were conducted until the experts were satisfied with the existence of the Indonesian circumstances, the RME theory, and the consistence link of the components in the materials.

In this second stage the conclusion of whether the practicality of the RME prototype existed depending on the teachers' impression towards the materials and the instructional process. In other words, the practicality was established if:

1. *the teachers had a good general impression of the lesson,*
2. *the teachers had a good opinion toward the learning climate,*
3. *the teachers perceived pupils' active engagement in the lesson,*
4. *the teachers had a good opinion toward the RME materials.*

Each proposition is described in the following section as the results of this stage.

6.2 RESULTS

6.2.1 Teachers' general impression

With 3 teachers completed 2 logbooks each, the results of teachers' impression of the RME lesson were illustrated as follows:

Table 6.3
Teachers' general impression of the lesson

	Chosen option for n = 6[a]					
Positive	*Fully agree 5*	*Agree 4*	*Fair 3*	*Not agree 2*	*Fully not agree 1*	**Negative**
Useful	3	3				Not useful
Interesting		4	2			Not interesting
Easy to use		2	3	1		Not easy to use
Comfortable		2	2	2		Not comfortable
Cumulative	3	11	7	3	-	Mean = 3.58
percentage	12.5%	46%	29%	12.5%		

Note: [a]Numbers of teachers.

It can be seen that teachers were in favors of positive answers (58.5% and the mean score was 3.8). It can be concluded that teachers found the RME materials were useful, interesting, comfortable, and easy to use. An example illustrating this result could be obtained from the interviews with a teacher. She said that it was easier to explain the strategies in dividing multi-digit numbers than that of the standard algorithm. She gave this idea when she was teaching the subject after being an observer in the instructional activities held by the researcher.

Few teachers found that the RME materials were not easy to use and they were not comfortable in conducting the learning process. The reasons were the teachers' lack of familiarity towards (1) the learning activities, (2) the use of contextual problems as the starting point of teaching, and (3) the interactive discussion during the learning. From the interviews it was found that the teachers needed more time to adjust with the RME approach. They also suggest having a guide, not only the book but also an expert to be facilitator for their practice.

6.2.2 Teachers' opinion toward the learning process

The logbook indicated that the teachers' opinion towards the learning climate is illustrated as follow:

Table 6.4
Teachers' opinion toward the learning process

	Chosen option for n = 6[a]					
Positive	*Fully agree 5*	*Agree 4*	*Fair 3*	*Not agree 2*	*Fully not agree 1*	**Negative**
Easy to apply	1	1	2	2		Not easy to apply
Run smoothly			3	3		Many problems
Time estimation OK			3	3		Time is too optimistic
Lesson aims met		2	2	2		Lesson aims not met
Cumulative percentage	1 4%	3 12%	10 42%	10 42%	-	Mean = 2.79

Note: [a]Numbers of teachers.

From the table above it can be concluded that teachers found out the learning process was fairly easy to apply and moderately met the aims. They thought that the learning process did not run smoothly and did not suit the time estimation. To find the reasons behind these answers, interviews with teachers were conducted.

Several reasons were found. *Firstly*, the learning process needed more time to be adjusted because of the teachers' unfamiliarity towards the approach. *Secondly*, during the learning process some problems emerged. The pupils' inability to read, to multiply 1-digit numbers, and to manipulate (add and subtract) multi-digit numbers had obstructed the pupils' engagement in the learning process.
Thirdly, teachers' definition on learning performance was not on pupils' understanding, but on pupils' correct answers in solving problems. For teachers the

correct answer meant the correct use of the standard algorithm in solving problems. Pupils' use of other strategies did not account. Meanwhile the researcher found that some pupils got correct answers using different strategies. The pupils said that the procedures they used were easier to understand than the standard algorithm.

The last reason was the teachers' resistance of effective teaching. They perceived that the conventional approach (teaching by telling algorithm) was more effective than the RME approach. Effective was in the meaning of less time to explain and to teach the subject but more time for drilling. De Lange (1994) reports that the conventional approach was not effective but easier for the teachers and in reverse for the pupils.

6.2.3 The teachers' opinion toward pupils' engagement

The pupils' engagement in the learning process is illustrated in the following table.

Table 6.5
Pupils' engagement in the learning process

	Chosen option for n = 6[a]					
Positive	*Fully agree* *5*	*Agree* *4*	*Fair* *3*	*Not agree* *2*	*Fully not agree* *1*	**Negative**
Active						*Passive*
▪ asking questions	2	2	1	1		
▪ finding the procedures	1	2	1	2		
▪ applying the procedures		3	2	1		
▪ giving opinions		2	2	2		
Independent						*Dependent*
▪ working individually	2	2	2			
▪ finding and reinventing	1	2	2	1		
▪ thinking individually	1	3	1	1		
▪ discussing with peers	1	1	3	1		
Interested	2	2	1	1		*Uninterested*
Cumulative percentage	10 18.5%	19 35%	15 28%	10 18.5%	-	Mean = 3.53

Note: [a]Numbers of teachers.

The table above showed that the teachers perceived that the pupils were actively involved in the learning activities, independently worked and discussed the subjects, and interestingly engaged in the learning process. Some pupils found inactive, dependent, and uninterested because of several reasons: their lack capability on multiplying numbers, adding and subtracting numbers, and understanding problems. For these pupils, guidance from teachers (giving hints and drawing pictures) and assistance from peers became the major help to enhance their learning engagement.

6.2.4 Teachers' opinion toward the RME prototypical materials

The RME prototype consisted of two materials: the teacher guide and the pupil book. The teacher guide included the guidelines (aims, table of contents, overview of the guide, planning, pacing and preparation), the structure of contextual problems in daily basis, and the appraisals. The teachers' opinion towards the teacher guide is illustrated in this table.

Table 6.6

Teachers' opinion toward the teacher guide

	Chosen option for n = 6[a]					
Positive	*Fully agree 5*	*Agree 4*	*Fair 3*	*Not agree 2*	*Fully not agree 1*	**Negative**
Content clear	3	2		1		Content unclear
Lay-out clear	1	3	2			Lay-out unclear
Information provided	1	1	2	2		Information not provided
Easy to use	2	1	1	2		Not easy to use
Easy to apply	1	2	1	2		Not easy to apply
Text too extensive	1	3	2			Text too concise
Cumulative percentage	9 25%	12 33%	8 22%	7 19%	-	Mean = 3.64

Note: [a]Numbers of teachers.

It can be seen that most teachers' answers were in favor of positive option (58% and the mean score was 3.64). The teacher guide was used and applied easily. The texts tended to be too extensive even though it had clear content and lay-out. The teachers found out that some information such as the pupils' mistakes and their reinvented procedures were significantly needed to understand pupils learning process. This suggestion was taken into account in improving the teacher guide for the next stage of the prototyping phase.

The pupil book consisted of the aims, the structure of contextual problems in daily basis, and the summary (examples of strategies). The teachers' impression towards the pupil book is illustrated in the following table.

Table 6.7
The teachers' opinion toward the pupil book

	Chosen option for n = 6[a]					
Positive	*Fully agree 5*	*Agree 4*	*Fair 3*	*Not agree 2*	*Fully not agree 1*	**Negative**
Content clear	3	2		1		Content unclear
Level of exercises ok	1	2	1	2		Level of exercises high
Language usage ok	1	2	2	1		Language usage difficult
Layout clear	1	2	3			Layout unclear
Text too extensive	1	2	2	1		Text too concise
Cumulative percentage	7 23%	10 33%	8 27%	5 17%	-	Mean = 3.63

Note: [a]Numbers of teachers.

The table above indicates that teachers found the pupil book were reasonably useful to be used in the learning process. The content and layout were clear, level of exercise and language (sentences) were good enough for the pupils to read and to understand the circumstances. From the interviews it was found out that pupils were familiar with the contexts involved. And they agreed that the book was usable and easy to use in learning the subject.

6.3 CONCLUSION: PRACTICALITY OF THE RME PROTOTYPE

The teaching experiments in the second stage of the prototyping phase revealed the practicality of the RME prototype in teaching multiplication and division of multi-digit numbers. Practicality was defined as the degree towards whether the RME prototype was useable and easy for the Indonesian teachers and pupils. This was analyzed based on the teachers' and pupils' initial impression on the RME materials and the instruction activities.

This stage found that the teachers perceived RME prototype as practically usable and moderately easy to be applied in the classroom. The learning activities proposed in the exemplary materials were found useful to lead pupils toward its aim. It also guided teachers to conduct a proper teaching performance.

However, few teachers found that the RME materials were not easy to use because of teachers' lack of familiarity. They convinced that applying the RME approach needed more time to meet the aims. Adjusting with the activities that was different than they used to conduct and pupils' dependence attitude in learning were the aspects to be accounted. Weaknesses such as pupils' ability to read, to multiply 1-digit numbers, and to manipulate (add and subtract) multi-digit numbers had obstructed the pupils' engagement in the learning process.

Having a guide from experts was very essential for improving teachers' competences in facilitating the learning activities in the classroom. In this case, competences were in the meaning of introducing contextual problems, delivering questions, guiding discussions, and defining pupils' performances. The last item referred to the teachers' definition on the pupils' performances that was the correct answers in solving problems. It was more reasonable if they perceived as the pupils' understanding because it described the progress the pupils had in learning the subject. The learning progress was in the meaning of how far the pupils were, what their difficulty and weaknesses were, and how they reasoned the subject.

Working with the experienced teachers in this stage became challenging adventures. In one hand the researcher, as well as the teachers had no experiences in conducting the RME approach in the classroom. On the other hand, the teachers' resistance of what effective teaching was still existed. They perceived that the conventional teaching process (teaching by telling algorithm) was more effective than the RME approach. Effective was in the meaning of less time to explain and to teach the subject but more time in drilling. This statement was inaccurate and against De Lange's (1994) report that the conventional approach put teachers in the easier side but in reverse for the pupils. It was not easy to convince teachers that the RME gave more opportunities for pupils to develop and restructure their own understanding. These would build good basic foundation for pupils, not only in their learning performances but also in their learning attitude. However, it seemed inconvincible because what they needed was an empirical fact concerning Indonesian pupils' achievement using the RME approach.

Nevertheless, this perception was understandable since in the first place De Lange (1997) had mentioned that in applying RME approach in the classroom the teachers will lose several aspects including loss of taking control of the learning process. This meant that teachers had mixed feeling on losing their control on the activities and being incompetence. Another aspect was that the teachers did not know what would happen during the learning process. Complexity of the RME approach and its differences from the conventional one made teachers feel loss of direction in conducting the learning activities.

Choosing teachers as sample of study was taken into account thoroughly in the third stage of the prototyping phase. In this stage, the teachers chosen varied in their competences and were asked on their willingness in conducting the RME approach. This element was essential because it was believed that teachers with a strong motivation would like to learn and to understand condition and activities being conducted in the classroom. The design and the results of the third stage of the prototyping phase are illustrated in the next chapter.

CHAPTER 7
THIRD STAGE OF THE PROTOTYPING PHASE: IMPROVING AN RME TRY-OUT VERSION

Chapter 5 and 6 illustrate the research design as well as the results of the first and second stage of the prototyping phase. It was found out that the RME prototype was valid (representing the Indonesian circumstances and the RME theory) and practical (usable and easy) for teaching multiplication and division of multi-digit numbers. This chapter describes the research design and the results of the third stage of the prototyping phase. This stage was conducted in the cyclic process of the teaching experiments and the reflections of the local instructional sequences. It was aimed at having conjectured local instructional sequences for teaching multiplication and division of multi-digit numbers in Indonesian primary schools. Focusing on the implementability of the prototypical materials, this stage produced the try-out version of the RME prototype.

7.1 RESEARCH DESIGN

This third stage of the prototyping phase focused mainly on the implementability of the RME prototypical materials. The implementability referred to whether the RME prototype could be used as intended. It analyzed the consistency between the intended and the operational curriculum also between the intended and experiential curriculum (see section 4.2.3). As an effect of implementing the RME prototype in the classroom, the pupils' performances were also analyzed. It referred to the initial effectiveness of the RME version. It examined the consistency between the intended and the attained curriculum. The phase was led by the following question:

To what extent was the teachers' implementation of the RME prototype and the pupils' performances in multiplication and division of multi-digit numbers in Indonesian Primary schools?

To address this question, the third stage of prototyping phase was conducted through the teaching experiments in 4 Indonesian primary schools, 2 primary schools in Medan (SD 101746 and SD 101748) and 2 other schools in Yogyakarta (SD Rejodadi and SD Sonosewu II). The schools were chosen purposively based on the teachers' willingness in applying the RME prototype and their qualifications (novice, moderate, and experienced teachers). These teachers were qualified in judging the quality aspects of the RME prototype. They had experiences of teaching the subjects for 1-6 years.

Table 7.1
Teachers' backgrounds

	Medan		**Jogyakarta**	
Items	*SDN 101746*	*SDN 101748*	*SDN Rejodadi*	*SDN Sonosewu*
Age	40-49	30-39	40-49	30-39
Gender	Female	Female	Female	Female
Education	BA or eq.	S1 Degree	S1 Degree	S1 Degree
Experience in teaching	5	1	6	3

Interviews with the teachers revealed that their approaches in teaching multiplication and division of multi-digit numbers were typical of teaching mathematics in Indonesia (see chapter 2). They believed that the mechanistic way of teaching (teaching by telling) was effective enough to transfer the mathematics concepts to the pupils. They also believed that practicing procedures to solve many problems improved pupils' understanding. This finding implied that (1) the pupils had been frequently confronted with the traditional standard word problems and (2) there was no involvement of the realistic problem situation to develop pupils' understanding.

The pupils' engagement in the learning process varied in terms of their capability, performance, gender, and socio-backgrounds. They were 10-11 years old and most of them were girls. The following table illustrates the pupils' backgrounds.

Table 7.2
Pupils involved in the third stage

Items	SDN 101746	SDN 101748	SDN Rejodadi	SDN Sonosewu
Numbers of pupils	*20 pupils 6 boys and 14 girls*	*16 pupils 7 boys and 9 girls*	*25 pupils 11 boys and 14 girls*	*33 pupils 14 boys and 19 girls*
Ownership of:				
calculator	25%	19%	60%	82%
table	20%	12.5%	60%	70%
room	20%	12.5%	60%	76%
computer	0%	0%	0%	3%

In Medan, most pupils were coming from the families of low socio-economic background in a rural plantation area. Their parents work as lower-paid temporary workers or manual-laborers. The pupils had no calculator in their home (75%) and only 20% of the pupils had their own room and table in the house. In Yogyakarta, pupils varied in their socio-economic background level but most pupils were coming from the middle economic background where more than 60% of pupils have their own room and table in their house as well as calculators. Only 1 student had computer in the house.

During the teaching experiments the data were collected using various appraisals (teachers' logbook, pupils' portfolios, interviews, quizzes, and tests) and from different individuals as evaluators and sources of data (experts, teachers, and pupils). Logbook addressed the implementability of the RME prototype in the classroom. Interviews with teachers were conducted during the learning activities to pinpoint the teachers' reasons when they were dealing with the problems occurred in the classroom. The interviews were also held with pupils with different abilities to analyze their understanding of the subjects. Subsequently pupils' portfolios were collected to analyze their weaknesses and their reinvented procedures. During the learning activities each day an item of daily quiz was given to pupils to analyze their learning progress. The weekly quiz was also given to examine pupils' achievement after learning the subject each week. Before the learning activities began, pre-test was delivered to find pupils' instinctive informal (or formal) mathematics forms that representing prerequisite knowledge the pupils' had before learning the subjects.

Then post-test was held at the end of the two-week learning activities aimed at analyzing pupils' achievement. In the first place, the appraisals and individuals involved in this phase aiming at ensuring the quality of the data in order to come into reasonable results. The following section 6.2 illustrates the results of this phase. The following table illustrates an overview of the instruments used in measuring each quality aspect. The use of all instruments was aimed at justifying the "goodness" of the study. Utilizing various sources (experts, teachers, and pupils), different observers, and various appraisals determined the objectivity of the study. This triangulation (data, observer, and methods) assured the quality of the data and provided the quality control of this study.

Table 7.3
The overview of instruments used in the third stage of prototyping phase

	Quality aspects and its criteria					
	Implementability			**Initial effectiveness**		
Instruments	*The use of contextual problems*	*Interactive teaching*	*Socio-mathematics norms*	*Learning progress*	*Level of under-standing*	*Achieve-ment*
Questionnaires	√	√	√			
Logbooks	√	√	√			
Interviews	√	√	√	√	√	√
Portfolios				√	√	
Quizzes				√	√	
Tests						√

7.2 RESULTS

7.2.1 The implementability of the RME prototype

As mentioned earlier, the implementability of the RME prototype was defined as the extent towards whether the RME prototype could be used as intended. This stage determined that the implementability of the RME prototype was established if (a) the teachers introduced the contextual problems properly, (b) the teachers conducted an interactive teaching approach, and (c) the teacher established socio-mathematical climate in the classroom. Each proposition will be described in the following section.

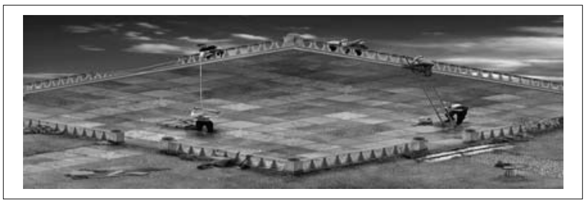
a. Introducing contextual problems

The RME teaching process started with introducing a contextual problem involving the concepts of multiplication and division. This approach was different from the conventional process where the teachers introducing the algorithm to the pupils in the first place. The contextual problems involved were designed to elicit two reactions from the pupils: informal or formal mathematics solutions. These reactions could be a serious consideration to the realistic situation involved in the problem or an attention towards the context in the problem.

The contextual problems of multiplication and division (see the student book page 8 and 14) allowed the pupils recall their addition and subtraction concepts to solve these problems. The context of the problems supplied the pupils a firm basis for learning the formal multiplication and division operations in conjunction with the addition and subtraction operations. In the end this reaction would produce a cognitive understanding towards the relationship between the everyday life situation and the arithmetic story-problem solving.

According to Treffers and Goffree (1985), contextual problems in RME fulfil four functions: (1) concept formation; (2) model formation; (3) applicability; and (4) practice. Contextual problems in the RME prototype were subjected to fulfill these aims, for instance the first day of contextual problem in learning multiplication.

Tiles
How many tiles do they need to build a square as in the picture?



Encountering this problem allowed the pupils to make use of various addition operations. This problem involved a picture of a square with 14 tiles in each of the 14 rows. The pupils answered this problem by reinventing and using their own-understanding of the multiple addition of 3, 4, or 5, or 10 consecutive numbers of 14 until they found the right answer. These models were the pupils' own-productions representing the multiplication of 14 x 14. In this case the pupils formatted their own concepts and model of addition for learning formal operations of multiplication.

Similar approach had been conducted in teaching division. In the first day the pupils encountered the opening problem about people leaving Jakarta to Surabaya using a train that carrying several wagons. Given a total number of 1400 people and a wagon can carry 86 passengers, how many wagons are needed. Using the context the pupils came up with different solutions, such as addition of 86, subtraction of 1400 - 86 consecutively, or using multiplication of several numbers to 86 to find out the exact solution. Encountering different contextual problems in each day (day-1 until day-4), the pupils learnt, discussed, developed and reinvented different division procedures, varied from the unstructured repeated subtraction until the standard division algorithm.

The context of the problem not only helped the pupils to develop their own solution and understanding towards the operations applied in solving the problem but also helped the teacher to manage the learning process. By observing the teacher delivering and introducing the problem in the classroom, the researcher found that the teacher did not get any difficulties since the task could be easily conducted by asking the pupils to read the opening problem in their pupils' book. The problems arose whenever the teacher started the discussion (discussed in section 7.2.1 item b.).

One of the weaknesses in developing pupils' understanding using the contextual problems was that most pupils did not realize the use of the context. Since they had the hint from the teacher that the problem was about the division then they tried to use and apply the division operation. It did not work well because they did not have any idea of how to work on the operation. In fact the teacher did not teach it yet. In this circumstances, guidance from the teacher (i.e. by asking to read the problem again in order to understand the context better, asking pupils' explaining the problem conditions, giving hints, drawing picture and discussing the problem condition) was the most constructive help the pupils needed. This made the discussion more active. Indeed the teacher had an essential role describing the situation, re-explaining or re-summarizing the solution to the pupils.

b. Interactive teaching approach

The teaching models proposed in the RME prototype differed significantly from those used in the conventional lessons in teaching multiplication and division of multi-digit numbers in Indonesia (see Figure 3.7 in section 3.4). The differences are illustrated as follow.

Firstly, as mentioned earlier the use of contextual problems was the starting point of the learning process. Meanwhile the conventional teaching introduced the standard algorithm procedures as the starting point.

Secondly, the contexts involved in the problems attracted pupils to apply their former knowledge of the subject being learnt. For instance, using the repeated addition to answer the multiplication problems. In this manner the pupils themselves reinvented the repeated addition procedures for solving the problems. On the contrary, the conventional approach related on memorizing and practicing the procedures.

Thirdly, the teachers' role was assisting and guiding the pupils by giving them questions and clues to understand the multiplication and division procedures. As it was in the conventional approach, the teachers did not give the answers to the pupils at once or describe the algorithm whenever pupils had difficulties in solving the problems. It was crucial because culturally the teacher have had a very significant role in managing the classroom as it used to be in the conventional classroom.

Fourthly, the central role of the teaching process was in the interactive discussion between the teacher and pupils and among the pupils as a whole class, individual, and in small group. The social interaction allowed the pupils to learn from the teacher and form other peers. It was significantly crucial considering the opportunity for the pupils to provoke the intended cognitive changes in learning the subjects.

As noted earlier, each day of teaching process of the subjects had an opening problem. The problem was encountered in the whole class. Each student read the problem (or the teacher read it out-loudly) and the teacher guided them to answer the problem using the reflective questions, such as: "What is the problem about?", "What do the numbers stand for?", "What is the question about?". The pupils were asked to solve the problem individually and later on they shared it with their peers as a small-group work. These activities were followed by a whole-class discussion, in which the answers of several pupils written on the blackboard and their comments toward the answers were collected, evaluated and analyzed. In addition the pupils were asked the following reflective questions: "What difficulties did you encounter when solving this problem?", "On what points did you disagree?", and "What did you learn from solving this problem?" (Verschaffel & De Corte, 1997). The contextual problems each day were encountered by the pupils in the same teaching approach in order to pursue the intended learning outcomes of the day (see the learning trajectory of the subjects in section 9.4).

Since the teachers had few experiences in managing the discussion in their previous teaching approach, the teacher did not make use of the questions proposed in the RME prototype. Moreover, their eagerness to manage the classroom quietly resulted in the teacher directed the pupils to work individually rather than in-groups. Finally, the process of learning in interactive discussion occurred several occasions. However, to some extent the teacher improved the teaching and learning process in the following session of introducing problem soon after having a small discussion with the researcher. The improvement significantly occurred after two days of the teacher conducting the teaching process. The teacher managed discussion interactively in which pupils presented their opinions and solutions and chose the best solution they understood. From the interview with the teacher, the study found that the need of having experiences in the actual conditions in the first place and having guidance from an expert would be an essential part of effectively conducting the RME approach in the classroom. This conclusion became the important aspect to be put into consideration.

The main questions the pupils asked in encountering the problem were: "*What should I do with this problem? Should I multiply them or add them all? What kind of operation should I use to solve the problem, multiplication or addition or division?*" It was certainly a prototype of pupils' attitude that always learnt mathematics in a conventional approach where the teacher played as a key figure in transferring the knowledge. Encountering these questions, rather than asking questions to other pupils or asking to read the problem carefully, the teacher in the conventional approach answered the questions directly. It was a type of hesitatingly resistant to the idea of playing the key figure in the classroom. However, there was a significant improvement of this condition. Having several times of familiarizing and experiencing the RME prototype the teacher grasped the idea of providing the pupils the opportunity to develop and enhance their understanding and knowledge by themselves. The essential effect was that whenever the pupils actively involved in the learning process then they had confidence in learning the subjects and in discussing the solutions as well as the mathematical reasons.

c. Socio-mathematical norms

Socio-mathematics norms referred to expected ways of explaining and acting in whole-class discussions that are specific to mathematics (Gravemeijer & Cobb, 2001). It included a different, sophisticated, and efficient mathematical solution, and also an acceptable mathematical explanation and justification. Pupils' personal beliefs and their personal way of judging whether a solution is different, sophisticated, or efficient are being continually structured by the negotiation and the discussion occurred in the classroom. The norms in this study were the efficient mathematical (multiplication and division) procedures the pupils used in encountering the contextual problems. The pupils had freedom to choose and apply a strategy they thought was understandable for them.

In establishing socio-mathematical norms in the classroom the teacher acted as a facilitator, in which the teacher asked pupils to interpret, to choose the best solution and to explain their reasons of choosing. The norms were based on the following ideas:

- solutions did not come from the teacher, but from the discussions
- no one perfect strategy to find solution
- no restriction with only one strategy
- the best solution was the most understood strategy that gave correct answers.

The teacher and pupils discussed these norms along with the learning activities. Based on these discussions the pupils built concepts, perceptions, ideas and beliefs about the learning process and also the mathematical understanding.

The RME teaching and learning process of multiplication and division started with the introduction of contextual problems. The problem contexts facilitated pupils to build various informal or formal mathematics forms bridging the contextual problems and the mathematics problem. For instance, in the case of teaching multiplication three problems given in the first day of teaching were aimed at reinventing the repeated addition (the problems can be seen on the Pupils' book page 8). Encountering these problems, pupils discovered various valid repeated addition strategies, namely repeated addition of 5-numbers or 10-numbers.

During the reinvention process, discussions among the pupils and with the teacher encompassed the route of finding the strategies and solutions. In this situation the teacher played a role as facilitator of discussion and analyzing correct and incorrect strategy applied by the pupils. The last role was significantly different than that of what was suggested by Gravemeijer and Cobb (2001). They argued that in building mathematical norms in the classroom the teacher does not outline specific guidelines for what solutions are acceptable. In this manner the researcher found out that the teacher still resisted to the idea of being a key figure in the learning process.

After finding the solutions using strategies, then discussion, negotiation, and explanations were carried out. It was aimed at facilitating pupils to compare and find the efficient and understandable solution for their own. During this discussion they realized that the repeated addition of ten numbers consecutively was the most efficient way of finding solution.

This stage also found that pupils' pre-requisite knowledge, such as multiplication facts had a significant influence on establishing mathematical norms in the classroom learning culture. Whenever difficulties occurred or a mistake took place in the discussion, pupils with strong background knowledge would give their opinion and reason right away. It accelerated the discussion. On the contrary, few interactive discussions occurred. This weakness made pupils unsuccessful to learn one strategy to another. Therefore they mastered only one strategy of solving problems. These pupils struggled to master other strategies and they failed to apply them correctly in solving the problems (see section 7.2.2 item c).

These pupils could be divided into two types. *First*, some pupils were still in the first level of thinking, in which they could manipulate the known characteristics of the multiplication pattern that were familiar with them, for instance the repeated addition of numbers. But at the same time the pupils could not manipulate the interrelatedness of the multiplication concepts from the repeated additions of numbers. *Second*, some pupils were still in the second level of thinking in which they understood the interrelatedness of the multiplication and addition concepts but could not manipulate the intrinsic characteristics of the multiplication operations. The interviews showed that the pupils had difficulties in understanding the attributes (traits) of the multiplication operations. For these pupils the need of addition learning time or remedial learning to develop their capability of mastering the multiplication facts and to grasp the intrinsic characteristics of the multiplication operations is very essential. The time prepared for the opening lesson proposed in the RME prototype was not enough to increase the pupils' ability in mastering the multiplication facts.

d. Concluding summary

This third stage of prototyping phase found that the teachers were properly able to introduce the contextual problems to the pupils in the learning process. Even so the teachers needed to practice more various activities of introducing the problems, such as asking the pupils (individually or together) to read the problems loudly. This activity will help the pupils to practice their reading and listening ability.

The teachers could also conduct the interactive teaching model in the classroom properly as being proposed in the RME prototype. The main weakness was in accelerating the discussion process. The teachers did not make use of the questions proposed in the RME prototype. They needed more time and guidance to go through the activities in the RME actual classroom approach.

The teachers were able to establish socio-mathematical norms in the classroom. However they should maintain the conditions needed to develop the climate: interactive discussions on (informal and formal) mathematics forms, mathematical strategies, and efficient mathematical solutions. The new socio-mathematical norms in the classroom culture should be established, negotiated, and redefined continually in order to attain the effective teaching and learning process.

Considering those results discussed above the phase concluded that the RME prototype was implementable as intended in the Indonesian primary schools for teaching multiplication and division of multi-digit numbers. The proper implementation of the RME prototype had essential influence on the pupils' performances in learning the subject. It is illustrated in the following section.

7.2.2 Initial effectiveness of the RME prototype

The initial effectiveness of the RME prototype was established if the pupils correctly (a) solved the contextual problems in the quizzes (learning progress), (b) performed in the expected level of understanding, and (c) obtained better achievement. Each of these points will be discussed in the following section.

a. Pupils' learning progress

Pupils' learning progress referred to the correct solutions the pupils obtained from applying valid strategies (multiplication and division procedures) to solve the daily and weekly quiz items. Pupils were allowed to choose a strategy that they understood most. The contextual items of 2 and 3 in each subject were prepared to examine pupils' learning progress in daily quiz and in weekly quiz respectively. The level of reliability and difficulty showed that the quizzes were good enough to examine the pupils' learning progress (see section 5.4.1). The results of pupils' learning progress in daily quiz are illustrated in the following table.

Table 7.4
Pupils' correct answers in the quizzes

	Daily quiz items						**Weekly quiz items**			
	Multiplication			*Division*			*Multiplication*		*Division*	
Schools	*1*	*2*	*3*	*1*	*2*	*3*	*1*	*2*	*1*	*2*
SDN 101746	7	5	7	8	10	11	6	15	6	8
SDN 101748	9	6	8	9	7	8	8	10	9	8
SDN Rejodadi	10	13	20	10	12	14	11	18	16	12
SDN Sonosewu	20	20	27	15	23	24	18	24	26	23
Overall (%)	46	44	62	42	52	57	43	68	57	51
n = 94[a]	49%	47%	66%	45%	55%	61%	46%	72%	61%	54%

Note: [a]Overall numbers of pupils. Numbers of pupils in each school can be seen in Table 4.3 section 4.3.3.

The table above shows that in overall the pupils progressed significantly after the third day of daily quiz (66% and 61%) in both subjects (multiplication and division of multi-digit numbers). In fact the table also shows that in the first and second day of learning multiplication and in the first day of learning division less than 50% of the pupils answered the items correctly. But in the second day of learning division, 55% of the pupils encountered the items correctly.

These results would be an indication that in the first day the pupils still adjusted with the learning circumstances (in the RME approach) that was very different from the one applied by their teacher before. Even so the data showed that 20 pupils of SDN Sonosewu (61% of 33 pupils) in the first and second day and 27 pupils (82%) in the third day got correct multiplication answers. In learning division, the smallest number of pupils who got correct answers occurred in the SDN 101746 and SDN 101748 Medan. The study found out several pupils' weaknesses: the careless addition of the numbers (see the left picture of Figure 7.1 below) and lack of multiplication of 1-digit numbers (see the right picture of the following figure).

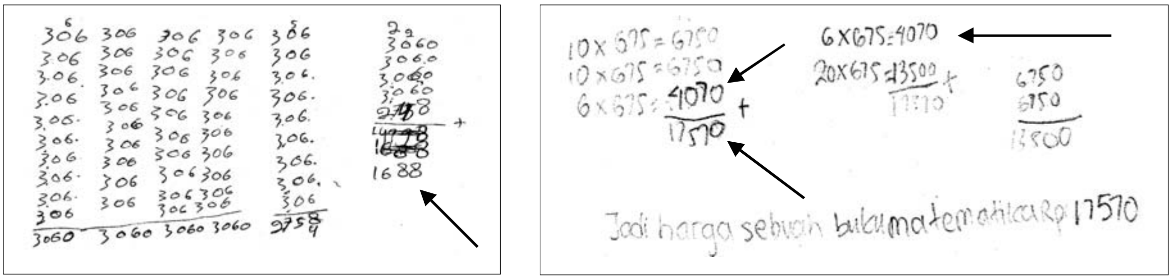
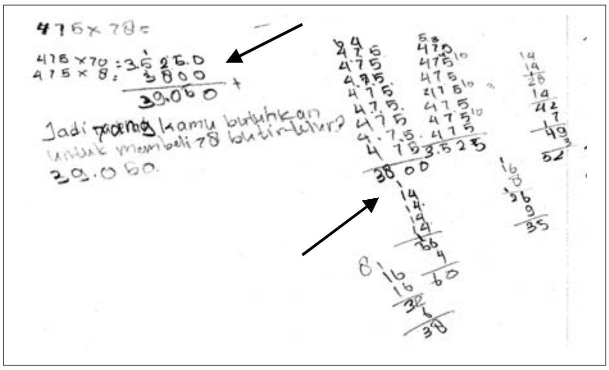


Figure 7.1
Pupils' mistakes in adding solution and multiplying numbers in daily quiz (pointed by author)

In weekly quiz, 43 pupils (46%) and 68 pupils (72%) gave the correct answer to the item 1 and 2 of the multiplication items respectively. Meanwhile in the division problems 57 (61%) and 51 (54%) pupils made the correct solutions to the item 1 and 2 respectively. It can be concluded that in average more than 58% of the pupils solve the weekly items correctly. It can also be seen that most pupils still had difficulty in solving item 1 of the multiplication. The reason was that pupils had difficulty in multiplying numbers of 8 x 475. The left picture of the figure below was an example of the errors. In solving item 2 of the division, most pupils incorrectly made an insubstantial mistake that was having the calculation correctly but did not put the answer into account. The right picture in the figure below was an example of the errors. Both mistakes were called the substance stage (S) where the pupils demonstrated sufficient detail of rational solution but major error (see Table 4.8 in section 4.5.2) obstructed the correct solution process.



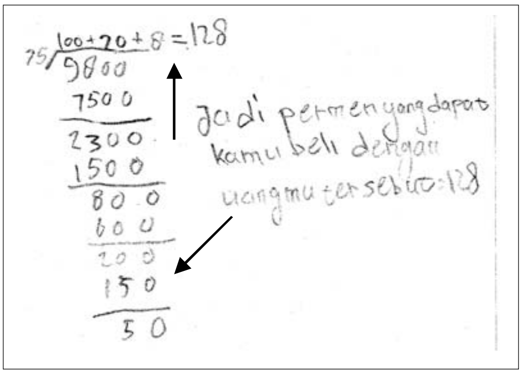


Figure 7.2
Errors in adding numbers and forget to include the last subtraction (pointed by author)

b. Pupils' level of understanding

In this study pupils' level of understanding referred to the solution stage (from Malone, et al., 1989; see Table 4.7 in section 4.5.2) the pupils attained in solving the daily quiz items. The data is illustrated in the following table.

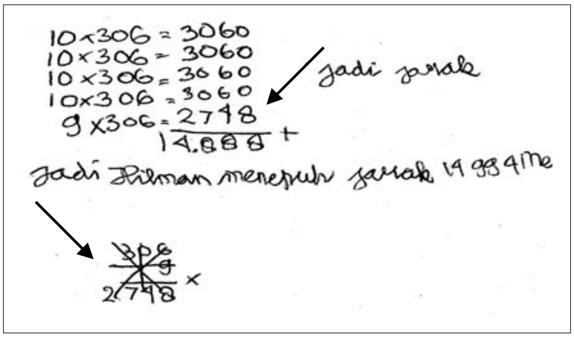
Table 7.5
Pupils' level of understanding in daily quiz for n = 94[a]

Level of understanding[b]	Multiplication items *1*	Multiplication items *2*	Multiplication items *3*	Division items *1*	Division items *2*	Division items *3*
Non-commencement	1	1	1	3	1	1
Approach	8	9	3	7	2	4
Substance	36	35	20	27	34	23
Result	3	5	8	15	5	9
Completion	46 (49%)	44 (47%)	62 (66%)	42 (45%)	52 (55%)	57 (61%)

Note: [a] Overall numbers of pupils; [b]The meaning of each level of understanding can be seen in Table 4.7 section 4.5.2.

The data in the table above show that most pupils were in the *substance level* of understanding (see the numbers in the *substance* row). It meant that the pupils applied sufficient detail of strategy that demonstrated their proceeding toward a rational solution, but a major error or misinterpretation obstructed the correct solution process. In the case of multiplication for instance, the major error found was whenever the pupils multiplied 1-digit numbers with multi-digit numbers (see Figure 7.3 below). From the figure it can be concluded that pupils understand how to use the strategy of multiplication by 10 but they conducted the multiplication of

306 by 9 mistakenly (see the arrow in the left picture). In doing the division using the strategy of limited structured repeated subtraction, they understood the route of procedures but they chose a number (40) that did not suit the numbers being divided (140; see the arrow in the right picture) and the multiplication result of 40 x 24 was not correct as well. These pupils still had lacked in multiplying numbers.



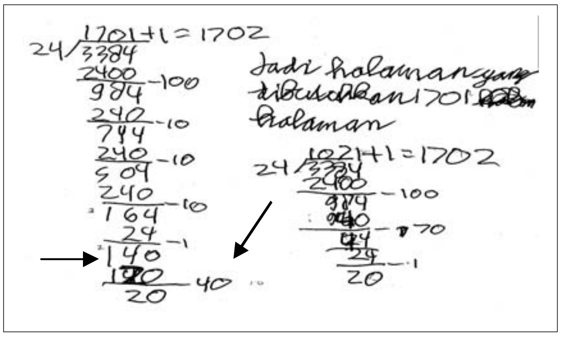


Figure 7.3
The pupils' incorrect multiplication (pointed by author)

From table 7.5 three important results could be summarized. *Firstly*, most pupils approached the problem with meaningful work indicating their understanding on the problem both in multiplication and in division. Only a student was unable to start answering the problem in each day. *Secondly*, there was a decreasing amount of pupils that conducted the major error toward the valid solution (from 36 to 20 pupils in multiplication and from 34 to 23 pupils in division). *Thirdly*, 10% of pupils reached the result level of solution stage (8 in multiplication and 9 in division on the third day). This meant that these pupils nearly solved the problem but a minor error produced an invalid final solution. It was believed that the pupils understood and were capable of solving the problem.

c. Pupils' achievement

The pupils' achievement referred to (1) the average score the pupils obtained from solving the post-test items, (2) the level of achievement the pupils get in the post-test, and (3) the significant different between the pupils' scores in the pre-test and in the post-test. Each aspect is described subsequently in the following section.

The average scores the pupils obtained from the tests is illustrated in Table 7.6.

Table 7.6
Pupils' average score in the pre-test and the post-test

Test	**N[a]**	**M**	**SD**
Pre-test	94	16,3	6,7
Post-test	94	25,1	8,9

Note: [a] Overall numbers of pupils.

The table above shows that in the post-test the pupils' mean achievement was 25.1 and in the pre-test it was 16.3. With the mean difference of the tests was 8.8, it can be said that the pupils' score improved after engaging in the RME learning process. The post-test mean score showed that the pupils were in the middle level of achievement.

To analyze the significant difference between the pupils' scores in the pre-test and in the post-test, the phase utilized the independent t-test in the SPSS program. It was found that the mean difference between pupils' performance in the pre-test and in the post-test was significant ($t = 7.675$, $p < 0.05$). It means that there was a significant difference between pupils' achievement before and after engaging in the RME teaching and learning approach.

As a whole the pupils' level of achievement in the pre-test and post-test is illustrated in the following table.

Table 7.7
Pupils' level of achievement for n = 94[a]

Score	**Level**	**Pre-test**	**Post-test**
0 - 13	Low	28 (30%)	11 (12%)
14 - 26	Middle	56 (59%)	36 (38%)
27 - 40	High	10 (11%)	47 (50%)

Note: [a] Overall numbers of pupils.

The table showed that 50% of the pupils were in the high level of achievement in solving the post-test items. Meanwhile in the pre-test 70% of the pupils were in the same level. Considering this fact the study concluded that there was a considerable improvement on the pupils' achievement in solving the problems. The table also proved that the pupils' in the low ability level decreased from 30% in the pre-test to 12% in the post-test. Most pupils progressed to the high level of achievement, which was from 10% in the pre-test to 50% in the post-test. The shift of pupils' achievement from pre-test to post-test can be seen in the following table.

Table 7.8
Pupils' shifting achievement from pre-test to post-test

		Post-test		
Pre-test	*Low*	*Middle*	*High*	∑
Low	11 (12%)	14 (15%)	3 (3%)	28 (30%)
Middle	-	22 (24%)	34 (36%)	56 (60%)
High	-	-	10 (11%)	10 (11%)
∑	11 (12%)	36 (38%)	47 (50%)	

The table shows that 39% of the pupils (3% from the low level and 36% from the middle level) progressed to the high level of achievement. These facts showed that the pupils from the middle level got the most benefit from the RME learning process compared to the pupils from the other levels.

d. Concluding summary

In the case of initial effectiveness, it was found out that in using the RME prototype in the classroom the pupils performed in the intended level of performances. Significant day-to-day learning progress occurred during the learning process. Within lack of multiplication facts, 66% and 61% of pupils had correct answers in solving daily quiz items of multiplication and division respectively. Most pupils had good level of understanding. They could proceed the problems with meaningful work to solve the problems (74% pupils in learning multiplication and 70% in learning division). In the post-test the pupils performed in the middle level of achievement. This achievement improved significantly compared to their performance in the pre-test.

7.3 CONCLUSION: IMPROVEMENTS IN THE RME TRY-OUT VERSION

The third stage of prototyping phase was focused on analyzing the implementability and the initial effectiveness of the RME prototype for teaching multiplication and division of multi-digit numbers. Implementability referred to whether the RME prototype could be used as intended. The initial effectiveness was related to whether the RME prototype improved pupils' performances on the expected level of understanding. It was found out that the RME prototype was used as intended in teaching multiplication and division of multi-digit numbers. During the RME learning process the pupils performed in the intended level of performances. These results informed that the teachers applied the early version of the RME prototype as intended and the prototype improved pupils' performances on the intended level of achievement.

Nevertheless several weaknesses were also found during the execution of the teaching experiments in this third stage of prototyping phase. In the case of implementability the teachers needed to practice various ways in introducing the problems. And they did not make use of the questions proposed in the RME prototype. They also could not establish, negotiate, and redefine the socio-mathematical norms properly because of their resistance to the idea of being a key figure in the classroom. In the case of effectiveness, pupils' lack of multiplication facts and addition of multi-digit numbers had obstructed their high performances. Having good level of understanding and proceeding problems with meaningful work to solve the problems did not guarantee that pupils would get the correct valid answers.

Considering those results the researcher believed that teachers and pupils needed more time to (1) get accustomed to the RME approach, (2) do the reinvention activities, and (3) improve those pre-requisites in order to engage actively in learning the multiplication of multi-digit numbers realistically.

During the reflection sessions of the third stage of prototyping phase, the early version of the RME prototype had been improved (the revision version was called the try-out version). From the reflection session several decisions were reached. *Firstly*, there was a need to conduct group discussions before and during the RME instruction process in the classroom to familiarize the RME approach to the teachers. *Secondly*, attention should be given closely on the two activities before and after the learning process in the try-out version: (1) preliminary game (manipulating numbers before the learning process began) and (2) homework (manipulating numbers after the learning process ended). These activities were aimed at attracting pupils' attention, motivation as well as their former knowledge and also practicing their understanding.

Thirdly, during the learning activities, it was found out in solving items in the daily and weekly quizzes (see Table 7.5 section 7.2.2 item b), some pupils (10%) made errors in interpreting the meaning of the "remainder" from the division calculation. The reason was that the pupils did not compare the mathematical solution with the problem situation. In order to give chances for pupils to deal with such problem, the RME try-out materials provide several similar items in the learning process of division. The following was an example of the items for giving more experiences dealing with the remainder.

I. Buying candies

9. A candy costs Rp. 75. How many candies do you get if you have Rp. 20000?

Fourthly, analyzing the quality of the tests showed that some improvements should be done in the items involved. The need of parallelism between the pre-test and the post-test made the researcher improved the items. Focusing on the conventional problems (CV), the researcher revised the conventional items of the pre-test and for the post-test (2 items in each subject - multiplication and division). The RME experts evaluated and justified the equivalence of the items regarding the numbers involved and its level of the difficulty.

The improvement also focused on the equivalence of the contextual problems. The judgement of equivalent items in both tests was based on the contexts employed, the numbers involved, and the pupils' familiarity toward the context. In both tests the materials involved contexts that were familiar to pupils and the Indonesian expert judged that it represented the Indonesian circumstances. The RME experts found out that the numbers involved were not significantly different based on its difficulty level.

Fifthly, the improvement of daily quiz items was related to the numbers involved in the items. Considering the need of analysing pupils' learning progress, the items should have a good quality to measure their performance. From the pupils' answers, the study found that the first item of division problem in the first day (**The Lebaran Day**) did not give enough information about the learning progress. the researcher then involved numbers that were realistic enough within the contexts. In this case the change was from the number of 4000 to 1400. In the multiplication items in the first day the change of numbers involved was from 604 to 204 in the **Skilful Mason** problem. The RME experts judged these changes as significant improvement for the learning trajectory.

The sixth improvement was related to the appraisal that was used to examine the implementability of the RME prototype. The RME experts suggested employing a classroom observation that would give more information about the learning activities held by the teachers. Relying on the experts' proposition, the researcher then adopted the Classroom Observation Form from Thijs (1999). The study employed the items that were related to the learner-centered orientation (see Teachers' Guide page 49). The main reasons were: (a) this classroom observation form had been tried-out several times to assure the reliability of the instruments; (b) this form designed for the learner-centered instruction process that was similar to the RME instruction approach; and (c) its quality had been judged by several experts in science education (including mathematics). Utilizing the RME experts in analyzing the use of the form, some revision and improvement had been made. The form was then called the teaching profile checklist (see section 4.4). This checklist was used in the assessment phase. The phase is illustrated in the next chapter.

CHAPTER 8
ASSESSMENT PHASE: TOWARD THE ULTIMATE RME PROTOTYPE

Chapter 5, 6, and 7 illustrate the stages of prototyping phase and its results. Conducting the cyclic process of front-end analysis, teaching experiments, and reflections to the instructional sequences, the prototyping phase resulted in a try-out version of the RME prototype. It was found out out that the RME prototype was implementable as intended (with several inappropriate learning activities) and pupils performed on the expected level of understanding. This chapter describes the teaching experiments using the RME try-out version in the assessment phase. The phase evaluated whether the teachers used the RME prototype as intended (implementability) and whether the RME prototype improved pupils' performances (effectiveness). This phase built a theory of conjectured RME local instructional sequences and structured the ultimate RME prototype for teaching multiplication and division of multi-digit numbers in Indonesian primary schools.

8.1 RESEARCH DESIGN

This phase was aimed at justifying the implementability and the effectiveness of the RME prototype for teaching multiplication and division of multi-digit numbers. It referred to whether the prototype was applied as intended (implementability) and whether the pupils performed on the intended level of achievement (effectiveness). Focusing on the consistence of the intended curriculum and the attained curriculum, the phase was led by the following sub-research question:

To what extent was the RME prototype implementable and effective for teaching multiplication and division of multi-digit numbers in Indonesian primary schools?

To address the research question given above this assessment phase conducted the teaching experiments in Medan, Indonesia (see Figure 4.2 section 4.2.3). Using the try-out version of the RME prototype, the teaching experiments were held in eight

(8) primary schools chosen as the experimental groups. All schools were chosen using purposive random sampling considering several reasons: (1) the teachers' willingness of applying the RME prototype in their classroom, (2) the teachers' qualification (novice, moderate, and experienced teachers), and (3) the schools' location area. As the control group, other eight primary schools were chosen randomly (see Table 4.3 in section 4.4). During the pretest, it was found out out that there was no significant differences on the pupils' achievement of the experiment group and those of the control one. The pupils from both groups were homogeneous in their performances before the teaching experiments started.

In teaching multiplication and division of multi-digit numbers, it was found out out that at the beginning the teachers believed that the mechanistic teaching (teaching by telling) was effective enough to improve pupils' understanding and achievement. These implied that (1) the pupils had frequently learnt mathematics with the traditional standard word problems and (2) the realistic problem situation was involved at the end of the learning process.

291 pupils were engaged in the RME learning process and 310 pupils learnt the subjects conventionally. These pupils age of 10-11 years varied in their capability, performance, gender, and socio-backgrounds. Most pupils were coming from the families of low socio-economic background in a rural plantation area. Their parents work as lower-paid temporary workers or manual-laborers. 88% of the pupils had no calculator in their home and only 14% of the pupils had their own room and table in the house. None of them had computer in their house.

In this phase the data were collected by using variety of appraisals (teachers' logbook, teaching profile checklist, pupils' portfolios, interviews, quizzes, and tests) and gathered from variety of individuals as evaluators and sources of data (experts, university pupils, teachers, and pupils). Logbook and checklist were used to address the implementability of the RME prototype in the classroom. Interviews with teachers were conducted during the learning activities to identify the teachers' reasons when they dealt with the problems that occurred in the classroom. The interviews were also held with pupils with different ability to analyze their understanding of the subjects. Subsequently pupils' portfolios were collected to analyze their weaknesses and their reinvented procedures. During the learning activities each day the daily quiz was given to pupils to analyze their learning progress. The weekly quiz was also given to examine pupils' achievement after learning the subject each week. Before the learning activities began, pretest was administered to find out pupils' instinctive informal (or formal) mathematics forms representing pupils' prerequisite knowledge. Then posttest was held at the end of the two-week learning activities that was aimed at analyzing the pupils' achievement. In the first place, the appraisals and individuals involved in this phase were to ensure the quality of the data in order to come into reasonable results. The results of this phase will be described in section 8.2 below. Utilizing various sources (experts, teachers, and pupils), different observers, and variety of appraisals determined the objectivity of the study. This triangulation (data, observer, and methods) assured the quality of data and provided quality control of this study. The following table is an overview of the instruments utilized in each quality aspects.

Table 8.1
Overview of instruments used in the assessment phase

	Quality aspects and its criteria					
	Implementability			**Effectiveness**		
Instruments	*The use of contextual problems*	*Interactive teaching*	*Socio-mathematics norms*	*Learning progress*	*Level of under-standing*	*Achieve-ment*
Questionnaires	√	√	√			
Logbooks	√	√	√			
Checklist	√	√	√			
Interviews	√	√	√	√	√	√
Portfolios				√	√	
Quizzes				√	√	
Tests						√

The data from those appraisals were analyzed quantitatively and qualitatively. Descriptive analysis instruments such as mean and standard deviation (sd) was used to describe the data from the tests and the quizzes. Data from logbooks and interviews were transcribed and re-coded qualitatively. Data from checklist and portfolios were tabulated and its frequency was analyzed descriptively. All these analysis aspects have been illustrated in section 4.5.2.

8.2 RESULTS

8.2.1 Implementability of the RME prototype

Implementability was defined as the degree to which the RME prototype was applied as intended. This phase determined that the implementability of the RME prototype was established if the teacher: (a) introduced contextual problems as intended; (b) conducted an interactive teaching approach; and (c) established socio-mathematical norms. Each aspect will be described in the following section.

a. Introducing contextual problems

In the RME approach the contexts involved in the problems were supposed to help pupils to develop their own understanding. The contexts attracted pupils to utilize their former mathematics knowledge to mathematize the problems and to structure mathematics forms until they found out solutions. The contexts also asked teachers to manage the learning process differently from that they used to practice in the classroom. introducing the problems, making them understood by the pupils, asking questions, inviting ideas, giving hints, and encouraging pupils to ask questions were several introduction activities the teachers should do to facilitate the learning process. The level of decency of these activities was observed to judge the implementability of the RME prototype. From the teaching profile observation it was found out out that in overall the mean score was 3.7 (see cumulative tabulation in the mean column in Table 8.2 below). Interpreting this mean score to the teachers' level of conducting the RME learning activities (Table 4.7 in section 4.5.2), it meant that the multiplication introduction activities were conducted in the range of fairly and good by the teachers. It can also be seen that the teacher introduced the contextual problems as intended (with the mean score was 4.1; see item 1 in the following Table 8.2).

Table 8.2
Introduction activities in learning multiplication

	Chosen options, n = 44[a]					
Items	*Very poor 1*	*2*	*3*	*4*	*Very good 5*	*Mean[b]*
1. Teacher introduces and formulates the problems			7	24	13	*4.1*
2. Teacher asks guided questions to introduce the lesson			16	24	4	*3.7*
3. Teacher asks pupils for their own idea		3	16	20	5	*3.6*
4. Teacher responds to pupils ideas		6	29	9		*3.1*
5. Teacher encourages pupils to ask questions			21	19	4	*3.6*
6. Teacher guides the pupils to the conclusion			12	23	9	*3.9*
Cumulative tabulation for N = 264	-	9	101	119	35	3.7

Note: [a] Numbers of observations held; [b]The mean scores that were compared to the teachers' level of conducting the RME activities in Table 4.7 section 4.5.2.

After having experiences in the RME approach, the teachers introduced the division problems fairly (the cumulative mean was 3.6, see Table 8.3 below). The introduction activities were conducted fairly easy by asking the pupils to read the opening problem individually or together. The teachers also guided them by asking such questions: what their idea of problems, whether they understand or not, and what their idea of the numbers involved. They facilitated pupils with drawing pictures or by giving an example with lesser numbers involved.

Table 8.3
Introduction activities in learning division

	Chosen options, n = 44[a]					
Items	*Very poor 1*	*2*	*3*	*4*	*Very good 5*	*Mean[b]*
1. Teacher introduces and formulates the problems			9	27	8	*3.5*
2. Teacher asks guided questions to introduce the lesson			19	25		*3.6*
3. Teacher asks pupils for their own idea		7	17	20		*3.3*
4. Teacher responds to pupils ideas		3	17	24		*3.5*
5. Teacher encourages pupils to ask questions		5	12	22	5	*3.6*
6. Teacher guides the pupils to the conclusion		4	5	27	8	*3.9*
Cumulative tabulation for N = 264	-	19	79	145	21	3.6

Note: [a] Numbers of observations held; [b]The mean scores that were compared to the teachers' level of conducting the RME activities in Table 4.7 section 4.5.2.

During the division introductory activities the teacher found out the difficulty of making pupils understood the contextual problems. The reason was that most pupils did not make use of the contexts as well as the numbers involved. In facts, as it used to be, the hints from the teacher, for instance saying that the problem was a division problem made them apply the standard procedure. Certainly they did not know how to work on the division operation because the teacher did not teach it yet. In these circumstances, teachers' guidance was the most constructive help the pupils needed. Rather than giving the hint mentioned above, guidance such as reading problems carefully, finding the relations of the numbers, discussing problem conditions, and drawing pictures would encourage pupils to formulate the problems. In this manner, the teachers had an essential role in describing the problem situation, re-explaining or re-summarizing the mathematical forms the pupils reinvented. These roles will be explored more in the next section.

b. Conducting interactive teaching approach

In the RME approach interactive teaching process involves opportunies for discussion in a small group or as a whole class (Streefland, 1990). In the discussion explicit negotiation, intervention, cooperation, and reflection are essential elements for building a constructive learning process in which the pupils' informal methods are used as a lever to attain the formal ones. During the discussion the pupils explore the problems in small groups or individually, make use of their former knowledge, and discuss the mathematics tools and strategies employed in solving the problems. Meanwhile teachers are asked to facilitate discussions by giving opportunities to choose their own strategies, focusing their attention on crucial aspects, interacting with pupils, assisting and guiding pupils with hints and questions. The teachers also encourage pupils to discuss the strategies and to draw their own conclusion.

The study found out out that the instructional activities in multiplication were conducted as intended (the mean was 3.5, see the cumulative tabulation in the mean column of Table 8.4 below). During the instruction activities, the teachers accelerated pupils' mathematizing process on the crucial aspects; such as doing multiplication of 1-digit numbers and adding numbers consecutively. The teachers interacted and assisted pupils with guiding questions such as "*How do you find it?*", "*read the problem carefully*" and "*what the numbers stands for?*". The pupils also justified the solutions and compared each other answers interactively. For instance, in the first day of learning multiplication pupils developed various models of repeated addition. Having these strategies then the teacher asked pupils to wrote down several strategies on the blackboard and discussed the strategies and its arithmetic tools. In this discussion, pupils analyze which strategy was the most efficient and understandable for them. To solve another problems, the teachers allowed pupils to choose the strategy they were comfortable with. It represented the development of mathematics norms in which the pupils discussed, learnt and reinvented the multiplication procedures actively (the detail will be explored in section 9.4.1).

On the other hand all items in the instruction activities were conducted fairly well (the mean scores were in the range of 3.1 – 4.0). The teachers gave opportunity for pupil to explore the problem in-group or individually, make use of their former knowledge, and discus mathematics strategies. However, the teachers had to be more active in encouraging pupils to discuss the problem situation as well as the strategies used with their peers (see item 9 and 4 in Table 8.4 below). It can be concluded that the teachers accelerated the discussion process fairly. To overcome pupils' dependency, encouragement are needed to motivate pupils' discussion.

Table 8.4
Instruction activities in learning multiplication

Items	Chosen options, n = 44[a]					
	Very poor 1	*2*	*3*	*4*	*Very good 5*	*Mean[b]*
1. Pupils explore the problems in the groups or individually			25	19		*3.4*
2. Teacher allows pupils to choose their own approach		3	15	19	7	*3.7*
3. Pupils actively make use of their mathematics knowledge		3	25	16		*3.3*
4. Pupils discuss the operation employed in the problems		6	26	12		*3.1*
5. Teacher focuses pupils attention on crucial aspects of mathematics			15	22	7	*3.8*
6. Teacher interacts with pupils during the activity		4	14	23	3	*3.6*
7. Teacher assists pupils when necessary			18	20	6	*3.7*
8. Teacher asks pupils guiding questions, but does not provide outcomes			14	25	5	*3.8*
9. Teacher encourages pupils to discuss with peers		7	30	7		*3.0*
10. Teacher allows learners to draw own conclusions		5	13	22	4	*3.6*
Cumulative tabulation for N = 440	-	28	195	185	32	3.5

Note: [a] Numbers of observations held; [b]The mean scores that were compared to the teachers' level of conducting the RME activities in Table 4.7 section 4.5.2.

In learning division, the teachers gradually improved their performances in conducting the instruction activities. It was found out out that in overall the mean score was 3.6, meaning that the instructional activities were conducted fairly well (see Table 8.5 below). The improvements were in all items, except in the teachers' asking guiding questions, but do not provide outcomes (compare the mean score given in Table 8.4 and 8.5 of each item mentioned). These improvements indicated that having experiences of conducting the RME activities in the first place (in multiplication) had a positive impact on the teachers' performances in the next activities. This meant that teachers needed practical and actual experiences to develop their ability in conducting better instructional process.

Table 8.5
Instructional activities in learning division

Items	Chosen options, n = 44[a]					
	Very poor 1	2	3	4	*Very good* 5	*Mean[b]*
1. Pupils explore the problems in the groups or individually			22	22		*3.5*
2. Teacher allows pupils to choose their own approach		5	7	25	7	*3.8*
3. Pupils actively make use of their mathematics knowledge			27	17		*3.4*
4. Pupils discuss the operation employed in the problems			24	20		*3.5*
5. Teacher focuses pupils attention on crucial aspects of math			11	26	7	*3.9*
6. Teacher interacts with pupils during the activity			20	16	8	*3.7*
7. Teacher assists pupils when necessary			18	21	5	*3.7*
8. Teacher asks pupils guiding questions, but does not provide outcomes			25	19		*3.4*
9. Teacher encourages pupils to discuss with peers		5	20	19		*3.3*
10. Teacher allows learners to draw own conclusions			15	22	7	*3.8*
Cumulative tabulation for N = 440	-	10	189	207	34	3.6

Note: [a] Numbers of observations held; [b]The mean scores that were compared to the teachers' level of conducting the RME activities in Table 4.7 section 4.5.2.

c. Establishing socio-mathematical norms

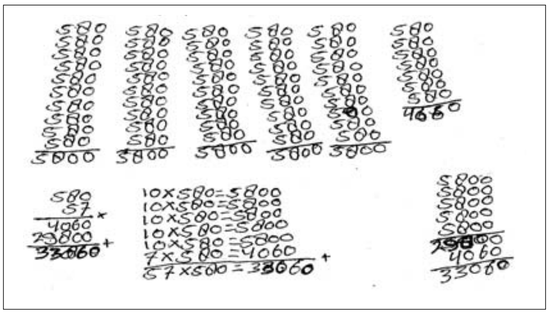
In the RME approach socio-mathematics norms are related to the intended ways of explaining and acting in whole-class discussions that specific to mathematics (Gravemeijer & Cobb, 2001). It included a different, sophisticated, and efficient mathematical solution, and also an acceptable mathematical explanation and justification. In this phase the norms were valid multiplication and division strategies pupils reinvented during the learning process (see Figure 8.1 and 8.2 below). Pupils' beliefs and their way of judging whether a valid solution is different or efficient were being continually structured through the negotiation and the discussion occurred in the classroom.

The mathematical norms in teaching multiplication and division were established started with describing, formulating and discussing the mathematical strategies they found out during the learning activities. They compared and discovered relations and regularities in mathematical strategies. This was a process of refining the mathematics understanding of the informal and formal strategies they used in solving problems (Gravemeijer, 1994). They discussed the mathematical tools that were used in the strategies. They represented the strategies, proved regularities in the strategies, refined and adjusted the strategies, and then they chose and used it in solving other problems.

The three problems in the first day of teaching multiplication were focused on motivating pupils to use the repeated addition of ten consecutive numbers. They represented the contextual problems in which multiplication concepts and operations were embedded. The problems could be solved using various models of repeated additions. For instance, in answering the "Tiles" problem pupils used repeated addition of 14 numbers consecutively. The discussion with other pupils and with the teacher guided pupils to compare, ask, and discuss the operations and finally they found out that the repeated addition of ten numbers was the most efficient way of solving the problems. They used this strategy to encounter the first daily quiz item (see example in section 9.4.1 item a).

In the second day the intention was in pupils' developing their understanding of multiplication by 10. They encountered the "Books" problem. Because they already learnt the repeated addition strategies in the first day, they applied it to solve the problem. After having the solution the teacher guided them to understand the other strategy; the multiplication by 10 (see citation in section 9.4.1 item b). It can be seen that during the learning process the teachers took in charge of the learning route. They asked questions, reminded of their former understanding, and discussed the mathematical tools for guiding pupils toward the shifting process from the repeated additions to the multiplication by 10. However, the teachers hardly encouraged pupils to comment on the strategies being learnt and its discrepancies. For some pupils these encouragement were needed because they still had conventional learning attitudes such as being afraid of making mistakes and made a fool out of by their friends.

Focusing on the multiplication by multiples of ten in the third day, the learning process started at encountering the "Water" problems. Having learnt the multiplication by 10, most pupils used it to solve the problems. Some of them applied the repeated addition. However, the teacher did not asked the pupils to compare these two strategies. The teacher assumed that pupils have already understood the discrepancies. Then guided by the teacher the discussion began toward the multiplication by multiples of ten (see excerpts in section 9.4.1 item c). During the learning process the pupils were using three multiplication strategies. It indicates that the pupils made their own choice of the procedures (observe the two pictures in Table 8.1). In many cases most pupils who still had difficulties in understanding those strategies applied only one strategy.



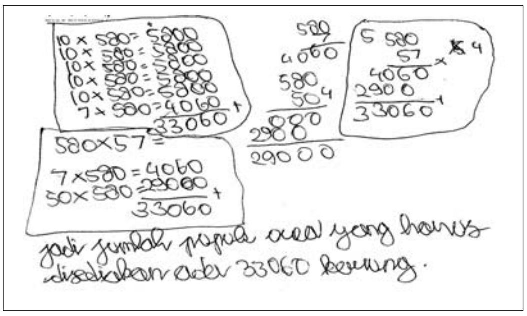


Figure 8.1
Pupils' valid multiplication solutions on the third day

Having similar approach in learning division, the pupils used the unstructured repeated subtraction in the first day, limited structured of repeated subtraction in the second day, structured repeated subtraction in the third day, and standard division algorithm in the fourth day. These reinvented strategies were the outcomes of utilizing the multiplication by 10 or 100, multiplication by multiples of ten, the standard multiplication, and random table of multiplication. They applied these strategies in solving items in the quizzes and the tests. The following figure indicates the examples of the strategies the pupils reinvented in learning division.

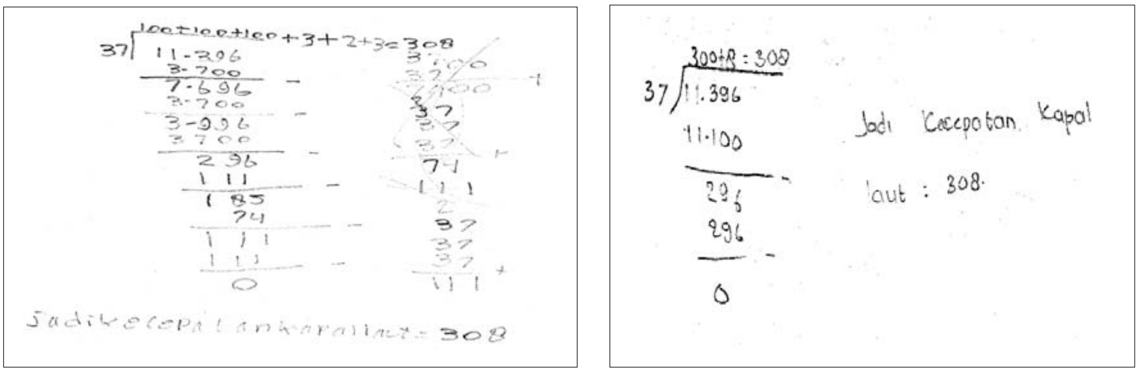


Figure 8.2
The unstructured and structured division algorithm

In learning division of multi-digit numbers the study found out that several prerequisites were needed to actively engage in the learning process: (1) memorizing multiplication facts; (2) multiplying 2-digit numbers; and (3) subtracting numbers. Even though these prerequisites were learnt and practiced in the previous lessons, some pupils still had many difficulties in doing the operations. These prerequisites became absolutely significant whenever they mathematized and treated the problems using mathematics tools to find the solutions. The better they understood the multiplication facts the more they actively engaged in the classroom. On the contrary, little interactive discussion occurred whenever pupils still had difficulties in manipulating numbers (for instance, using fingers to add the numbers). They failed in advancing their learning from one step to another. Therefore they mastered only one strategy of solving problems and applied it in solving the problems. Although some pupils tried very hard to master other strategies they failed to apply them correctly in solving problems. These pupils needed more time to practice their understanding.

To engage pupils actively in the learning process the RME experts suggested giving them the table of multiplication facts (Gravemeijer, 2000, in an interview concerning the pupils' ability of multiplication facts). Most teachers did not agree with this idea. They argued that using table of multiplication would distract pupils' skill of memorization. Only one teacher agreed with this idea. He mentioned that the table helped pupils to accelerate their active engagement in the learning process without distracting their understanding.
As a summary the following table illustrates how the teachers conducted regulation activities in facilitating the mathematical norms (see the following Table 8.6).

Table 8.6
Regulation activities in learning multiplication

	Chosen options, n = 44[a]					
Items	*Very poor 1*	*2*	*3*	*4*	*Very good 5*	*Mean[b]*
1. Teacher asks several groups/individual to report their results to the class		20	6	10	8	*3.1*
2. Teacher invites and encourages pupils to comment on the outcomes		22	13	9		*2.7*
3. Teacher asks critical open-ended questions regarding the outcomes		22	22			*2.5*
4. Teacher compares pupils outcomes and its discrepancies		26	5	13		*2.7*
5. Teacher guides pupils to understand discrepancies in their results		29	15			*2.3*
6. Teacher draws conclusions from the activity with the pupils		28	16			*2.4*
Cumulative tabulation for N = 264	-	147	77	32	8	2.3

Note: [a] Numbers of observations held; [b]The mean scores that were compared to the teachers' level of conducting the RME activities in Table 4.7 section 4.5.2.

From Table 4.7, it can be concluded that the multiplication regulation activities were conducted poorly (the cumulative mean was 2.3). The teachers hardly accelerated regulation activities in learning multiplication. The classroom observation indicated that most teachers did not monitor and evaluate pupils' learning progress at the end of learning process. They did not take recovery actions (asking comments, comparing outcomes and its discrepancies, and drawing conclusions) to improve pupils' understanding. In addition, they did not maintain pupils' motivation, generate the feedback, and ask pupils' doing self-assessment.

There were many reasons behind these facts. During the reflection session after the learning activities (see section 4.4) it was found out that the teachers did not realize the essential of conducting regulation activities. They thought that by asking a pupil to write his/her answer on the blackboard then other pupils could learn and compare their answers and understood the differences without teachers' guidance. In fact these regulation activities helped pupils realize what the main aspects to be understood, what mathematical tools to be applied, and how the calculation being conducted properly. Other reason related to the teachers' knowledge of pupils' cognition. In this manner they needed to understand pupils' weaknesses and how they learn the subjects (learning trajectory).

After having these reflection and discussion the teachers performed better in the next teaching division experiments. It was found out that they improved their performances in doing the regulation activities of learning division (the cumulative mean was 3.0). Most activities were conducted poorly (see the mean score of item 3, 4, 5, and 6 in Table 8.7). In conducting activities such as asking individual or groups to report and encouraging pupils to comment, the teacher performed it fairly.

Table 8.7
Regulation activities in learning division

	Chosen options, n = 44[a]					
Items	*Very poor 1*	*2*	*3*	*4*	*Very good 5*	*Mean[b]*
1. Teacher asks several groups/individual to report their results to the class		12	5	22	5	*3. 5*
2. Teacher invites and encourages pupils to comment on the outcomes		14	14	16		*3.1*
3. Teacher asks critical open-ended questions regarding the outcomes		17	19	8		*2.8*
4. Teacher compares pupils outcomes and its discrepancies		19	16	9		*2.8*
5. Teacher guides pupils to understand discrepancies in their results		21	10	13		*2.8*
6. Teacher draws conclusions from the pupils' activity		18	13	13		*2.9*
Cumulative tabulation for N = 264	-	101	77	81	5	3,0

Note: [a] Numbers of observations held; [b]The mean scores that were then compared to the teachers' level of conducting the RME activities in Table 4.7 section 4.5.2.

d. Concluding summary

Considering the three criteria of the implementability of RME prototype, the phase concluded that the teachers could introduce the contextual problems as intended in the learning process. The teachers were able to conduct the interactive teaching model in the classroom properly as being proposed in the RME prototype. The weakness was in teachers' encouraging pupils to discuss with peers. They hardly made use of the questions proposed in the RME prototype. The teachers were not able to establish the socio-mathematical climate as intended in the classroom. They conducted the regulation activities (cognitive and affective) poorly. In other words, the teachers hardly conducted the activities, reflecting on pupils' learning progress, taking recovery actions, maintaining motivation, and generating feedback. These activities considered as integral actions of maintaining the mathematical norms.

In summary, this study concluded that in general the activities in learning multiplication of multi-digit numbers were conducted poorly (the mean score was 2.9). The teacher acknowledged pupils' idea and asked open-ended questions fairly but they conducted other activities (discuss pupils' idea and summarize pupils' answers) poorly. It seemed that the classroom atmosphere had a little impact on encouraging pupils to raise and discuss their questions (see the following Table 8.8).

Table 8.8
General impression in learning multiplication

	Chosen options, n = 44[a]					
Items	*Very poor* *1*	*2*	*3*	*4*	*Very good* *5*	*Mean[b]*
1. Teacher acknowledges pupils' ideas		15	20	6	4	*3.1*
2. Teacher uses and discusses pupils' ideas		17	14	13		*2.9*
3. Teacher summaries pupils answers		20	15	8		*2.7*
4. Teacher asks open-ended questions to individual pupils		13	18	7	6	*3.1*
5. Classroom atmosphere seems to encou-rage pupils to ask and answer questions		14	19	11		*2.9*
Cumulative percentage for N = 220	-	79	86	45	10	2.9

Note: [a]Numbers of observations held; [b]The mean scores that were compared to the teachers' level of conducting the RME activities in Table 4.6 section 4.5.2.

Similar results were also found in the general impression of learning division using the RME approach which was conducted poorly (the mean was 2.9). The following Table 8.9 summarizes the results.

Table 8.9
General impression in learning division

	Chosen options, n = 44[a]					
Items	*Very poor 1*	*2*	*3*	*4*	*Very good 5*	*Mean[b]*
1. Teacher acknowledges pupils' ideas						
2. Teacher uses and discusses pupils' ideas		15	16	13		*2.7*
3. Teacher summaries pupils answers		16	13	15		*3.0*
4. Teacher asks open-ended questions to individual pupils		15	17	12		*2.9*
5. Classroom atmosphere seems to encourage pupils to ask and answer questions		14	16	14		*3.0*
Cumulative percentage for N = 220	-	72	72	66	-	2.9

Note: [a] Numbers of observations held; [b]The mean scores that were then compared to the teachers' level of conducting the RME activities in Table 4.6 section 4.5.2.

These conclusions had significant impact on the pupils' performances (learning progress, understanding, and achievement). It can be seen in the following section.

8.2.2 The effectiveness of the RME prototype

The effectiveness of the RME prototype was established if the pupils: (a) reached the intended learning progress; (b) performed in the expected level of understanding; and (c) obtained better achievement. Each point is discussed in the following section.

a. Pupils' learning progress

The intended learning progress dealt with the knowledge and the skills the pupils mastered daily after engaging in learning multiplication and division in RME approach. In this phase it was verified by the pupils' correctness in solving contextual problems of the daily and weekly quiz.

The RME prototype developed 3 items for the daily quiz in each subject and 2 items for the weekly quiz (see the items in the Teacher Guide). The pupils solved the items and its correctness can be seen in the following Table 8.10.

Table 8.10
The pupils' correct answers for the daily and weekly quiz

	Daily quiz items						**Weekly quiz items**			
	Multiplication			*Division*			*Multiplication*		*Division*	
Schools	*1*	*2*	*3*	*1*	*2*	*3*	*1*	*2*	*1*	*2*
SDN 101746	7	12	15	5	9	12	12	16	12	10
SDN 101747	4	6	9	5	6	9	10	11	10	10
SDN 101748	9	14	20	5	11	18	20	22	20	18
SDN 101749	10	15	22	12	12	17	25	23	20	18
SDN 101750	12	19	30	11	20	26	22	30	18	23
SDN 101751	17	27	31	24	22	33	23	26	32	31
SDN 101752	7	23	33	18	13	28	28	30	30	29
SDN 106153	22	25	47	15	16	37	41	46	42	35
Overall (%),	88	141	207	95	109	180	181	204	194	174
N = 291[a]	30%	48%	71%	33%	37%	62%	62%	70%	67%	60%

Note: [a] Overall numbers of pupils. Numbers of pupils in each school can be seen in Table 4.5 section 4.4.

Table 8.10 indicates that in overall the pupils progressed significantly in the third day of learning. 71% And 62% of the pupils solved correctly the daily items. This progress in the third day was beyond the teachers' predictions (see the 3rd day of learning in the following figure). In the weekly quiz more than 60% of pupils got correct answers in solving the items. The problems in the formative phase emerged again in this phase: lack of multiplication facts and careless subtraction.

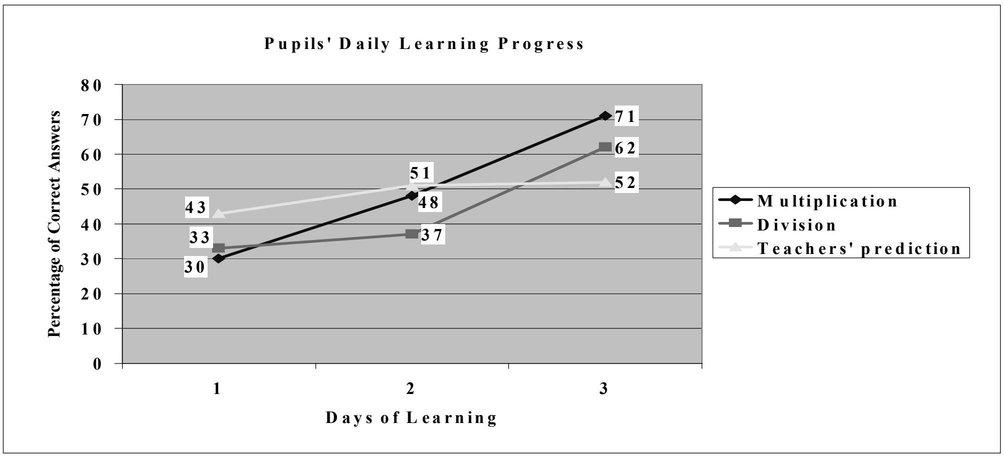


Figure 8.3
The pupils' learning progress in the daily quiz

In contrast, Table 8.10 also indicates that in the first and second day, less than 50% of the pupils answered the items correctly. In one hand, it would be an indication that pupils needed more time to adjust to the learning conditions in the RME approach that was very different than the one used to be applied by their teacher. On the other hand, it indicated that most pupils demonstrated that they proceeded toward a rational solution, but a calculational mistakes (multiplication and subtraction error) obstructed the correct solution process (see Figure 8.4).

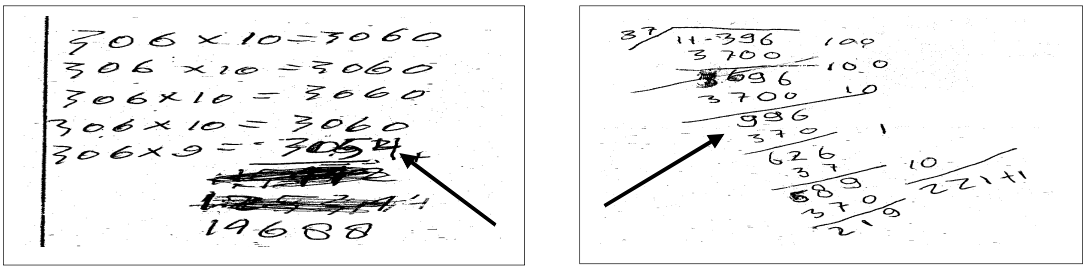


Figure 8.4
Multiplication and subtraction error (pointed by the author)

b. The pupils' level of understanding

In this assessment phase the pupils' level of understanding related to the solution stage the pupils reached in solving daily items. The table below illustrates the data.

Table 8.11
The progress toward the level of understanding

Level of understanding[b]	Daily Quiz for n = 291[a]					
	Multiplication items			*Division items*		
	1	*2*	*3*	*1*	*2*	*3*
Noncommencement	8	7	5	7	5	5
Approach	38	22	18	36	46	28
Substance	104	77	37	102	88	52
Result	53	44	24	41	43	26
Completion	<u>88</u> (30%)	141 (48%)	<u>207</u> (71%)	<u>95</u> (30%)	109 (37%)	<u>180</u> (62%)

Note: The values represent numbers of pupils who were in each level; [a]Overall numbers of pupils; [b]The meaning of each level of understanding can be seen in Table 4.8 section 4.5.2.

Table 8.11 summarizes five important results of the study. *Firstly*, there was an increasing amount of pupils who completed their understanding (see and compare the underline numbers). *Secondly*, only few pupils (less than 8) were unable to start answering the problem in each day (see the numbers in "Non-commencement" rows). It meant that most pupils approached the problem with meaningful work that indicated their understanding of the problem both in multiplication and in division. *Thirdly*, few pupils reached an early impasse in the first day and it decreased in the third day (see the numbers in 'Approach' rows). *Fourthly*, there was a decreasing amount of pupils that conducted the major error (multiplicational error) toward the valid solution (see the numbers in 'substance' rows). Fifthly, many pupils nearly solved the problem but a minor error (addition error and failed to include the last subtraction) produced an invalid final solution. The mistakes are as follow.

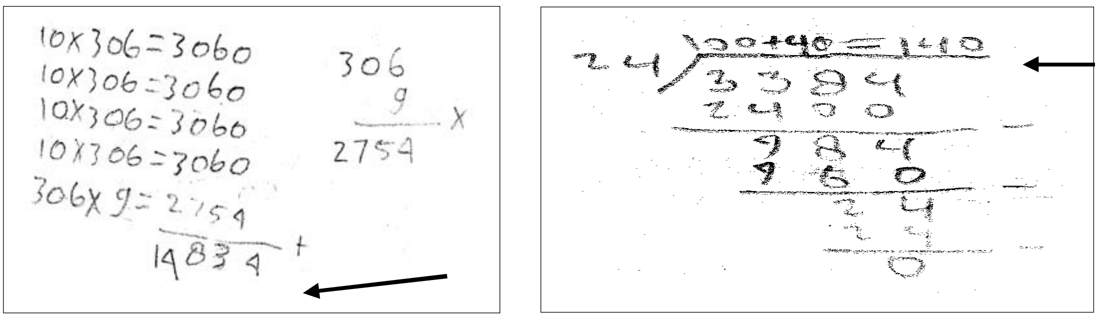


Figure 8.5
The addition error and failed in including the last number (pointed by the author)

Because the pupils conducted a minor careless error in solving the problem, the study believed that pupils understood the problem and were capable to solve the problem. If so the study concluded that after the third day of learning multiplication and division of multi-digit numbers in RME approach, more than 70% pupils progressed toward the completion of understanding (see the underlined numbers in Table 8.12 below).

Table 8.12
The progress toward the level of understanding (a summary)

Level of understanding[b]	Daily Quiz for n = 291[a]					
	Multiplication			*Division*		
	1	*2*	*3*	*1*	*2*	*3*
Completion	141 48%	185 64%	<u>231</u> <u>79%</u>	136 47%	152 52%	<u>206</u> <u>71%</u>

Note: [a] Overall numbers of pupils; [b]The meaning of each level of understanding can be seen in Table 4.8 section 4.5.2.

c. Pupils' learning achievement

In this phase pupils' learning achievement referred to the pupils' cognitive performance in the posttest. This related to the overall impact of the RME prototype in teaching multiplication and division of multi-digit numbers. A good cognitive performance was established if:

1. *Most pupils achieve middle and high level performance in the posttest*
2. *There is a significant difference between pupils' performance before (pretest) and after (posttest) engaging in the RME approach and between pupils who engaging in RME approach and that in the conventional approach.*

Each of these points will be discussed in the following sections.

1. Pupils' level of achievement in the posttest

In this phase pupils' level of achievement was analyzed from the score of posttest that was given after the learning activities had been conducted. The categories of level are explained in Table 4.9 section 4.5.2. The following table illustrates the levels.

Table 8.13

Pupils' level of achievement for n = 291[a]

Score	Level	Pretest	Posttest
0 - 13	Low	264 (91%)	16 (5%)
14 - 26	Middle	24 (8%)	155 (53%)
27 - 40	High	3 (1%)	120 (42%)

Note: [a]Numbers of pupils.

It can be seen that most pupils (91%) were in the low level of achievement when they started the learning process and only 1% of them was in the high level. After engaging in the RME learning process 42% of pupils reach the high level and 53% performed in the middle level. Only 5% of the pupils was still in the low level of achievement, 4% of them from the low level and 1% from the middle level (see Table 8.14 below). The table below also indicates that 106 pupils (36%) from the low level reached the high level after learning the subject using the RME approach. There was 88% of the pupils who were in the high level achievement. In addition, 49% of them reached the middle level of achievement. It can be concluded that the low-level pupils got the most benefit from engaging in the RME approach.

Table 8.14

Pupils' shifting achievement from pretest to posttest

	Posttest			
Pretest	*Low*	*Middle*	*High*	*Σ*
Low	14	144 (49%)	106 (36%)	264
Middle	2	9	13	24
High	-	2	1	3
Σ	16 (5%)	155 (53%)	120 (42%)	n = 291[a]

Note: [a] Numbers of pupils.

2. *Pupils' achievement differences in pretest and posttest*

In this phase pupils' performance was analyzed based on their average score in the pretest and posttest. Each of the 10 items in the posttest was scored from 0-4 and each pupil had an interval score of 0 - 40. The following table describes the pupils' mean score in the pretest and posttest from both groups (experiment group or EG and control group or CG)

Table 8.15
Pupils' mean score in the pretest and posttest

Score	N[a]	M	SD	Minimum[b]	Maximum[c]
Pretest of the EG	291	8,44	5,90	0	37
Posttest of the EG	291	25,07	7,75	9	40
Pretest of the CG	310	9,56	7,60	0	30
Posttest of the CG	310	20,42	9,98	0	38

Note: [a] Numbers of pupils involved; [b]The minimum score is 0; [c]The maximum score is 40.

With a mean score of 25.07 and 20.42 respectively, pupils of experiment group (EG) and control group (CG) were in the moderate level of achievement (compare those means with pupils' level of achievement in Table 4.9 in section 4.5.2). In both groups, pupils' achievement in pretest was in the low level.

Analyzing those means using one-way of ANOVA it was found out that there was a significant difference among the mean scores of the pretest and posttest from both groups ($F(3, 1201) = 311.597$, $p < 0.05$). Then the post hoc test of Tukey HSD was utilized to analyze which of those means were significantly different. The following results were found out out.

Firstly, with a mean different 16.63 (25.07 in the pretest and 8.44 in the posttest), it was found out that there was a significant difference between the score of the pretest and posttest of the pupils taught using EG ($p < .05$). This meant that pupils in the EG had a significant improvement in their score after learning the subjects in the RME approach. With a mean difference of 10.85 (20.42 in the pretest and 9.56 in the posttest), the same results were also found out for the pupils in the CG.

Secondly, with a mean difference of 4.65, pupils of EG scored significantly different from the pupils of CG in the posttest ($p < .05$). It can be concluded that pupils who learnt in the RME approach performed better than those pupils who learnt in the conventional approach. Another substance illustrating this result was the interval of minimum and maximum score the pupils achieved (see the third and fourth column of Table 8.12). The pupils of EG (with an interval of 9 – 40) performed better than the pupils of CG (with an interval of 0 – 38).

Thirdly, in the pretest pupils from EG and CG got the mean score of 8.44 and 9.56 respectively. In the post hoc test of Tukey HSD, the mean difference of 1.12 was not significant at the .05 level. It meant that there was no significant difference of the two groups (EG and CG) score in the pretest ($p = 0.312$). It can be concluded then, that pupils from EG and CG had equivalent score in the pretest.

To have a deep picture of pupils' achievement, pupils' mean scores in different type of problems (contextual and conventional) is illustrated in the following Figure 8.6. The figure was constructed based on the way pupils' solving the problems in the pretest and posttest and between experiment group (EG) and control group (CG).

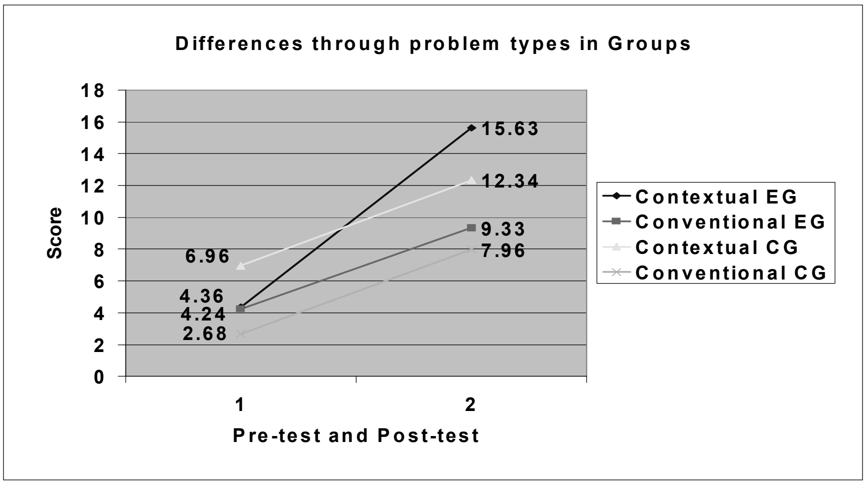


Figure 8.6
Pupils' mean scores in each type of problems

The EG pupils in the pretest scored 4.36 and 4.24 in the contextual and conventional problems respectively. Meanwhile the CG pupils got 6.36 and 2.68. After involving in the learning process the EG pupils got 15.63 and 9.33 in the posttest. The CG pupils scored 12.34 and 7.96. Comparing these posttest means to the level of achievement mentioned in Table 4.10 section 4.5.2, it could be said that the EG pupils scored moderately in both types of problems as well as the CG pupils.
Using ANOVA multivariate, several results were found. *Firstly,* significant score differences were found in the posttest of contextual problems between the EG pupils and the CG pupils ($f(1, 599) = 61.087$, $p < .05$). *Secondly,* significant score differences were also found out out in the posttest conventional problems between the EG pupils and the CG pupils ($f(1, 599) = 12.380$, $p < .05$). However, pupils'

score in the pretest for both types of problems were also significantly different (with f (1, 599) = 47.011, p < .05 and f (1, 599) = 31.124, p < .05 for contextual and conventional problems respectively).

In order to differentiate the increasement score in the contextual problems between the EG pupils and the CG pupils, the differences of the two scores (pretest and posttest) are illustrated in the following figure.

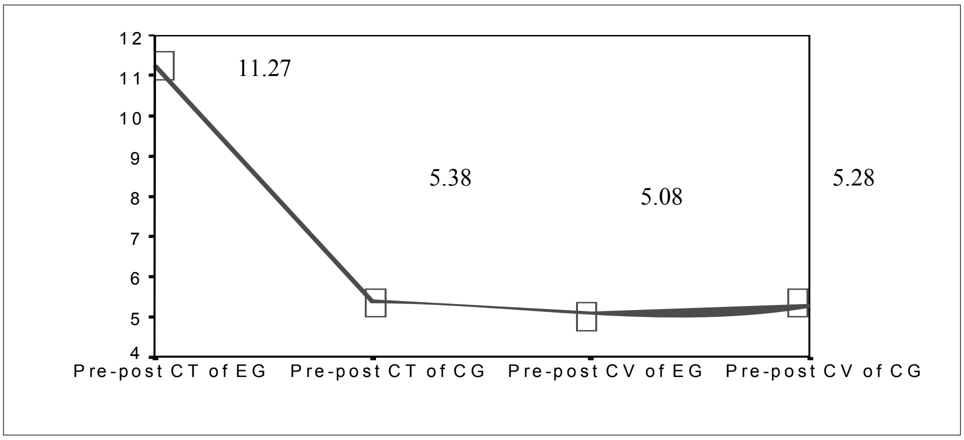


Figure 8.7
Pupils' mean score difference in each type of problems

It can be seen that in contextual problems, the EG pupils' mean difference of pretest and posttest was 11.27. The CG pupils' mean score difference was 5.38. Meanwhile in conventional problems, the EG pupil mean score difference was 5.08 and the CG pupils was 5.28. Using independent t-test, it was found out that: (1) in contextual problems, there was a significant difference between mean scores of the EG pupils and the CG pupils (t = 11.002, df =599, p < .05) and (2) in conventional problems, there was no significant difference between mean scores of the EG pupils and the CG pupils (t = 0.421, df = 599, p > .05).

d. Concluding summary

In overall the pupils progressed significantly in daily learning and the progress was beyond the teachers' predictions. In one hand most pupils demonstrated that they preceded toward a rational solution, but in another hand a major substantial error obstructed the correct solution process. Lack of multiplication facts and careless subtractions interrupted their high performances. The pupils performed in the moderate level of achievement. However, they performed better than those pupils who learnt in the conventional approach.

8.3 THE ORNATE VERSION: TOWARD THE RME ULTIMATE PROTOTYPE

In the assessment phase it was found out that the teachers introduced the contextual problems as intended. They could conduct interactive teaching model using the RME prototype. However, they were not able to establish socio-mathematical norms properly in the classroom. Learning in the RME approach pupils performed on the expected level of achievement. Conversely, pupils' lack of multiplication facts, addition and subtraction of multi-digit numbers had obstructed their high performances. They needed to improve those pre-requisites in order to engage actively in the learning process.

This conclusion was taken into account in revising the tryout version of the RME prototype. The Indonesian and RME experts were involved in accomplishing this revision. This became the ornate version: the ultimate RME materials for teaching multiplication and division of multi-digit numbers in Indonesian primary schools.

The RME prototype comprised of exemplary materials of a teacher guide, a pupil book, and a compilation of the assessment materials. The teacher guide provided information about the contents and its objectives, the planning and its pacing, preparation and its activities to conduct the instruction. The pupil book provided contextual problems that are useful to give the pupils a trajectory to develop their understanding of the subjects. The assessment materials included quizzes, tests, questionnaires, and logbook for pupils and teachers. These appraisals were structured compactly and available in the teacher guide.

The exemplary materials of RME prototype were developed in accordance with the 1994 mathematics curriculum of Indonesian primary school. The subjects (multiplication and division of multi-digit numbers) were developed based on the RME theory and its instruction principles. In the first place, the contextual problems were structured culturally and environmentally from Indonesia that teachers and pupils were familiar with. The contexts were aimed at facilitating pupils' understanding towards the reinvention process of multiplication and division strategies. The numbers involved in the problems were structurally appropriate for the contexts, the content, and the zone of thinking of the pupils. For all that development process, formative evaluation (with experts) should be utilized in order to judge the validity of the RME prototype. This process should be done intensively and the interviews were transcribed sincerely. In order to decrease bias during the formative process, the audio-tape recorder would be another essential facility that could be utilized. This study recommends that involving more experts in evaluating the prototype and utilizing formal appraisals (for instance, observation checklist) would increase the internal validity of the prototype.

The contextual problems structured in the materials considered several instructional aspects: (1) pupils' learning trajectory and its interactive discussions; (2) its level of difficulty (the length of sentence and the numbers involved); (3) pupils' familiarity with the problems; and (4) pace of hours provided in each session. It was found out that the pupils were comfortable with encountering 3-5 problems during the 2 x 40 minutes session (see Armanto, 2000). Considering pupils' difficulty in reading, the length of sentences becomes the most essential point to be taken into account. This study suggests illustrating some problems into pictures and varying the types of contextual problems. In order to encounter the accessible hours to teach the subject, this study recommends integrating the learning process of multiplication and division of multi-digit numbers with the introduction part (Numbers and its symbols) and the mixed part (Multiplication and division). These two parts were elaborated in section 8.2. This study also proposes that the instructional process of multiplication and division of multi-digit numbers (held in the second trimester) should be conducted after the learning of multiplication and division of 1-digit numbers (held in the first trimester). Indeed, all those instructional processes should be conducted in the RME approach.

CHAPTER 9
CONJECTURED LOCAL INSTRUCTIONAL THEORY FOR TEACHING MULTIPLICATION AND DIVISION OF MULTI-DIGIT NUMBERS IN INDONESIA

The development and the evaluation of the RME prototype have been illustrated in chapter 5, 6, 7, and 8 referring to whether the materials were valid, practical, implementable, and effective for teaching multiplication and division of multi-digit numbers in Indonesian primary schools. It was found out that the RME materials were representing the RME theory and the Indonesian condition, usable and easy to teachers and pupils, used as intended in the classroom (with several moderate activities), and improving pupils' performances. This chapter describes a theory of the local Indonesian instructional sequences for teaching multiplication and division of multi-digit numbers as it has been developed during the execution of the research. It focuses on the pupils' trajectory of learning multiplication and division and teachers' role and confusions during the process. It ends with an ideal of RME real-life classroom in the future.

9.1 INTRODUCTION

Chapter 5, 6, 7, and 8 described the characteristics of the RME prototype in the curriculum level referring to the quality aspects of the materials (validity, practicality, implementability, and effectiveness). It was found that the RME prototype was valid (its content representing the Indonesian circumstances and the RME theory and its components were linked each other) and practical (usable and easy) for teaching multiplication and division. The Indonesian teachers introduced contextual problems and conducted the interactive teaching model as intended. But they did not establish socio-mathematical norms properly. For instance, in conducting the instructional activities they did not make use of the guiding questions proposed in the teachers' guide materials. In addition, 55% activities in regulation activities of teaching multiplication were conducted inappropriately.

Even though teachers' performances improved in teaching division, most teachers did not reevaluate pupils' understanding, did not reflect on the results, did not take recovery actions to improve pupils' understanding.

This chapter describes an intended conjectured local instructional theory for teaching multiplication and division of multi-digit numbers in Indonesian primary schools. It is a local theory, the initial theory that is not a well-developed theory but is open for adaptation and functions as a guideline and inspiration for the next developmental research. It is called a reconstruction of theory in action (Gravemeijer, 1997) in the learning level. It was developed by confronting the hypothetical learning trajectory (see section 3.5) with the pupils' learning trajectory as found during the fulfillment of the research and the learning activities that occurred in the classroom (elaborated in section 9.4). Most activities and actions were taken from the teaching experiments in the assessment phase. It was the intended learning activities that conducted by the teachers with an analysis of how it was conducted, the reasons of doing the activities, what difficulties encountered, and how the teacher dealt with those difficulties.

In section 9.2, a concise picture of structuring contextual problems that were involved in the learning process is illustrated. Next, a brief illustration of pupils' initial strategies, as well as their weaknesses in solving (contextual and conventional) problems in the pretest are enlightened in section 9.3. Then, in addition to pupils' learning trajectory of multiplication and division (section 9.4), the teachers' role and confusions experienced during the learning activities were elaborated in section 9.5. This chapter ends up with the illustration of the conjectured realistic mathematics classroom in Indonesian primary schools (section 9.6).

9.2 RESTRUCTURING CONTEXTUAL PROBLEMS

In this study the RME formal curriculum included: (1) the learning route, (2) the teachers guide, and (3) the pupil book. The learning route was constructed based on the pupils' reinvented procedures in learning multiplication and division of multi-digit numbers. It comprised the objectives and its sub-objectives and the learning activities: preliminary games and encountering contextual problems (see section 5.4). The games were aimed at attracting pupils' attention and motivation, as well as practicing multiplication facts. It was based on the fact that many pupils still had difficulties in memorizing multiplication facts. It was also conducted as a reminder to what pupils had to use and apply in the learning process. Meanwhile, encountering contextual problems was the main starting point in the RME approach to lead pupils to use their former (informal and formal) knowledge towards the reinvention of "new" skills and understanding.

To facilitate this reinvention process, this study constructed the contextual problems that were experientially real to pupils. It considered that the RME theory and the Indonesian contexts should be embedded in the contexts used. These contextual problems were constructed, developed, and revised during the preliminary phase and formative phase. As mentioned earlier, the process of developing and revising the contextual problems involving the RME experts, the Indonesian mathematics education expert, the teachers, and the pupils. It guided pupils to a learning trajectory of multiplication and division of multi-digit numbers.

Considering Greers' suggestion (1992), the contextual problems were constructed in several types: equal groups (a number of groups of objects having the same number in each group), rate (the number of group is multiplied by the number of groups to find the total number), and multiplicative comparison (n times as many as). The RME prototype distributed the contextual problems of the subjects as follow:

Table 9.1
Distribution of multiplication and division problems

Type of problem	**Multiplication**				**Division**			
	Day 1	*Day 2*	*Day 3*	*Day 4*	*Day 1*	*Day 2*	*Day 3*	*Day 4*
Equal groups	3	4	3	4	3	3	4	5
Rate	1	1	1	1	1	1	1	1
Multiplicative comparison	2	2	1	2	2	2	-	3
Total [a]	6	7	6	7	6	6	5	9

Note: [a] Including the miscellaneous problems.

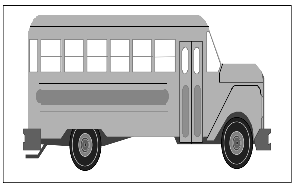
It can be seen that in each day the pupils encountered 6 - 7 contextual problems (see the total amount of problems). The first 3 problems were aimed at leading the pupils through the guided reinvention process of the multiplication and division procedures. The next 3-4 problems were aimed at giving the pupils homework to practice their skills (see Pupil Book p. 8-11). The contextual problems were not distributed equally because several reasons: (1) the difficulties in creating appropriate contextual problems relating to Indonesian circumstances and the mathematics concepts for the learning trajectory; (2) the pupils' needs to have experiences in solving these kinds of problems; and (3) the difficulty to create the problems for other types than the equal groups. These reasons influenced how the researcher created, developed, and evaluated the contextual problems that were included in the pupils' book and the teachers guide.

The contextual problems were created during the preliminary and formative phase involving experts (Indonesian and RME experts) and the primary school teachers. The experts and the teachers agreed with three main concerns to be taken into account.

First, the contexts should be familiar with the Indonesian pupils and teachers. Familiar is in the meaning of experientially real for them. The contexts such as snow and four different seasons for instance were not in the lists of contexts that were involved. They were not typical for Indonesian pupils even though they were recognizable and well-known. However, some contexts such as "train" could still be argued. It was because in some parts of Indonesian there was no train used as transportation. For this reason the study propose to use any kind of transportation to be involved in the context, indeed with a change in the numbers. The following is the example of a contextual problem used in this study that was considered familiar with the pupils.

To the zoo

In a weekend pupils from a school are going to the zoo using 75 buses. A bus can carry 42 pupils. How many pupils can the buses carry?



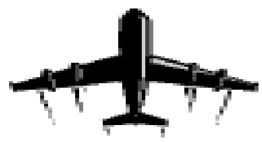
Second, the contexts attracted pupils to use their former mathematical knowledge by modeling (formal or informal) mathematical forms. It should fulfill four functions (Treffers & Goffree, 1985): concept formation, model formation, applicability, and practice. The contexts helped pupils to create their own understanding and attracted them to sculpt the mathematical forms, for instance the repeated addition. This study also found that this problem encouraged pupils to take an opportunity to create their own strategies without having order from the teacher. And the next problem did not attracted pupils to structure their repeated addition strategies.

Tiles

How many tiles do they need to build a square as like in the picture?

A plane

For flying from Jakarta to Medan a plane needs 604 liters of gas. If the plane flies 52 times a year, how many liter of gas does the plane need?



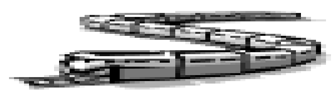
Third, the numbers involved should be in harmony with the contexts and the formal curriculum being developed. The numbers involved in the problem should be multi-digit numbers. However, the 1994 mathematics curriculum restricted them with the numbers until 100.000 (see Table 5.9 in section 5.3). The numbers were chosen considering several thoughts: (1) pupils' difficulties in multiplying these types of numbers, (2) variation of numbers, and (3) the need of practicing to multiply or divide numbers. For instance, the following contextual problems were involved the numbers that were not in harmony with the contexts; too big and unrealistic.

Skillful mason

Pak Budi is a skillful mason. He is asked to build a huge wall that needs 604 bricks in each layer. The wall contains 52 layers. How many bricks does the wall need?

Lebaran Day

A day before the Lebaran day, there are 5,400 people crowded in the Gambir Station to go back (mudik) to Surabaya. How many train-wagons do they need to go back to their place if each wagon can carry 86 people?



However, the experts convinced that these contexts were applicable, useful, and practical. In the first problem, the pupils would think about the layers of the bricks and then they would create the informal mathematics to bridge the problems towards the formal mathematics forms. In the second problem the pupils might think of putting people in each wagon and then they subtracted them to apply the repeated subtraction strategies (see section 3.5) Then, the decision of change was made by involving lower numbers such as 204 and 52 for the first context and 1400 and 86 for the second.

Considering those three aspects this study structured, developed, and revised all contextual problems of multiplication and division of multi-digit numbers. The problems were used in the learning trajectory, the quizzes, and the tests. The experts and the teachers reviewed the problems and the decision of changing or involving the problems was decided. Their satisfaction toward the problems that involved in this study judged the quality of the problems, the data being collected, and for the good of the study. Having these problems in hand the study structured the problems orderly in line with the learning trajectory (elaborated in section 9.4).

9.3 PUPILS' INITIAL STRATEGIES IN THE PRETEST

In this study pupils' initial strategies referred to the valid and correct mathematical forms (informal or formal) the pupils applied to solve the contextual problems in the pretest. In the RME point of view the pupils' initial strategies are in the intuitive phenomenological level, the lowest level in which the pupils' relational framework is not yet existent. However, the exploration on this level may lead to the formation of fundamental relations, which may, in turn, be interconnected in such a way that a learning framework is created (Gravemeijer, 1997). In this manner, the pupils' initial strategies in multiplication and division of multi-digit numbers would be a basic foundation for building and establishing pupils' understanding in the learning process.

This section analyzes the pupils' reinvented strategies that were collected from pupils' solving the pretest problems in the assessment phase. It focuses on two different aspects: pupils' reinvented strategies and their typical mistakes. These aspects were examined from the types of problems' point of view (contextual and conventional items) in each subject (multiplication and division).

Earlier, as a part of the results in the pretest the following table illustrates the pupils from the experimental groups who got correct answers in solving each item in the pretest. It was found that less than 25% of pupils got correct answers in each contextual multiplication item. In division item, less than 9% of the pupils got correct answers. In conventional item, less than 18% of pupils got correct answers in multiplication and less than 9% of pupils got correct answers in division. The results are in Table 9.2 below.

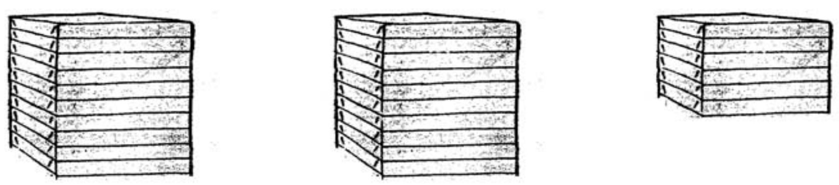
Table 9.2
Pupils' correct answers in each item of the pretest

		Contextual item						**Conventional item**			
		Multiplication			*Division*			*Multiplication*		*Division*	
Schools	**n[a]**	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
SDN 101746	23	7	2	-	-	-	-	1	2	-	-
SDN 101747	12	3	1	-	-	-	-	1	1	-	-
SDN 101748	26	3	-	-	-	-	-	-	-	-	-
SDN 101749	31	1	-	2	-	-	-	1	-	-	-
SDN 101750	37	4	2	1	-	-	-	2	-	-	5
SDN 101751	45	32	12	10	3	3	3	17	3	2	1
SDN 101752	52	2	4	4	3	2	-	6	3	8	2
SDN 106153	65	21	16	13	20	7	8	23	14	17	7
Overall (N[b])	29	73	37	30	26	12	11	51	23	27	15
Percent (%)	1	25%	13%	10%	9%	4%	4%	18%	8%	9%	5%

Note: [a] Number of pupils in each school; [b]Overall numbers of pupils.

The first contextual problem in multiplication was about the amount of cards in 26 boxes where there are 125 cards in each of the boxes. The problem was as follows.

Playing cards



26 boxes in the picture contain 125 playing cards in each box. How many cards are there all?

The context and the picture involved in the item were supposed to attract pupils to use the repeated addition for multiplying 125 by 26. These strategies were elaborated from the definition of multiplication and the pupils had learnt them previously in the first trimester. It was found that 80% of pupils who got correct answers in this item used and applied the repeated addition. The following figure illustrated the repeated addition the pupils reinvented.

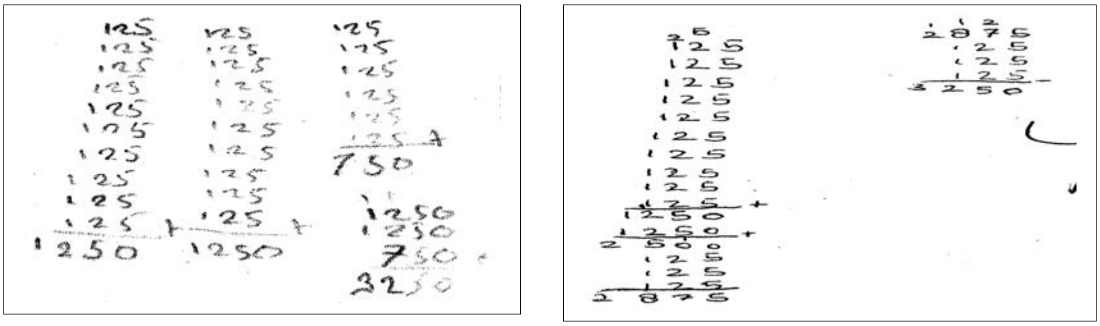


Figure 9.1
Pupils' repeated addition strategies

Other pupils who got correct answers applied the different type of repeated addition (see the left picture of the following figure) and the standard algorithm. The following figure illustrates those types of strategies.

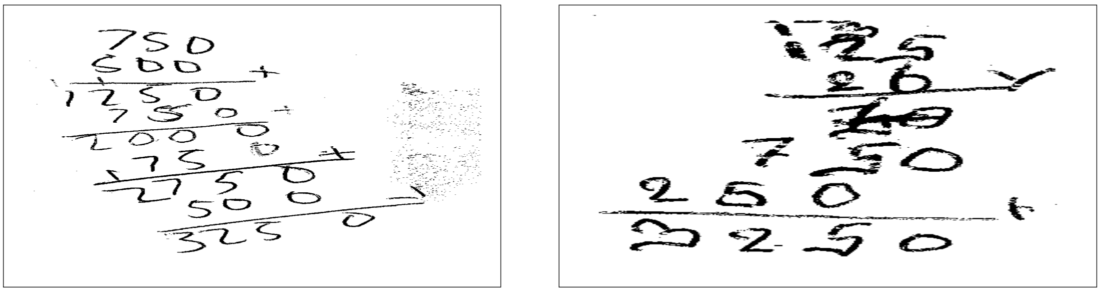


Figure 9.2
Pupils' addition strategies and standard algorithm

However, some pupils who got incorrect answers made mistakes in the repeated addition. The reason was that they added all 26 numbers of 125 together at one time (see the left picture in the following figure). It made the repeated addition more complicated than when it was done by ten numbers subsequently (compare it with the left picture Figure 9.1 above). However, some pupils made mistakes as well when they added six numbers of 125 together (see the right picture in the following figure).

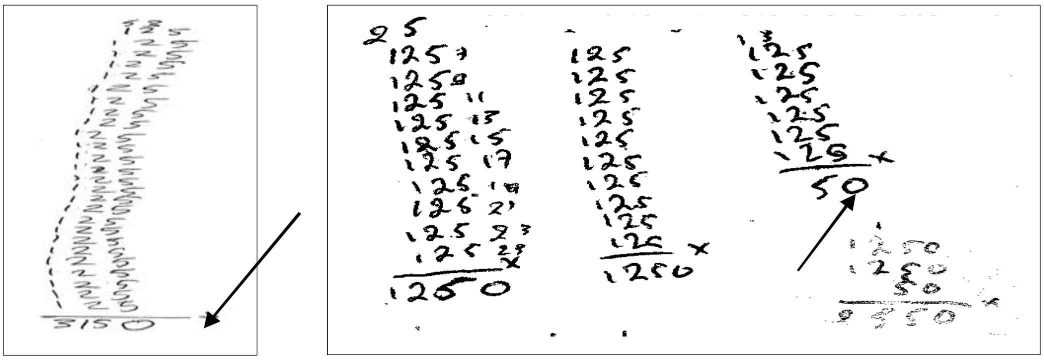


Figure 9.3
Pupils' incorrect repeated addition strategies (pointed by the author)

Other mistakes were coming from what operation to be used and how to do the calculation. To answer the problem some pupils just added the two numbers (26 and 125) they found in the problem (see the left picture in the following figure). Others used the standard algorithm inappropriately. And other pupils miscalculated the multiplication because of their lack of multiplication facts. The following figure illustrates the mistakes.

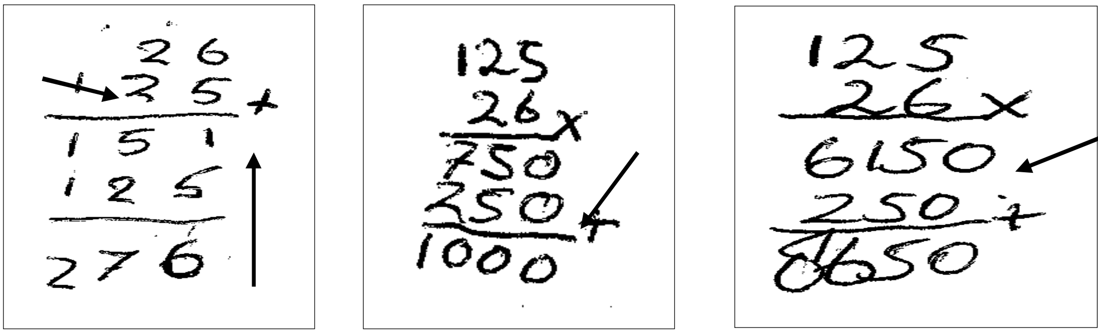


Figure 9.4
Incorrect operation, corrupt procedure, and error multiplication (pointed by the author)

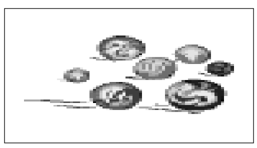
These various types of strategies were also found whenever the pupils solve the second and third item of the pretest. The problems were as follows.

1. Drinking water

Budi drinks 35 glasses of water every week. How many glasses of water does he drink for 108 weeks?

2. Marbles

Haqim has 230 marbles and Hilmy has 46 times as many. How many marbles does Hilmy have?



Pupils applied many types of strategies; the repeated addition and the standard algorithm. The followings were the strategies.

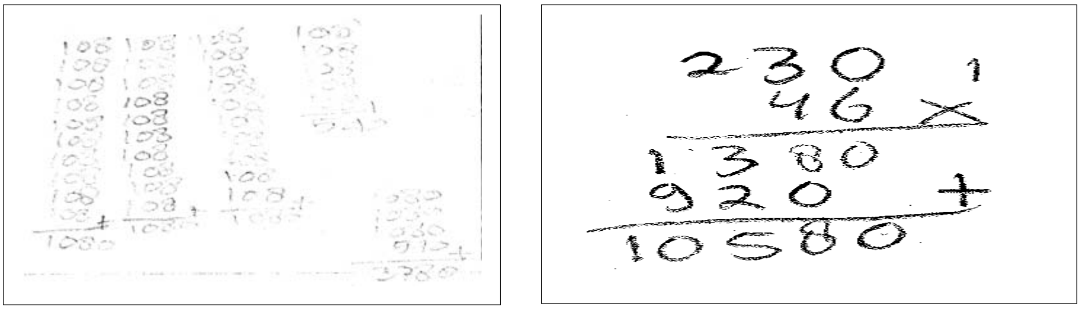


Figure 9.5
Pupils' valid strategies for item 2 and 3

However most pupils still lacked of calculating addition of numbers and conducting error standard algorithm. The lack of multiplication facts was also found in the calculation.

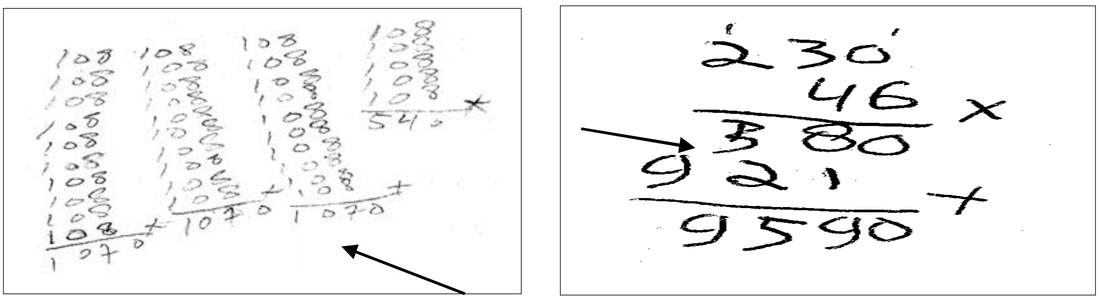


Figure 9.6
Pupils' incorrect addition and multiplication (pointed by the author)

In answering division contextual items most pupils (more than 91%) did not get the correct answers (see Table 9.2 above). However, most pupils who got correct answers applied the standard algorithm to solve the problems. They said that they learnt this algorithm from the addition learning outside the classroom. The following items were the contextual problems used in the pretest.

4. Changes

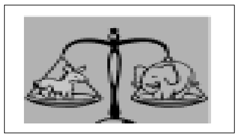
The length of a Rp. 1000-curency is 14 cm. Khalid lines up all his Rp. 1000-money together. The length of all his money is 1330 cm. How many notes does he have?

5. Good reader

To be a good reader, one could read 3404 words in 57 minutes. How many words should one read in a minute?

6. Elephant and goat

An elephant weights 94 times as much as the weight of a dog. If the weight of an elephant is 5452 kg, how much is the weight of a dog?



The first item of the division problems, "the changes" asked the pupils to line up the 1000-change together until the length was 1330. In solving this problem the pupils were attracted to use repeated addition or repeated subtraction. For instance, having 10 of 1000-notes for 8 times resulted 1260, that was minus 70 more from the 1330. Then the last could be 5 more of the notes.

However, most pupils tried to use the standard form. The following figure describes how pupils got the right answer.

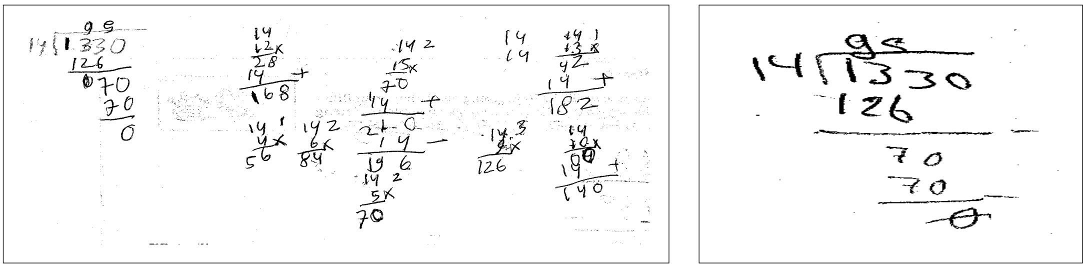


Figure 9.7
Pupils' correct strategies

In using the standard algorithm many pupils made incorrect calculation because the teacher did not teach the strategy yet (see the two left pictures in the following figure). Some pupils just multiplied or added the two numbers together (see the two right pictures below). It was believed that pupils did not use the contexts involved in the problems. Several reasons were identified: (1) pupils did not understand the problem because of their weak reading ability; (2) pupils did not get used to encounter contextual problems; and (3) pupils got used to learn and apply a correct strategy, the teachers' strategy. These conditions made pupils did not have any other strategy to fall back on. They created buggy procedures as follow.

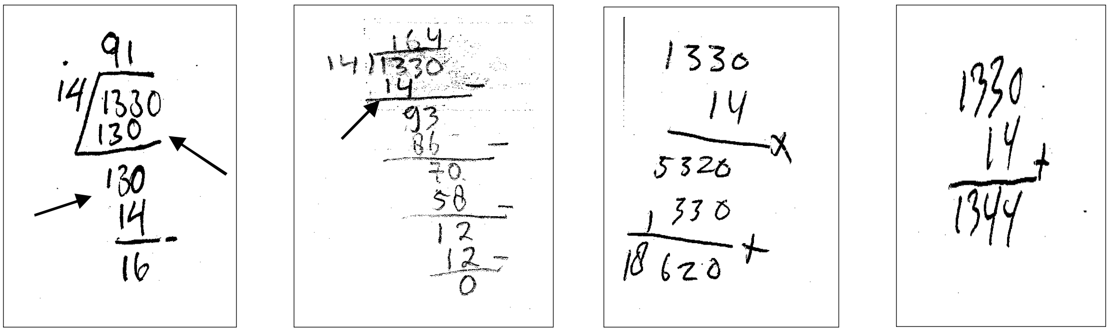


Figure 9.8
Incorrect subtraction, error number, incorrect operations (pointed by the author)

In solving the division item no 5 and 6, most pupils who got correct answers applied the standard algorithm as well. They developed a type of table of multiplication using the repeated addition. The following is the figure.

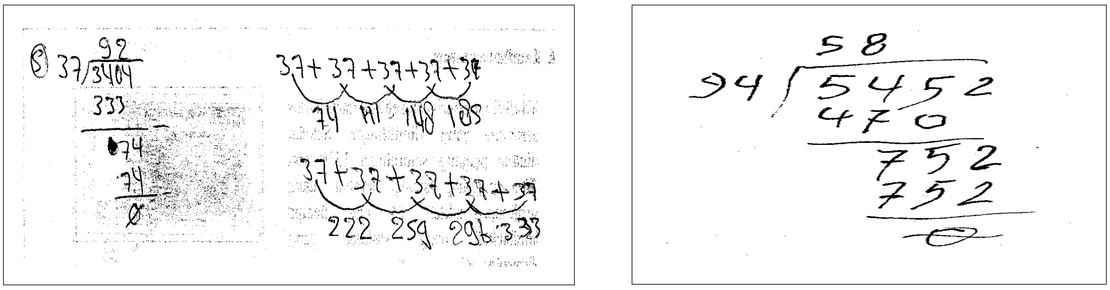


Figure 9.9
Pupils' correct solutions

In some cases of solving the item number 5 and 6, pupils created the multiplication table incorrectly. It made them calculated the standard algorithm incorrectly (see the left picture below). However, some pupils made a correct start to solve the problem (see the middle and right pictures in the following figure). But since they had no idea, knowledge, and experience of doing the calculation, then they just stopped at the first step of conducting the repeated subtraction.

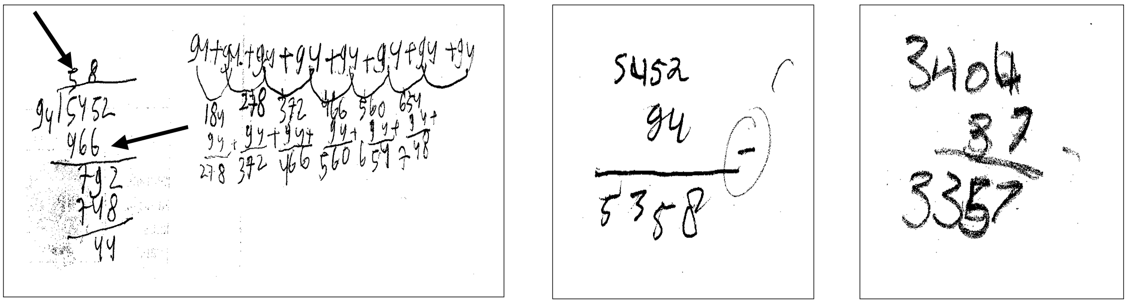


Figure 9.10
Incorrect multiplication and early impasse solutions (pointed by the author)

In solving conventional problem in the pretest (item 7) most pupils applied the standard algorithm of multiplication to get the right solutions (see the left picture below). However, some pupils still reinvented and applied repeated addition to get the solutions (see the right picture below). They believed that using the repeated addition strategies was easy and understandable to get the correct solutions.

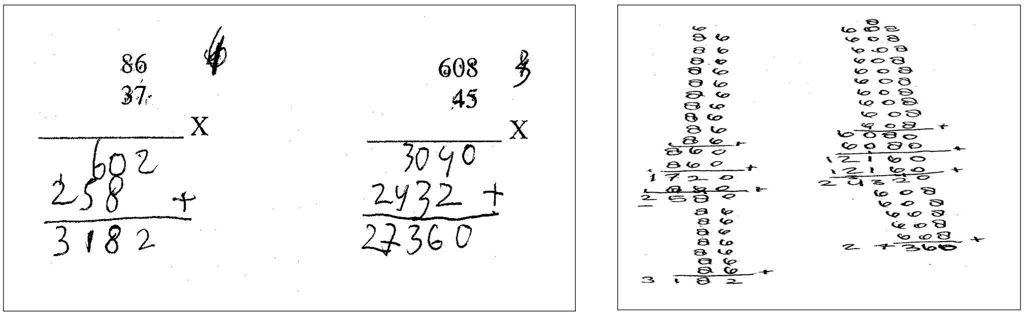


Figure 9.11
Pupils' correct strategies in solving conventional problems

In most of cases pupils tried to applied the standard algorithm desperately. Most of them made mistakes, some because of their lack of multiplication facts (see the first three left picture below) and others because of misunderstanding the strategy (see the right picture below). It can be concluded that the multiplication facts and the understandable strategy became the main aspects to be taken into account in the learning process.

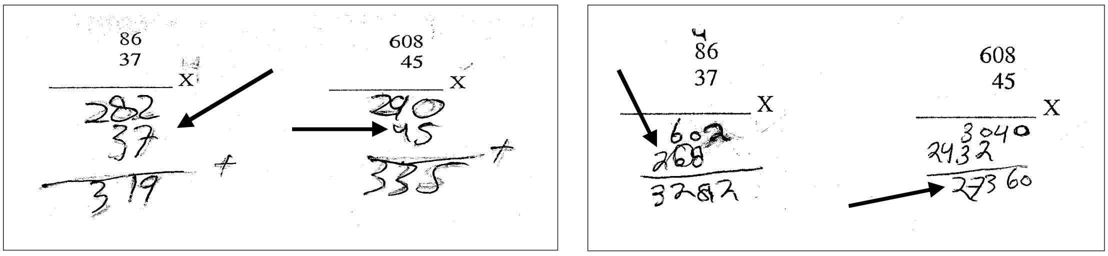


Figure 9.12
Pupils' incorrect multiplication and addition (pointed by the author)

In solving the conventional division items pupils also used the standard algorithm (see the left picture below). Most pupils tried to apply it by their own understanding. However, most of them failed because of lack of understanding of the strategies (see the right picture below). These incorrect strategies the pupils applied showed that pupils needed to know and understand the strategy (how to start, where to end, reason to do the calculation) better in order to apply them correctly.

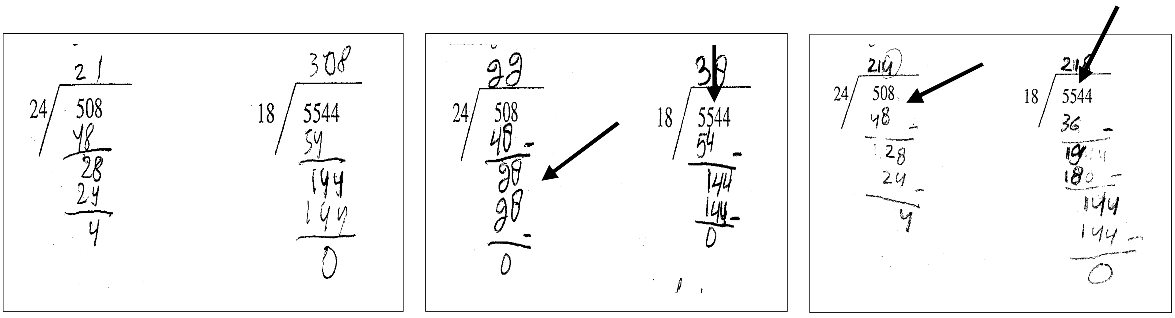


Figure 9.13
Pupils' correct solution, incorrect multiplication and procedure (pointed by the author)

Having those pupils' correct and incorrect strategies in solving multiplication and division item in the pretest, it can be concluded that pupils had weak ability in multiplication facts, in adding numbers subsequently, and in subtracting numbers. These weaknesses made the researcher aware of the need of practicing these prerequisites in order to accelerate the learning activities.

Other aspect to be taken into account was related to pupils' use of the repeated addition and subtraction to solve the problems. Some pupils use these strategies spontaneously. Their reinvention showed that pupils have had their former knowledge (repeated addition and subtraction) before they learnt the subjects. This indicated that the learning activities could be started at the structuring the repeated addition strategy.

The given current situation of the pupils involved in the RME learning process was discussed with the teachers and the observers in the small group discussion. These discussions assured the presence of teachers' awareness toward those adequate and inadequate aspects during the learning activities. It was concluded that questions, help, guidance, and hints should be appropriately given to the pupils to lead them to engage in the proposed RME learning trajectory. The pupils' learning trajectory is illustrated in the next section.

9.4 PUPILS' LEARNING TRAJECTORY

In this study a learning trajectory was defined as a description of the path of learning activities the pupils can follow to construct their understanding of multiplication and division of multi-digit numbers. This path considers the learning goals, the learning activities, and the teaching process that occurs in a short term of instructional process in which the pupils might engage (Simon, 1995). Simon called it a hypothetical learning trajectory (HTL) because the actual learning trajectory is unknowable in advance. Even though pupils' learning trajectory vary but the learning sequence often proceeds along similar path.

In this manner the teachers can construct a HTL based on the expected path they believe will occur. The teachers then can find out the actual learning path and adjust the HTL in the actual learning process. These activities will lead the teachers towards the new understanding of pupils' learning cognition. For generating the HTL, teachers rely on their 'domain-specific' knowledge (Gravemeijer, 1997; see also the teachers' competence in Grouws & Koehler, 1992). It includes their competencies on mathematics and on the mathematical activities and representations. This knowledge along with the HTL guides the teachers interact with pupils towards the objectives of learning the subjects.

The following sections (9.4.1 and 9.4.2) illustrate the pupils' learning sequences of multiplication and division of multi-digit numbers. It illustrates pupils' developing conceptions and the teachers' decision in posing problems during the learning process. It also describes how the teachers conducted the discussions to establish mathematics norms in the classrooms. It was compiled from many learning activities in different schools during the assessment phase.

9.4.1 Learning multiplication of multi-digit numbers

As mentioned earlier the learning process of multiplication was conducted in four days, 2x 40 minutes each. Each day had different aim to achieve. The first day was aimed at reinventing repeated addition of multi-digit numbers. The second day until the fourth day aimed at reinventing the multiplication by 10, multiplication by multiples of ten, and standard multiplication algorithm respectively. The following sections describe the activities towards the learning trajectory.

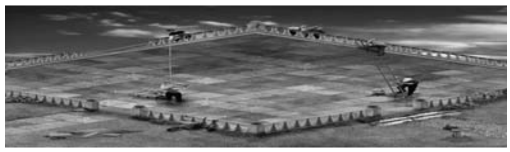
a. Day 1

The class began by asking 10 questions about multiplication facts and the pupils wrote down their answers in the portfolios. The activities were conducted for 10 minutes aiming at practicing their multiplication facts and attracting attention and motivation. Then teachers discussed the answers and asked how many pupils got how many right answers by raising their hands. It was found out that 60-70% of pupils got right answers, but less percentage was found whenever pupils lacked of multiplication facts. Almost 20% of them used their fingers to find the solutions.

Then the lesson began with introducing "the Tile" problem. The pupils sat in pairs and each had their own book. They were asked to solve the following problem:

A. ***Tiles***

1. *How many tiles do they need to build a square as in the picture?*



The teacher asked them to read the problem carefully and find how many tiles they need to build the square like the picture. Different reactions pupils acted upon depended on their understanding of the problems. Many pupils tried to count them all, but some pupils count just how many tiles in each row or in each column. However, some pupils raised their hands asking how they were going to do, multiply them or add them all. It can be seen that the pupils still had dependency on their learning attitude. The teacher then asked them to count them all or to multiply them (whenever the pupils knew how many rows and columns were there in the square). These teachers' answers represented the resistance of taking control in the learning process. In this matter the teacher made an inappropriate activity. The teacher should have answered pupils' question by asking what he/she (or other pupils) thought about it. The teachers' question made them think and discuss the problem situation.

The discussions among pupils began with the mistake in counting tiles in rows or in columns. Pupils asked: "How many tiles were there in each row?" It should be 14 but some pupils found them 13. Without having any answer from teacher or having answers such as: "count them again carefully" or "What do you think", then pupils tried to find the exact numbers of tiles together with his/her pairs. These showed that pupils could manage themselves whenever the teacher gave hints or just asked them to do it carefully. The pupils started counting again and discussed it with their friends. The interactive discussion occurred and the problem understanding progressed. It can be seen that the teacher managed good introduction activities in mathematizing a contextual problem (see section 8.2.1 item a.).

The discussion in counting the tiles in row or column was a core goal of the use of the "Tiles" problem. Pupils were attracted to utilize their own knowledge to find the solution of this problem. They developed many strategies, such as:

- counting tiles in each row and adding up all together
- counting all tiles up to the end
- repeated addition of five numbers
- repeated addition of ten numbers

The following figures illustrate the repeated addition of 10 numbers pupils reinvented during the learning process.

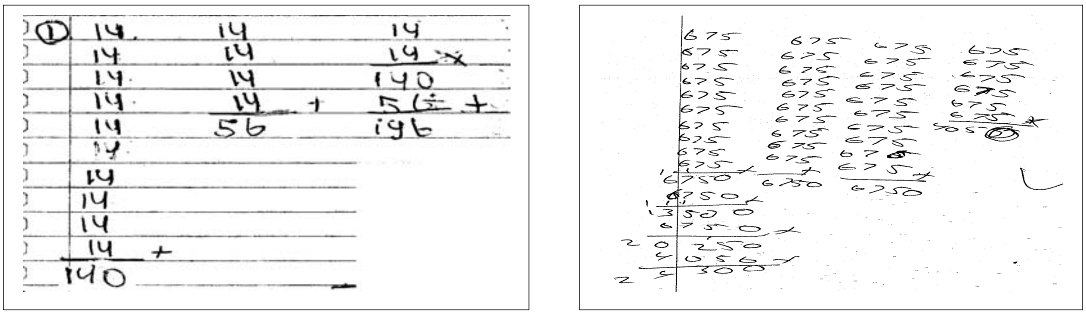


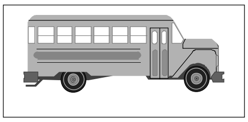
Figure 9.14
Pupils' problem solving strategies

Having several pupils' reinvented strategies, pupils realized that many strategies could be used to solve the problems. Then teachers conducted a discussion to which strategy the pupils were in favor of and understood better. Pupils then compared and gave opinion and reasons in discussing the strategies. Reasons such as "I like it", "I understand it better", "it is easier", or "too long" were some of the reasons given. They negotiated and established mathematical norms of the valid multiplication strategies. It led them to believe that repeated addition of ten numbers was easy strategy to find the solution (see the left picture in Figure 9.14). Then in solving the first item of the daily quiz they applied it (see the right picture of Figure 9.14). It was the aim of the first day of teaching multiplication.

However, as mentioned earlier (see section 8.2.1 item c) in managing discussions towards establishment of socio-mathematical norms, teachers conducted the activities inappropriately. In some cases, for example after solving the "To the zoo" and "Skillful mason" problems, most teachers did not carry out the regulation activities at all. These made the mathematical norms were not established properly. Some misunderstanding occurred because these activities were conducted in the first day of learning using the RME approach. For instance, because pupils still had dependency on the teachers' orders, they did not make use of the effective strategy unless the teacher asked them to apply it.

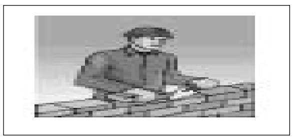
Having this strategy, pupils then encountered two contextual problems followed, "To the zoo" and "Skillful mason". These problems were aimed at reiterating reinvention of the repeated addition. Its purpose was also to justify the repeated addition of ten numbers as the effective strategy to solve the problems. The numbers in the problems that were bigger than those of the "Tile" problem helped pupils to believe this justification. The followings were the contextual problems.

B. To the zoo



2. *In a weekend pupils from a school are going to the zoo using 35 buses. A bus can carry 42 pupils. How many pupils the buses can carry?*

C. Skillful mason



3. *Pak Budi is a skillful mason. He is asked to build a huge wall that needs 204 bricks in each layer. The wall contains 52 layers. How many bricks does the wall need?*

Having discussion of which strategy was effective in solving "the tile" problem, most pupils solved those problems using the repeated addition of ten numbers. The following figure illustrates the pupils' strategies.

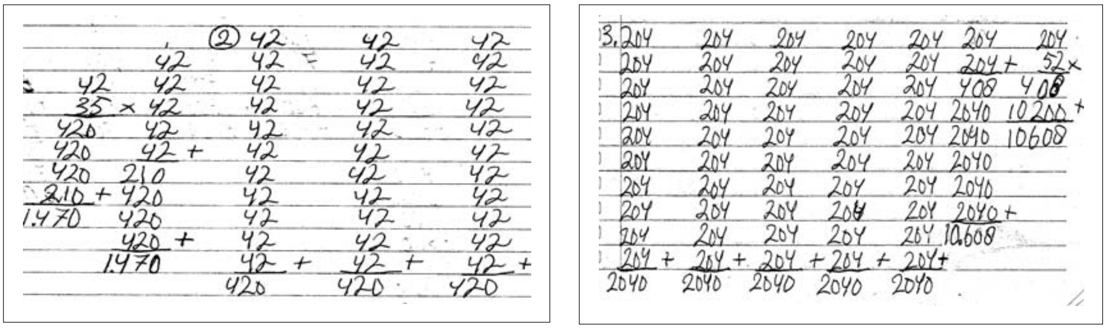


Figure 9.15
Pupils' repeated addition strategies in the first day

In some cases for instance in SDN Sonosewu, SD Kanisius, and SDN 106153 in which the teachers dealt with high ability pupils, most pupils were motivated to have a more effective strategy to solve the problems. Pupils raised questions such as "I found this strategy is very long, do you have another strategy that is shorter and easier?". There were two answers for this question the teacher offered. *First*, "Yes, but you will learn it later tomorrow" and "Yes, do you want to know it?". The second answer led the pupils to the reinvention of the multiplication by 10 (see the learning process in the second day). Teachers' answers made pupils interested to learn more about the multiplication. This maintained the pupils' motivation.

At the end of the instructional activities, the pupils encountered a daily quiz item, "The price". The problem was as follow.

A. The price

> *The price of a math book is 36 times as much as that of a pencil. A pencil costs Rp. 675. How much is the price of a math book?*

As mentioned in Table 8.10 section 8.2.2 item b, it was found out that 30% of pupils got correct answers. Most pupils made substantial mistakes; related to the lack of multiplication facts and careless addition of multi-digit numbers (see Figure 9.16 below).

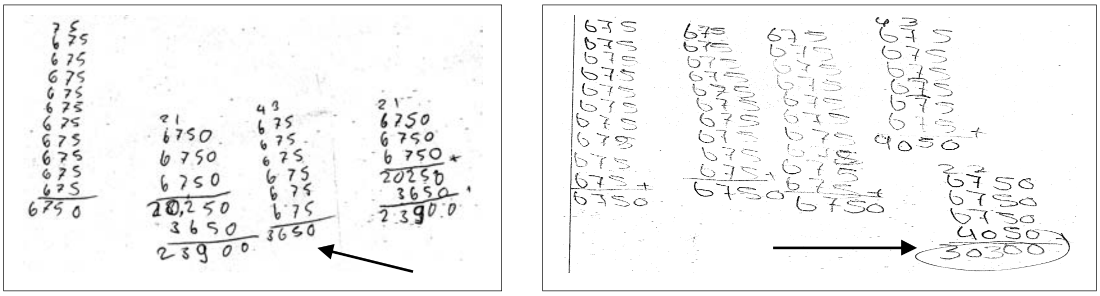


Figure 9.16
Pupils' addition errors in daily quiz (pointed by the author)

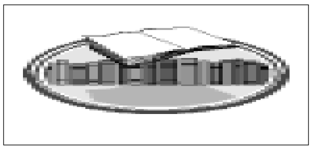
b. Day 2

The second day of learning started with practicing multiplication facts and multiplication by 10. It was aimed at improving pupils' skill on multiplication. It was hoped that during the learning process later they used the skill to find the solution. The data showed that pupils improved on their skills of multiplication facts. The impact of this improvement would be seen in the learning process.

The second day of learning was aimed at reinventing multiplication by 10. The learning process focused on the process of how the pupils came up with multiplication by 10. This process supposed to take place when pupils encountered the first problem of the second day, "Books".

G. Books

7. *A bookshop "CINTA BUKU" bought 96 boxes of Geography books. Each box contains 24 books. How many books are there in the boxes?*

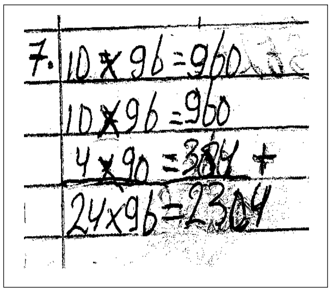


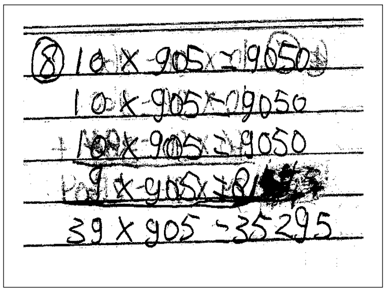
In the first place hypothetically pupils encountered the problems by repeated addition strategies, mostly by addition of ten numbers because they learnt it in the previous day. By asking a pupil to write down his/her strategy then the discussions began.

Teacher: How many of you used this strategy?
Pupils: .... (Many pupils raised their hands)
Teacher: Do you think this is the effective strategy, the easiest and the shortest?
Pupils: ... silent ...
Teacher: Do you have any idea how to find the solution in different way?
Maybe shorter than this strategy (pointing the pupils' written strategy)
*Pupils: ... silent ... (*it means no pupil developed other strategy*)*

Teacher: Do you want to know the other way?
Pupils: yes ... (together, accordingly, and smiling)
Teacher: Let's do it together. (Pointing a column of the numbers)
See these numbers on this column. How many numbers are there?
Pupils: ten
Teacher: What are the numbers being added?
Pupils: 96
Teacher: What is the result?
Pupils: 960
Teacher: Do you know how to write the connections among these numbers in mathematics? (the teacher asked to write the equation of those numbers)
Pupils: ... silent ...
*Teacher: OK, if you have 3 of number 5, you write down them in column like this (*the teacher writes number 5 in column). *If we add them together, how much is the result?*
Pupils: 15
Teacher: Good. Do you know how to write these numbers in mathematics?
There are 3 numbers of 5, and the result is 15.
Ani: I know it. 3 Times 5 is 15 or 3 x 5 = 15 (She raises her hand)
Teacher: Good. Thus, 3 x 5 = 15 (the teacher writes the equation)
Lets' go back to this column (pointing the former written strategy).
There are 10 numbers of 96, the result is 960.
Do you know how to write these numbers in mathematics?
Ani: Yes, 10 x 96 = 960
Teacher: Good. Thus, (writing) 10 x 96 = 960. This is from this column (pointing a column in the written strategy). We have another column, 10 numbers of 96, the result is 960, thus ....?
Pupils: 10 x 96 = 960
Teacher: (writing below the first equation) 10 x 96 = 960.
We still have another column. Four numbers of 96, the result is 384, thus ....?
Pupils: 4 x 96 = 384
Teacher: (writing below the second equation) 4 x 96 = 384.
(While pointing the numbers) Now we have 960, 960, and 384.
If we add them all, the result is 2304. This result is just the same as this result (while pointing the numbers). What do you think now.
Which way is the shortest and maybe easiest?
Pupils: The second one
Teacher: Now you have two strategies to solve a problem. Which one you understand better, you can use it to answer the next problem.

These learning activities imply that teacher shifted from the multi-digit numbers to 1-digit numbers to reorganize pupils' understanding. This approach helped pupils to refresh their memory of multiplication concepts. However, this shifting process is commonly used in the conventional mechanistic approach; which the teacher was familiar with. In RME theory the shifting process should also be started with contextual as well, in which the pupils reproduced their former understanding. During these activities the teacher and pupils also discussed and negotiated the norms of multiplication strategies. By discussing them, pupils had opportunity to compare which strategy was appropriate and efficient to solve the problem. The teacher facilitated these establishment activities by giving encouragement and questions to discuss the strategy with the whole class. The following left picture was the pupils' reinvented strategy in solving the "books' item.





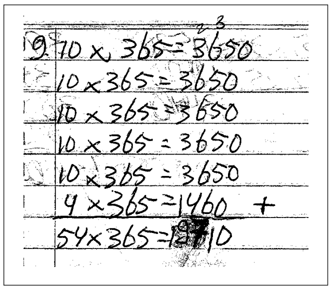
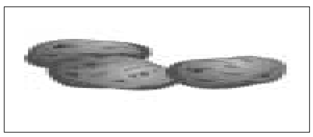


Figure 9.17
Pupils' multiplication by 10 in the second day

Then pupils started solving next two problems, "Potatoes" and "The teacher" problems (see the problems below). The main idea of encountering these problems was that pupils dealt with a rate problem that involved multiplication numbers with zero. The problem also gave opportunities for pupils to practice and use their understanding of multiplication by 10. They developed their understanding while establishing the multiplication by 10 as the efficient strategy to solve the problem. The pupils' reinvented strategies can be seen in the middle and right pictures of Figure 9.17.

H. Potatoes



8. *If the price of 1-kg potatoes is Rp. 905, how much should you pay for 39 kg of potatoes?*

I. The teacher

9. In a year there are 365 days. How many days do your teacher live if she is 54 year-old now?

As mentioned in section 8.2.1, having been learnt the main weaknesses encountered when the teachers conducted the instruction and conclusion activities, particularly in performing discussion process of reinventing the strategies. They thought that discussions about the strategies were not vital activities for enhancing pupils' understanding. In fact the pupils still needed more support to make sure that their manipulating number strategies were correct. It could be seen from their answers in the daily quiz, only 48% of pupils got correct answers. It would have been higher if the teacher conducted the discussions properly. The following figure illustrates the pupils' weaknesses in solving the daily quiz item in the second day.

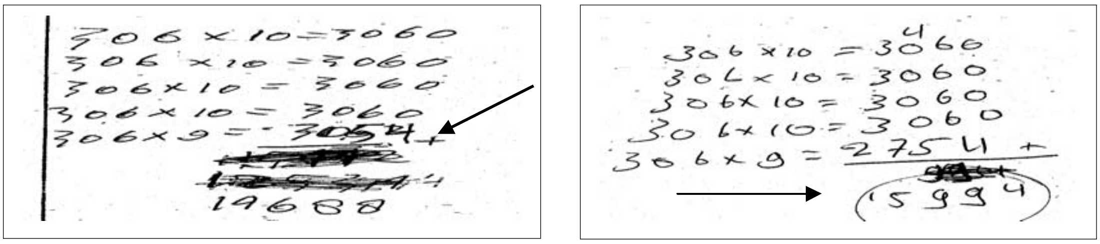


Figure 9.18
Pupils' error in multiplication and addition numbers in the second day (pointed by the author)

c. Day 3

The third day of learning multiplication began with the preliminary activities of practicing multiplication facts, multiplication by 10, and multiplication by multiples of ten. Each pupil solved 10 conventional problems of multiplication. The problems spread over the 3 types of the multiplication mentioned before. The aim was to practice pupils' ability of multiplying numbers using zeros.

The actual learning process started by encountering the contextual problems about "water". The pupils should read the contexts, understood them and solved the problem. The first "water" problem asked the pupils to multiply 35 x 384, which was from: "*An adult can stay alive and clean using 35 liters of water a week; how many liters of water do 384 adults to stay alive and clean every week?*"

Having discussion with teachers and pupils it was found out that pupils had never experienced this situation before. Pupils took more time than expected to understand the situation. The teacher asked them to reread the first problem (about water that people need to live and clean every week) and got connected with which context the problem was related to. Soon after pupils understood the problem and the context related, they used multiplication by 10 to solve the problem. It was found that several pupils still had problem in multiplying 1-digit numbers, they used repeated addition.

After having pupils' answers and asked a pupil to write his strategy, the discussion began. It was aimed at shifting pupils' understanding towards the multiplication by multiples of ten.

Teacher: Do you know how to make this strategy shorter?
Pupils: No …
Teacher: Do you want to know how to do it?
Pupils: Yes ..
Teacher: Let's do it together. (Pointing to a row of the multiplication by 10 of the strategy) How many numbers of 384 are in this multiplication?
Pupils: ………… (silent)
Teacher: Do you still remember numbers in each column two days ago? How many numbers are there in each column?
Pupils: ten
Teacher: Good. If there is a multiplication like this one, how many numbers of 384 in this multiplication?
Pupils: ten.
Teacher: Good. Now there are three row of this multiplication, in all how many numbers of 384 are there?
Pupils: 30.
Teacher: Good. Now we have 30 numbers of 384. The result is 3840 + 3840 + 3840 = 11520 (telling while writing the numbers). Again we have 30 numbers of 384 and the result is 11520, do you know how to write them in mathematics?
Pupil: 30 x 384 = 11520
Teacher: (While writing the equation on the blackboard) 30 x 384 = 11520. Do you know how to find this result if you do it by yourselves? We learnt it already.
Pupils: 3 x 384 first and put 0 behind the result.
Teacher: How many of you can do this calculation? We had learnt it already in the first trimester.
Pupils: (some pupils raise their hands) …
Teacher: Good. Look at this multiplication again. We have 30 x 384 = 11520. Now we can put the last one, 5 x 384 = 1920, below this one and we can add them all, we get 13440. Now we have three ways of solving a problem, repeated addition, multiplication by 10, and multiplication by multiples of ten. Any of you can use any way you understand it better to answer the next problems.

In this learning process pupils who had problems in multiplication facts needed more time in doing the multiplication by multiples of ten. It was found that 20% of pupils who got correct answers still used multiplication by 10 in solving the third item of daily quiz. After all RME learning activities in the third day 71% of pupils solved the item correctly (see Table 8.11 in section 8.2.2 item b). The pupils progressed significantly after the third day of learning, up to 23% from the second day. The following figure illustrates the pupils' reinvented strategies during the learning process.

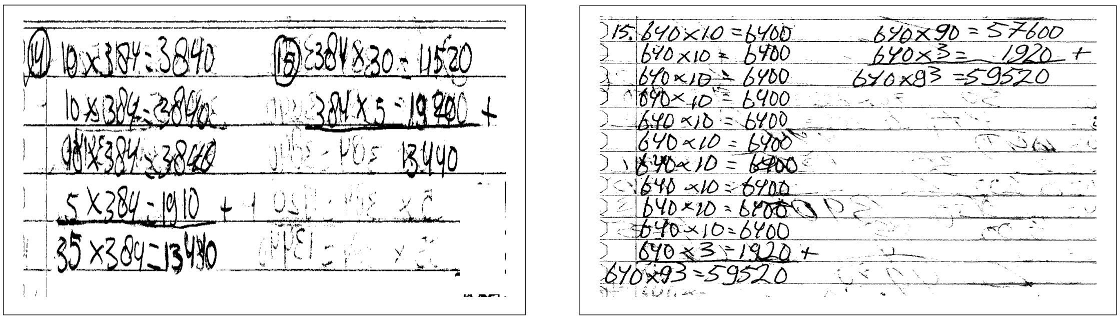
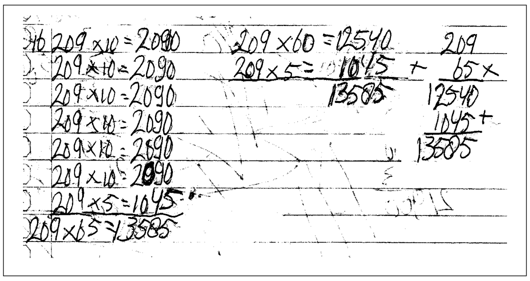


Figure 9.19
Pupils' problem solving procedures in the third day

For some teachers it was not easy to conduct the learning process in the third day. Being afraid of misunderstood by the pupils a teacher shifted from the multiplication by 10 to multiplication by 20. The idea was to use the doubling strategy in multiplying the numbers. According to the teacher this idea would help pupils to ease the multiplication process especially for pupils who had difficulty in multiplication of 1-digit numbers. The main problem was that this idea needed more time to be applied and the pupils still had to adjust the doubling strategy to the numbers that were being multiplied. For instance, in solving the given problem to find 35 x 384 the equation became: 20 x 384 = 7680, 10 x 384 = 3840, and 5 x 384 = 1920. These calculations were less efficient than the following strategy: 30 x 384 = 11520 and 5 x 384 = 1920. The second strategy would easily lead pupils to standard multiplication algorithm (this progress is illustrated in the next section).

During these learning activities, the teacher allowed pupils to use and apply the strategy they understood. The second and third item related to how many liters of water the families needed considering their house condition: 93 families needed 640 liters a day and 209 families needed 65 liters a day. In solving these problems pupils should multiply 640 by 93 and 209 by 65 respectively. These problems required the pupils to deal with multiplication of 0, in some cases some pupils miscalculated it (Armanto, 2000). Most pupils used two strategies (multiplication by 10 and by multiples of ten) to assure its correctness (see the right picture of Figure 9.19). Some pupils structured the standard algorithm while multiplying 209 by 65 (see the left picture of Figure 9.20). This condition made the learning activities accelerate a step ahead toward the fourth day of learning multiplication (see the next section).



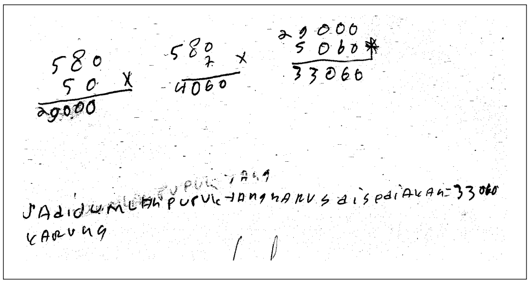
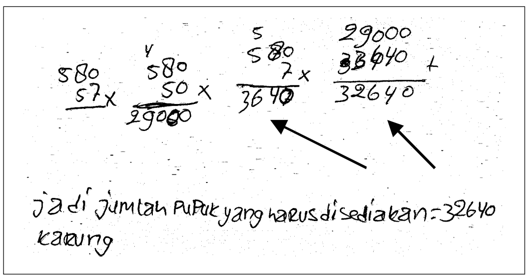


Figure 9.20
Pupils' multiplication by multiples of ten in the third day

At the end of the learning activities pupils encounter the third item of the daily quiz: "Donation". 57 Villages were about to get donation from government. Each village got 580 sacks of urea. Pupils were asked how many sacks of urea the government needed to supply to all villages. Using the multiplication by tents (see the left picture of Figure 9.20 above), 71% of the pupils got correct answers (see Table 7.11 in section 7.2.2 item b). Meanwhile some pupils still had difficulty in multiplying 1-digit numbers, others struggled on multiplying by multiples of ten (see the figure below).



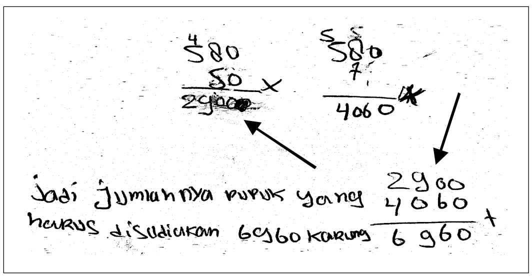


Figure 9.21
Pupils' error in doing multiplication in the third day (pointed by the author)

d. Day 4

The learning activities began with preliminary game of practicing multiplication facts, multiplication by 10, and multiplication by multiples of ten. It was aimed at enhancing pupils' multiplication ability as well as their understanding of several multiplication strategies in solving problems. These activities seemed to be essential since some pupils still lacked of multiplication facts (see Figure 9.21 above). These activities gave opportunities to pupils to rehearse their skills while answering conventional problems. They could also discuss the solutions and tricks (for instance, put the zero behind) at the end of the game. The teacher could also analyze which pupils still had difficulty on multiplying numbers.

Then the learning activities started. Pupils solved the first problem of the day, "A fan". As learnt previously they utilized the multiplication by multiples of ten; however some of them (the low and middle ability pupils) used the multiplication by 10. They preferred this strategy because it was easier and understandable. In order to facilitate the learning process towards the standard multiplication algorithm, the teacher asked two pupils with different strategies to write down on the blackboard. Next, the discussions began.

Teacher: What do you think of these two strategies? Which one is the easiest one? How many of you prefer the multiplication by 10?
Pupils: (Most pupils prefer this strategy)
Teacher: How many of you prefer the multiplication by multiples of ten?
Pupils: (Few pupils raise their hands).
Teacher: Good. The main thing you should consider is the strategy that is easiest and understandable for you. Now we have two strategies of doing multiplication. Do any of you know another strategy that can be used to multiply numbers?
Pupils: ………… (silent).
Teacher: this strategy is called the short method. It is modified from the multiplication by multiples of ten. We have known that 80 x 129 = 10320 and 4 x 129 = 516 (pointing the results from the multiplication by multiples of ten strategy). We can do these calculations down ward like this (writing down the calculation):

	129
	84
(Pupils' written strategies)	*---------- x*
80 x 129 = 10320	*516*
4 x 129 = 516	*10320*
------------------------ +	*-----------+*
84 x 129 = 10836	*10836*

First, we multiply 4 x 129 = 516 and then 80 x 129 = 10320. Then we can add them all, we get 10836. The calculations are the same but the strategies are different. You can use any of them to calculate multi-digit numbers.

In other school the process of learning the standard multiplication algorithm was even easier because some pupils could apply the short method. They learnt it from their extra lessons (scramming schools). The followings were the discussions after two pupils were asked to write their strategies (the multiplication by multiples of ten and the short method).

Pupil: How does she come up with this strategy?
Teacher: I don't' know, ask her and discuss with her.
Pupil: (after the discussion) I do understand the multiplication by multiples of ten but I still don't understand the short method.
Teacher: OK. Let's do it together. How many of you understand the short method?
Pupils: (Few hands rise)
Teacher: Let's compare these two strategies.

	129
	84
	----------- ×
80 × 129 = 10320	*516*
4 × 129 = 516	*1032*
----------------------- +	------------+
84 × 129 = 10836	*10836*

What is the different now? Do you see the differences? Or do you see the similarity?
Pupil: I know that 4 × 129 = 516 and 80 × 129 = 10320. But where is the 10320 in the short method? I see only 1032, not 10320. Where is this 1032 coming from?
Teacher: Any body knows?
Pupil: 8 × 129 = 1032.
Teacher: Why do you multiply 8 times 129? What is the meaning of 8 here?
Pupils: 8 is 80 from the 84.
Teacher: Well, if 8 is 80 then the 8 × 129 should be ....
Pupil: 80 × 129 = 10320, but why does she put only 1032, not 10320?
Teacher: Any body knows?
Pupils: .... (silent)
Teacher: If I put 0 in this 10320 (writing the zero), is there any differences?
Pupils: .... (shake their head)
Teacher: Well ... that is why it is called the short method. To make it short you don't have to multiply the 80 but only the 8 and the result should be put like this pretending you multiply the 80.

After these activities pupils solved the other problems ("A jumping frog" and "Plane and Car") using their own understandable strategies. The teacher asked them to use many strategies they understood to make sure that their result was correct.

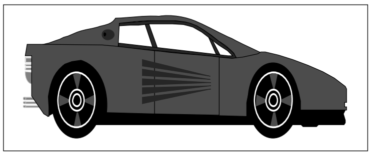
The following were the problems.

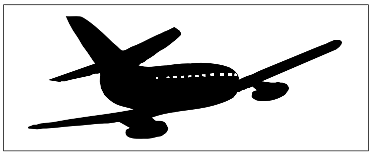
N. A Jumping Frog

21. A frog can jump 27 cm long each time. If the frog jumps 106 times, how long does he jump?

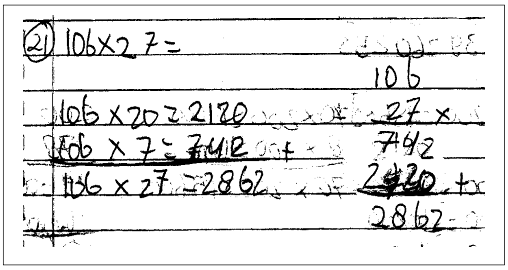
O. Plane and Car

22. Going by a plane is 34 times as quick as going by a car. If in average the speed of a car is 74 km for an hour, how much is the speed of the plane?





The following figure illustrates pupils' strategies in solving those problems.



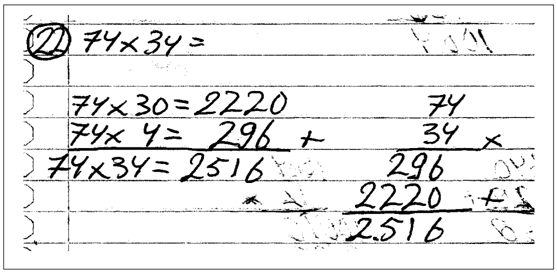
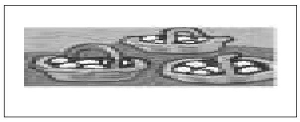


Figure 9.22
Pupils' standard multiplication strategies in the fourth day

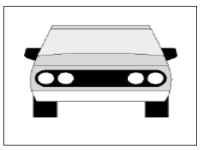
At the end of the day pupils solved 2 items of the weekly quiz ("Buying eggs" and "Biking and walking"). The problems were as follows.

1. Buying eggs



Hamim buys 78 eggs. An egg costs Rp. 475. How many rupiahs does he have to pay?

2. Biking and walking



Riding a car is 207 times faster than walking on foot. What is the speed of the bike if a man walks 6 m per minute?

As mentioned earlier, 62% and 70% of pupils solved the items 1 and 2 correctly (see Table 8.11 in section 8.2.2 item b). From the pupils' answers it was found out that most pupils (41%) applied the multiplication by multiples of ten. Other pupils (38% and 21%) applied standard multiplication algorithm and multiplication by 10 respectively. None of them applied the repeated addition. In addition, most pupils who got correct answer applied multiplication by multiples of ten. They argued that the multiplication by multiples of ten was easier to understand and shorter to calculate. However, most pupils got incorrect answers because of their lack of multiplication of 1-digit numbers. They were in the substance stage (S) where they demonstrated sufficient detail of rational solution but multiplicational error (see arrows in the following figure) obstructed the correct solution process. The weaknesses are illustrated in the following figure.

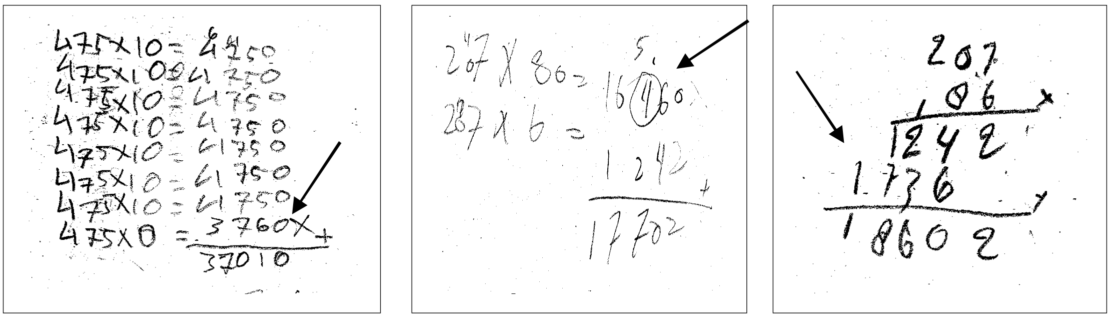


Figure 9.23
Pupils' multiplication errors in the weekly quiz (pointed by the author)

e. Concluding summary

The whole pictures of the learning activities of multiplication mentioned before gave a concise illustration of how the teachers played its role in guiding pupils toward the reinvention process of the strategies. It seemed that the teachers dominated the reinvention process by which the interactive discussion occurred. The mathematical norms was negotiated but with the agreement of the teacher. It was a guided reinvention process; however, it was not the intended ideal RME instructional activities in which pupils pushed the discussion process of reinventing the strategies by themselves. After all, these learning activities could be seen as the bridging instruments toward the ideal RME learning activities. The teachers' competence background hindrance their performances in conducting the learning process as intended.

9.4.2 Learning division of multi-digit numbers

The learning process of division was conducted in four days, 2x 40 minutes each. Each day had different aim to achieve. The first day was aimed at reinventing the unstructured repeated subtraction strategy of multi-digit numbers. The second day until the fourth day was aimed at reinventing the limited structured repeated subtraction, the structured repeated subtraction, and the standard division algorithm. The following sections illustrate the activities toward the learning trajectory.

a. Day 1

In the first day of teaching division the activities began by practicing multiplication facts and multiplication by 10. It was aimed at improving pupils' ability in those prerequisites as well as attracting pupils' attention to the learning process. In the end it motivated pupils to utilize the skills in doing the division strategies.

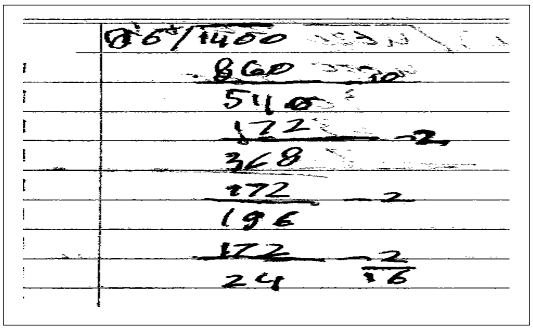
Starting the learning process, the "Lebaran day" problem was introduced (see the problem in section 8.2). The problem related to bringing 1400 people from Jakarta to Surabaya in wagons of train, each wagon could carry 86 people. By reading the problem pupils were attracted to model the contextual problem into formal or informal mathematical forms, for instance drawing picture of wagons.

However, the main distraction was coming from pupils' reaction towards the problem. Their dependency on teachers' order still remained so that they waited teacher to solve the problem. To break the ice of the learning process the teacher guided them to start modeling the mathematical form.

Teacher: Firstly, how many people are going to go back?
Pupils: 1400
Teacher: How many people are in each wagon?
Pupils: 86
Teacher: What is the question?
Pupils: (After a while…) How many wagons are needed?
Teacher: Anybody knows how to start answering the problem?
Pupils: ……… (silent)
Teacher: Let's make a drawing. (After drawing picture of a wagon) How many people can be in?
Pupils: 86
Teacher: (After writing 86 in the picture) 86 in the first wagon. How many wagon needed for the rest? Can you find it?

Pupils: ……………… (silent)
Teacher: OK. If there is one wagon, how many people can go back?
Pupils: 86
Teacher: How many people remain?
Pupils: 1400 – 86 = 1314
Teacher: How many wagons are needed? Can you count them?
Pupils: ……… (silent but working and discussing with their peers. Some of them had the idea of solving the problem)

From these activities it can be perceived that pupils still had dependency on teachers' orders. Considering this condition the teacher attracted them to build their mathematical forms, the unstructured repeated subtraction (see the left picture of the figure below). However some pupils reinvented and used the structured strategy in which they multiply the unit and the ten respectively (compare the right with the left one of the pictures in the figure below).



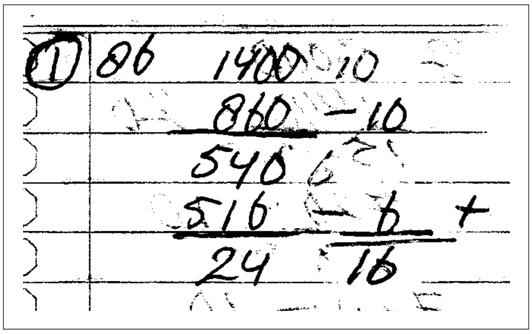


Figure 9.24
The pupils' unstructured and structured strategy

Then pupils improved their understanding by applying this strategy to solve the next two problems in the first day: "*Reading a book*" and "*Students in line*" (see the problems in section 9.2). One of the methods of accelerating the learning process the teacher asking the pupils to build a table of multiplication. This table helped pupils to guess and choose a relative close multiplication and carried out the subtraction several times until they found the solution. However this study found that one third of pupils were lack of subtraction facts, most of them used their hands in subtracting numbers such as 13 – 8 (the learning process of this kind of problems was in first grade). They also lacked of multiplication facts. The teacher should give attention to this prerequisite knowledge. Discussing of these weaknesses with pupils would be reasonable activities to improve their awareness.

The following figure illustrates the pupils' reinvented strategies in solving the "Reading a book" and "Students in line" problems.

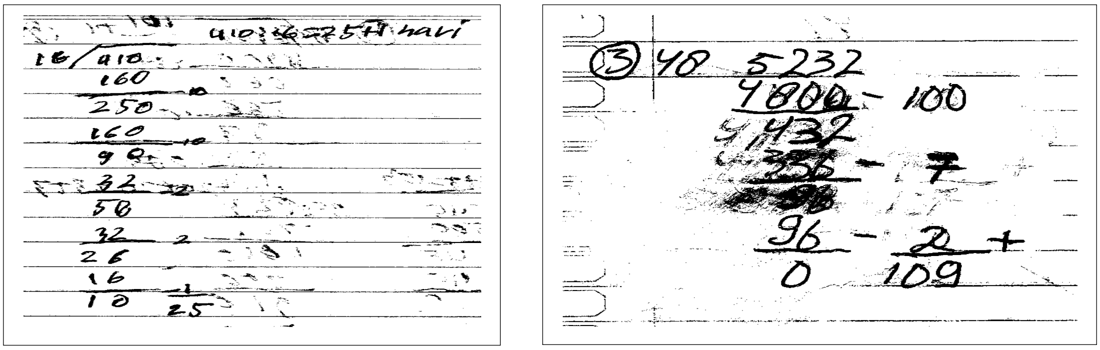


Figure 9.25
Pupils' unstructured and limited structured strategy

The learning process ended with discussing "The supporters" item of the daily quiz. The problem was as follows.

A. The supporters

3000 supporters are going to Jakarta from Bandung by bus to see the football match. Each bus can carry 72 supporters. How many buses are needed?

It was found that 33% of pupils got correct answers. Most pupils still struggled in substantial level, meaning that they were able to do the strategy but lack of multiplication and subtraction made them incorrectly conduct the calculation. The following figure illustrates the mistakes.

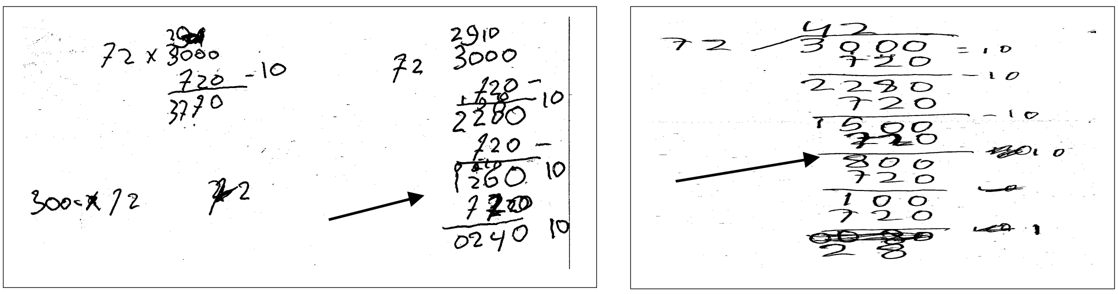


Figure 9.26
The pupils' weaknesses in the first day (pointed by the author)

b. Day 2

The second day of learning began with practicing multiplication by 10, 100, and by 1-digit numbers. It was aimed at improving pupils' lack of multiplication that was found in the first day (see mistakes in the figure 26 above). The learning of division started at encountering the "Graduation" problem. The problem is as follow.

G. Graduation

7. *There are 4630 people in the graduation ceremony. The OC should serve them with water. A bottle of instant water serves 23 persons. How many bottles must be prepared for the people?*

The above problem required pupils to use their former unstructured repeated addition strategy. During the learning process of reinventing the solution, teacher browsed around the classroom asking questions and giving hints. The teacher discussed the problem with pupils who still got difficulties in understanding the problem. Suggestions such as "read it again carefully" or "read it sentence by sentence" were heard several times indicating the teacher asking the pupil to reread the problem and try to understand it again. It was followed by teachers' questions: "what does the number 4630 stand for?" or "what is the meaning of number 23?" denoting that teachers' tried to build pupils' understanding of the problem. In encountering pupils' difficulties in understanding the contextual problems, the RME theory suggests to ask pupils to read and tell the problem on their own words. Another suggestion related drawing a picture and making a connection of numbers that involved in the context.

After a while, the teacher reminded pupils of the significant of being careful in multiplying and subtracting numbers and asked pupils to reassure the correctness of their solution. It was because of pupils' careless in doing the calculation.

Next, the teacher asked a pupil to write down her answer on the blackboard. The teacher guided the discussion toward the aim of the second day of learning, the limited structured of repeated subtraction.

Teacher: How many of you get this solution? 202 bottles.
Pupils: ……… (Some hands rise)
Teacher: Do you have any other solution or strategy rather than 100 + 100 + 1 + 1?
Pupil: Yes, I get 201 with 7 left over. Why is it 202?
Teacher: What is the meaning of 7 here?
Pupil: 7 Mean the people who did not get water yet.
Teacher: Good … if there are 7 people left, how many more bottle needed to serve these people?

Pupil: 1 bottle.
Teacher: if the OC needs 1 more bottle than how many bottle needed together?
Pupils: 201 + 1 = 202.
Teacher: Good. Now you have the answer why it is 202. (After a while …) Do you know how to make this strategy shorter?
Pupils: ………… (silent)

Teacher: Try to unite the hundreds together, or the tens together, or the unit together, you will find the answer.
Pupils: …………… (silent and try to do the teachers' suggestion)

This time the teacher did not guide pupils through out the process but just asked them to figure out the strategy themselves. It was the teachers' way of guiding pupils to reorganize their understanding. And most pupils came up with a strategy that unites the hundreds together. This strategy was a type of limited structured of repeated subtraction (see the left picture in the following figure). However, most pupils still used the unstructured strategy in solving the daily quiz (see the right picture of Figure 27).

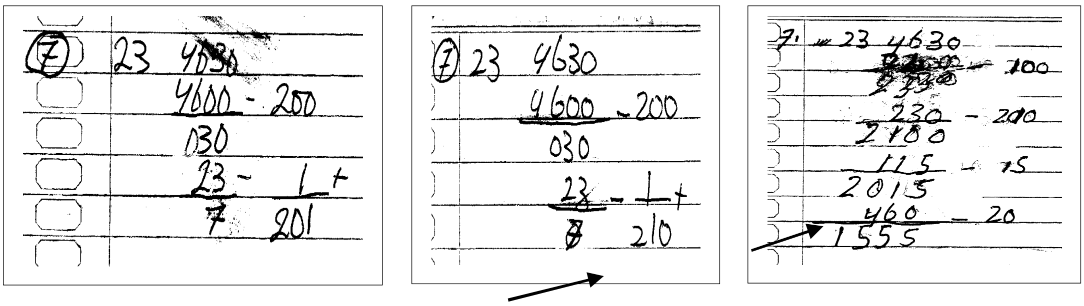
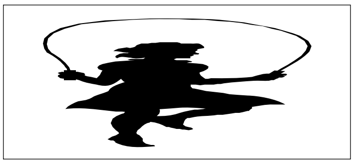


Figure 9.27
The pupils' limited structured and its mistakes (pointed by the author)

During the discussion it was found out that only several pupils gave their opinions. It means that most pupils were still reluctant to voice their idea and way of understanding the problem. Pupils' attitude towards the discussion process did not improve much. Most pupils were still afraid of giving their opinion and reluctantly asking questions. From the learning activities and the discussions illustrated above it can be seen that the teacher left little room for personal intervention. The teacher dominated the discussions. As a consequence the pupils expected their proposals to be judged by the teacher as right or wrong.

After the discussions illustrated above the pupils then encountered the "Jumping on the rope" and "Buying candies" problems. These problems were provided in order to give pupils chances to practice their understanding of the limited structured strategy that being learnt. These were also aimed at reiterating the meaning of "left-over" in the calculation. Pupils had to use the context to answer the problems correctly.

H. Jumping on the rope



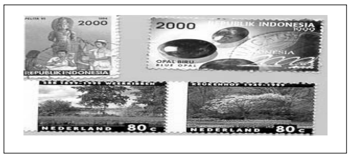
8. The record for the greatest number of consecutive jumping on the rope is 8960 jumps. There were 56 jumps in a minute. How many minutes does the record take place?

I. Buying candies

9. A candy costs Rp. 75,-. How many candies do you get if you have Rp. 20000?

At the end the pupils solve an item of the daily quiz, "The price" problem. The problem was as follow.

B. The price



Jagi has 5712 stamps in his album. There are two dozen stamps on each page. How many pages are there?

It was found that 37% of the pupils got correct answer (see Table 7.10 in section 7.2.2 item b). There was a 4% increasement of pupils' correct answers; however, most pupils still had substance difficulties in conducting the calculation. Their lack of multiplication and subtraction subtracted their understanding of the strategy.

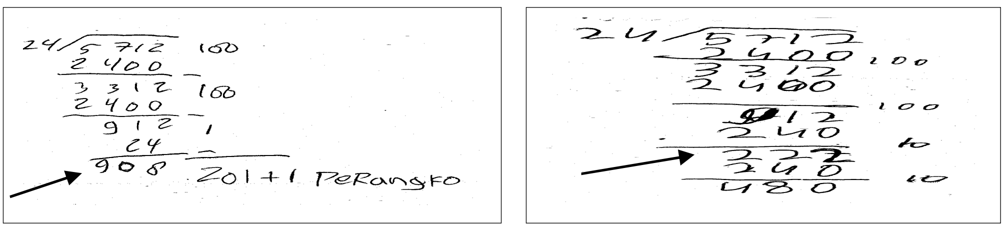


Figure 9.28
The pupils' subtraction errors in the second day (pointed by the author)

c. Day 3

Starting at practicing the multiplication by 10, 100, and tens, pupils were given 10 items of the multiplication facts. It rehearsed pupils' capability in multiplying numbers. This was aimed at helping pupils to develop their own understanding of multiplication by applying strategy, such as moving one or two zeros in multiplying numbers by 10 or 100 respectively. It also attracted and motivated pupils toward the learning activities being conducted in the classroom.

The first problem encountered in the third day of learning division was "Using water" problem. The problem was about the amount of water a chicken needed for a day before it can be sold in the market. In fact in 15 days before it can be sold, a chicken needed 1545 liters of water. Having this problem in the context of "water", many pupils still had difficulties in finding which context for what problems. For helping to understand the problem the teacher then discussed the problem together. Asking several questions, such as "what is the question here?" and "which item is related to this question?" encouraged pupils to answers, especially the pupils who understood and knew the relation in the first place after reading the context.

After the discussion of understanding the problem the teacher then asked pupils to find the solution by trying the strategy they understood well. The teacher then browsed around to see which pupil still had problems in understanding the problem and had difficulties in starting the calculation. The following was the conversation between the teacher and a pupil (Wahyu) to help her to understand the problem and finding the solution.

Teacher: what is 1545?
Wahyu: The amount of water.
Teacher: What is 15?
Wahyu: The number of day.
Teacher: What is being asked?
Wahyu: The amount of water needed in each day.
(In this condition it seemed that Wahyu did not have any difficulty in understanding the problem).

Teacher: What are you going to do to find the solution?
Wahyu: Divide them.
Teacher: OK … can you do it? I want to see you calculate them.

Wahyu: Writing:

```
15    1545
      1500
    ---------- --   100
        45
```

Teacher: What do you think of 100? What is 100 in this calculation?
Wahyu: …………… (silent)
Teacher: What is 1500? How do you find it?
Wahyu: …………… (silent)
(This conversation implies that Wahyu understood the problem but calculated blindly without understanding. Then the teacher asked him to read the problem again).

Teacher: If there are 100 chickens, how many liters of water needed?
Wahyu: ………… (silent)
(In this condition Wahyu had no multiplication understanding yet).

Teacher: OK … Let see another problem first. If there are 5 boxes, each box contains 2 books. How many books are there all?
Wahyu: 10 books.
Teacher: How do you find it?
Wahyu: ………… (silent).
Teacher: Draw pictures of boxes. How many boxes? How many books inside?
Wahyu: 5 boxes (then he draws five boxes) with two books.

2	2	2	2	2

Teacher: How many boos are there all?
Wahyu: 10
Teacher: How do you find it?
Wahyu: I add them all (2 + 2 + 2 + 2 + 2 = 10)
Teacher: Can you use multiplication?
Wahyu: Yes … 5 x 2 = 10.
(It means that Wahyu understood the multiplication concept. The researcher believed that it was because of the small numbers involved in the calculation. The teacher utilized Wahyu's difficulty of multiplication concepts by reminding him about the relation between multiplication and addition involving small numbers).

Teacher: Good … Lets go back to our former problem.

(Using Wahyu's hand writing the teacher asked the following questions again: what is 1545?; what is 15?; what is 100?; what is 1500?; and how do you find it?)

Wahyu: (at last…) … 100 × 15 = 1500.
Teacher: Good. Then you subtract them. You find this number (pointing to the 45). What is the 45?
Wahyu: The rest of the water.
Teacher: What are you going to do with it?
Wahyu: Divide it with 15.
Teacher: Why?
Wahyu: I want to find the water needed more.
Teacher: Good… How much is it?
Wahyu: 3 … I think.
Teacher: Why?
Wahyu: Because 3 × 15 = 45.
Teacher: How much is it for the answer?
Wahyu: 103 liters.

These long conversations showed that there was a need of asking pupils a guiding question to analyze their understanding of the problem, their understanding what they were doing in the calculation, and their understanding of the mathematical tools being used in the calculation. Another aspect to be account was to take the pupils back to the mathematical concepts; decreasing numbers that involved in the calculation was one of the ways. It helped pupils to realize its connection to the multi-digit numbers. The following was the pupils' reinvented strategy.

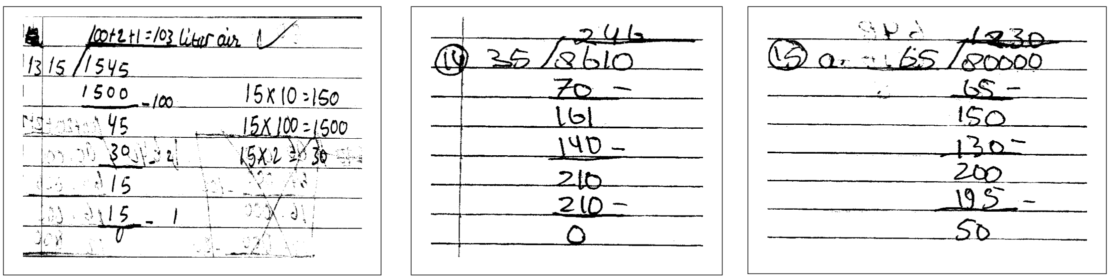
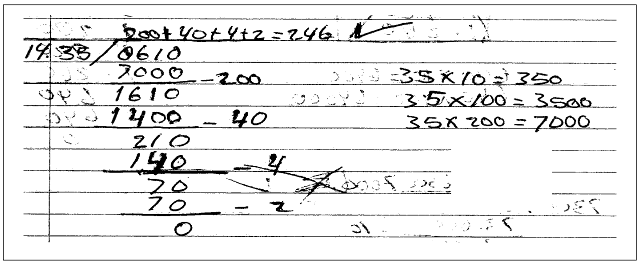


Figure 9.29
Pupils' problem solving strategies in the third day

After having these long conversations the teacher asked a pupil to write her solution on the blackboard. Asking how many pupils got the right answer then the teacher asked them to find the solution of the next problem. The problem asked about how many weeks an adult can stay alive and clean using 8610 liters of water if it was known that 35 liters of water was needed to be alive and clean. For this problem most pupils applied unstructured repeated subtraction. Some of them used the limited repeated subtraction.



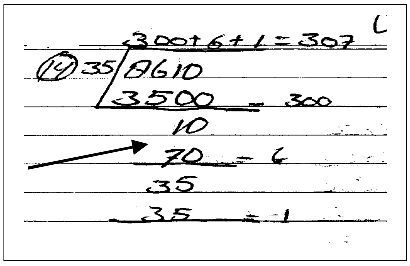


Figure 9.30
Pupils' strategies and its mistakes (pointed by the author)

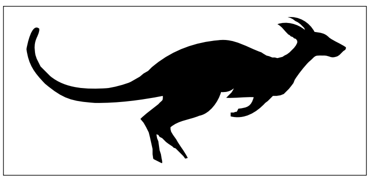
d. Day 4

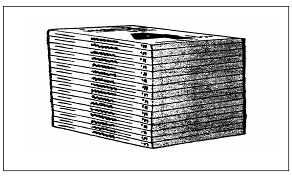
The fourth day of learning was aimed at finding out the standard division algorithm. It began with having a game of multiplication facts, by 10, tens, 100, and 1000. This strategy was aimed at improving pupils' capability of multiplying numbers. This strategy was useful and easier for most pupils to understand. However, for some pupils this game was not reliable enough because they still had difficulty in understanding the multiplication facts. Their ability did not improve as intended. It could be seen from their answers in solving problems in weekly quiz.

The contextual problems the pupils encountered were about "Kangaroo's Jump" and "Stack of Papers". The pupils were asked to answer the problems by dividing 7700 with 72 and 45240 with 29. The first item involved the numbers that there was still left over (remainder). These problems are structured based on the idea of giving pupils opportunity of encountering various problems (see Table 9.1 in section 9.2) that can be solved using the repeated strategy.

A. Kangaroo's Jump

1. *Kangaroos live in Australia. A kangaroo can jump approximately 72 cm long. How many times he should jump in order to pass a street that is 7700 cm long?*





B. Stack of papers

2. *The stack of papers in right figure above equals 29 times as many as a book. If the stack contains 45240 pieces of papers, how many pieces of papers are there in the book?*

In order to answer the first problem most pupils applied the limited structured strategy (see the left picture of the following figure) and some pupils applied the standard algorithm (see the right picture of Figure 9.31 below).

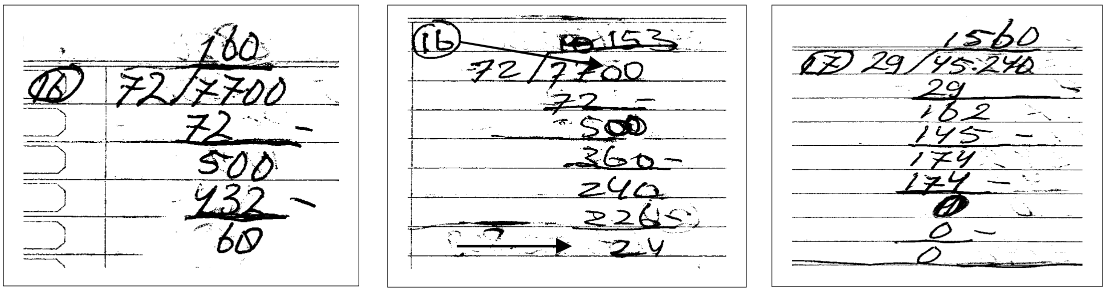


Figure 9.31
Pupils' standard algorithm and multiplicational error in the fourth day (pointed by the author)

The middle picture of Figure 9.31 above showed that pupils still made mistakes in dividing the numbers using the standard algorithm (see the middle picture of Figure 9.31 above). It showed that pupils did not understand the meaning of the first number of the solution as 100 and the second number of the solution was 6 with 68 meters as the leftovers. It would have not been calculated that way if they had used the structured repeated subtraction. Using this structured strategy, pupils realized the meaning of 1 was 100. Another aspect found out that the teacher did not discuss this problem during the learning process. It was because the time available was not enough to discuss this condition.
The discussion began whenever the teacher asked a pupil to write down her strategy on the blackboard.

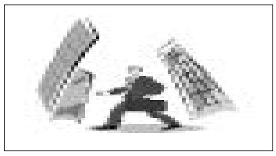
Teacher: How do you find the solution here? (While pointing to the pupils' answer).
Ani: I construct these multiplication tables (Showing her calculations from her portfolios).
Teacher: Would you write down the multiplication table on the blackboard? Put them at the side of the solution.
Ani: (Write down the multiplication table).
Teacher: How do you build them?
Ani: I just multiply the divisor by several numbers, for instance: 2 – 9. After I got the answer, I can put 0 on them if I multiply them by 10, or two 0s if I multiply them by 100.

*Teacher: This is a very good idea. (*Talk to other pupils) *Is there any of you have any other idea, different from this multiplication table?*
Pupils: …… (Silent)
Teacher: Alright. Do you have any questions about this?
Pupils: …… (Silent).
Teacher: If you get the idea of building the multiplication table than I think you would better to use them to answer the next problem.

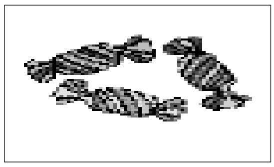
The conversation showed that some pupils built multiplication table to answer the problems. The table helped them to guess the numbers to be put as answers for the division problem.

At the end of the learning process the pupils encountered two items of the weekly quiz. The items were as follow.

1. ***Walking***
   Pak Amat always walks to his office. The distance is 1800 m from his house. If he walks for 82 m in a minute, how many minutes does he walk to his office?



2. ***Candys***
   The price of a "COKELAT" candy is Rp. 75. If you have Rp. 9800, how many candies do you get using all your money?



The results mentioned in Table 8.10 showed that 67% and 60% of pupils got correct answers for answering item 1 and 2 respectively. It meant that most pupils still had problems in associating the contexts with the numbers that were leftovers after the calculation. The other problems related to how to guess the number being used as the answer (see the middle picture in the figure below) and the correlation between the number of the answer and the results of the multiplication (see the arrows in the right picture of the following figure). It can be concluded that pupils' mistakes of multiplying numbers became the most considerable problems for the pupils in getting the correct answers. These weaknesses obstructed pupils' performances as well as their engagement in the learning activities.

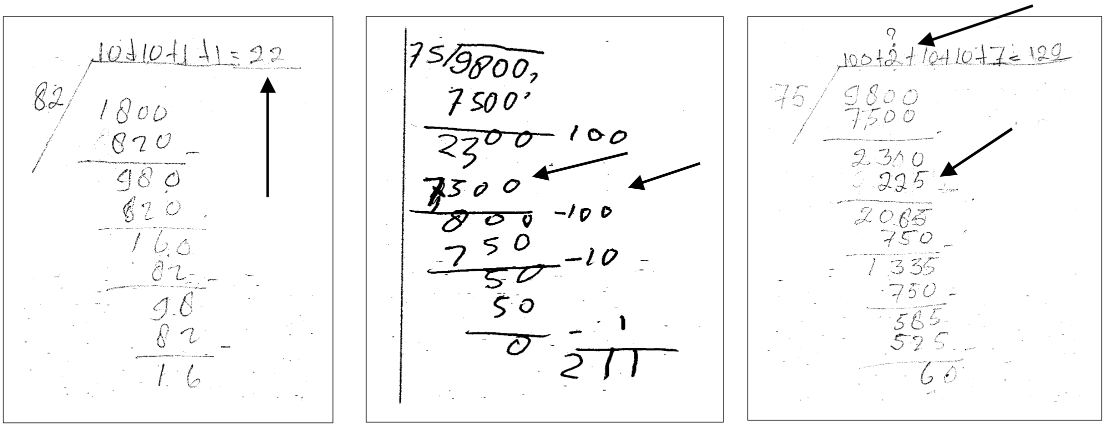


Figure 9.32
Pupils' standard algorithm and its multiplicational errors in weekly quiz (pointed by the author)

9.5 CONCLUSIONS TOWARD A REAL-LIFE CLASSROOM

The proposed learning route in teaching multiplication and division of multi-digit numbers began with the preliminary game in which pupils practiced their multiplication facts, multiplication by 10, tens, 100, and 1000. It was found out that this games attracted pupils' attention, motivation, and former knowledge and helped them to improve their ability of multiplying numbers. However, pupils who had difficulty of multiplication facts got less improvement. They needed more time to perform better and to engage actively in the learning activities.

Pupils' learning trajectories of learning multiplication of multi-digit numbers began with reforming the repeated addition strategies. The interactive discussions guided them toward figuring out the efficient strategy: the repeated addition of ten numbers. Then the multiplication by 10 took place. It was guided by comparing the strategies that were more efficient and understandable. The teacher asked pupils to focus on the representation of the amount of numbers in each column being added. Next, the multiplication by multiples of ten was represented as the amount of the multiplication by 10 that involved in the calculation. The last strategy was the standard multiplication algorithm (called the short method). It was realized by: (1) arranging the calculations downward and (2) comparing the multiplication by multiples of ten with the standard form. The important aspect in these learning activities was that the pupils were allowed to use the strategy they were comfortable with. It was also essential that pupils used connections between repeated addition and other strategies.

Because the pupils' learning attitude (dependency toward teachers' order and tightly learn only one strategy at a time) still existed. It was considerably essential to guide pupils toward those learning trajectories. Interactive discussion were also conducted and accelerated by giving guided questions. Encouragement was also needed to build pupils' self-confidence in learning the subjects individually or in small groups.

The learning activities of division of multi-digit numbers started at the repeated subtraction. It was the unstructured repeated subtraction in which pupils' prediction of the numbers used was developed randomly. Then the limited repeated subtraction was represented by the numbers involved which was gathered as units, tens, or hundreds separately. For instance, 9870 : 35 = 200 + 50 + 30 + 1 + 1 (the hundreds are gathered together). Next, the structured repeated subtraction was understood by gathering the units, tens, or hundreds together, for instance: 9870 : 35 = 200 + 80 + 2. The last reinvention was the standard division algorithm, in which the pupils conducted the short method in which the pupils divided the numbers separately and the answer was as a whole, for instance: 9870 : 35 = 282. It was found out that pupils mostly used the unstructured repeated strategy because it was easier and understandable without having to guess the number that was closed to the intended one. Another finding was that those strategies could be reinvented disorderly; however it was found that the order proposed in this study was the reasonable way to learn the long division algorithm. For accelerating pupils' understanding, structuring table of multiplication for the divisor was found useful and important. Most pupils who got correct answers applied this strategy.

During the teaching experiments in the classroom the role of the teacher was found essential to facilitate pupils' learning activities. To make pupils familiar with the RME teaching approach was one of the reasons. Another one related to pupils' attitude toward learning. Dependency of teachers' orders made pupils' was hesitantly engaging in the learning activities. Used to learn, to apply, to strict to only one strategy that obstructed pupils' creativity to develop their own understanding by using their former knowledge. For encouraging pupils to actively participate in the learning process, teachers' guidance questions and hints were needed significantly. It led pupils to discuss, comment, answer, and refine their own understanding and build the mathematical norms for their own knowledge.

However, many teachers did not realize the essential of the regulation activities reflecting on pupils' learning progress, taking recovery actions, maintaining motivation, and generating feedback. These activities were considered as integral actions of maintaining and establishing mathematical norms in the classroom during the learning process. One of which they always did was giving correction to pupils' answers (right or wrong). It was believed that more pupils would have performed on the expected level of understanding if the teacher had conducted the regulation activities regularly.

The ideal RME real-life classroom in the future would begin with the discussion of contextual problems that involved in learning the subject. The teachers' guided questions and hints helped pupils to create their own strategies to solve the problem. All strategies are reinvented by the pupils. Then the strategies are discussed and refined to find the best, understandable, and efficient one to solve the problems. The teachers' role is for facilitating and accelerating the discussion interactively in which every pupil got opportunities to give their opinion, give and answer the questions, give reasons and rationale of the strategies, and apply them in different situation. The questions such as *"What kind of operation should I use to solve this problem?"* and *"How should I start to calculate the numbers?"* are answered by asking the following guidance questions:

- What is the context about? Would you describe it in your own word?
- What do the numbers stand for and what is its correlation?
- Does anyone have a plan or strategy to solve the problem?
- Why does this work?
- Would it always work?

These questions would guide the pupils to organize their thinking process in reinventing the mathematical forms to solve the problems. The pupils can also learn from other pupils when they discuss and answer the questions. This is an interactive activity to bridge from contextual problem towards the model of situation. It is an activity of "model-of situation" in the third principle of RME (Gravemeijer, 1994). This would guide the pupils to start to reinvent the situational problem into the formal or informal mathematical form (see the figure of the long term learning activity). De Lange (1992) described it as "to organize, structure the problem, to identify the mathematical aspects of the problems, and to discover regularities and relations". This exploration leads to develop, enlarge and enrich the reinvention of mathematics concepts.

After having the (informal or formal) mathematics forms the pupils have been in the second step (referential level) of emergent model (Gravemeijer, 1997). The pupils create and reinvent the "model-of" the contextual problem, in which they figure out the connection between the context and numbers and put it into the informal mathematics figures. It is a model and strategy that emerged as a result of progressive mathematization from intuitive informal strategies to more abstract procedures (bridging by vertical instruments, the second characteristics of progressive mathematization, in Treffers, 1991 and Gravemeijer, 1994). And then the pupils develop their own thinking and understanding by building on the "model-for" problems until they have the formal knowledge of the multiplication and division procedures.

The learning trajectory will guide the pupils to learn the procedures on their own pace under the guidance of the teacher. The teacher can consider the learning trajectory as a guide to facilitate the pupils' process of learning multiplication and division. For instance the teacher can conduct the discussion as follow:

1. Teacher provides time and opportunity for pupils to distinguish individual solutions. It is better to ask several pupils to write down their solution on the blackboard.
2. Ask them to describe their solutions or procedures they used
3. After having 3-4 pupils writing down their solution, ask the following questions:
   - *How many pupils understand each procedure?*
     This question is used to find out the percentage of the pupils that understand the procedures used by their peers and to compare the procedures the pupils favor of.
   - *Why do they understand it and how?*
     This question is aimed at finding reasons mathematically (formal or informal). By answering this question pupils justify and explain the contextual problems using their own words. It refers to the context and numbers involved in the problem. They might illustrate it by using pictures or figures.

- *Why do you like it?*
  These questions are to find the didactical thinking process of the pupils when they use the procedures to solve the problem.
- *How many pupils understand and use several procedures? How do you understand them? How do you compare them? Why does it work? Would it always work?*
  These questions are used to find the percentage of the pupils' understanding several procedures. This percentage will help the teacher to see the development process of pupils' understanding. By answering those questions, pupils will describe the mathematical contents, procedures, strategies, and reasons they utilize. The teacher also has opportunity to analyze and improve pupils' understanding. This discussion has a significant effect to other pupils' understanding.
- *Ask pupils to explain their procedures in their own words.*
  This question will ask the pupils to describe their thinking process and their understanding of the mathematics procedures.

These teaching activities (including the questions being proposed) should be conducted interactively. It needs teachers' good competencies, not only in asking the questions but also in managing the classroom (as a whole class or individually or as small groups). The interactive discussions will develop and establish pupils' understanding as well as the classroom mathematical norms. These aspects are essential for building pupils' attitude toward learning mathematics in the future, decreasing their dependency on the teachers' orders, enhancing their creativity, and improving their performances. Indeed, these main objectives of teaching and learning mathematics are in line with the intended goals declared in the 1994-mathematics curriculum for the Indonesian primary schools. However, the most important aspect to be taken into account for conducting the RME teaching approach is accessible time; for the pupils to get familiar with various strategies, to develop a need for improvements, to relate the various strategies being used (e.g. Figure 8.1 on page 126). It is also essential for teachers to get familiar with the teaching approach; to practice their understanding of pupils' learning cognition, and to build their own teaching knowledge.

CHAPTER 10

CONCLUSIONS AND RECOMMENDATIONS

Chapter 5, 6, 7, 8, and 9 presented the research findings of this study concerning the quality aspects of the RME prototype and the local instructional sequences of teaching multiplication and division of multi-digit numbers in Indonesian primary schools. This chapter discussed the conclusions of this study; starting at summarizing the research question, the research design, and the main findings. The sections include giving reflection on pupils' learning performance, teachers' teaching performance, and other developmental research study. This chapter ends up with recommending further suggestions regarding pupils' performances, teacher education, curriculum developer, and policy makers in Indonesia.

10.1 SUMMARY

The implementation of the 1994 mathematics curriculum in Indonesian primary schools is focusing on the teaching and learning of arithmetic. The goals are to prepare the students to use and apply their mathematics knowledge and mathematical way of thinking in solving problems in their life and in the learning other different subjects (Depdikbud, 1995). In conducting the learning process, the curriculum suggests the application of the student centered teaching model in which the teaching activities give pupils opportunities to develop their own understanding and skills.

Contrarily, in realities most teachers utilized the chalk and talk strategy combined with the concepts-operations-example-drilling approach (Suyono, 1996). This model of teaching is called the mechanistic way of teaching (Treffers, 1987). The teachers teach mathematics with practicing mathematics symbols and emphasize on giving information and application of mathematics algorithms. During the instructional process this typical teaching and learning in developing country (Feiter & Van Den Akker, 1995; Romberg, 1998) occurs regularly (see section 1.2.2).

There were many reasons found. One of which was because the mechanistic character of the instructional textbooks used by the teachers (see section 2.5). The textbooks had essential influences on teachers' way of teaching. Their dependency to the textbook as the teaching materials in the classroom structured their performances in teaching the subject. The mechanistic mathematical contents in the textbooks guided them to perform the conventional teaching approach (see Figure 2.1 in section 2.5).

The other reason was the teachers' low quality of knowledge and skills (BPPN, 1996; Suyono, 1996). It refers to teachers' low quality of understanding mathematics contents and teachers' pedagogical knowledge and skills of teaching mathematics. Suyono (1996) found that teachers (1) have low ability in using variety of teaching methods and (2) teach using conventional methods without considering the logical thinking, critical and creativity aspects of the subject matter. The second aspect relates to the pupils' learning cognition. It can be concluded that the instructional materials and the teachers' competencies (understanding mathematics contents, pedagogical aspects, and pupils' learning cognition) was the most essential element that should be improved progressively (see section 3.2).

Those teachers' weaknesses influence the learning of mathematics in Indonesian primary schools into a deep annoyance. The students' drawbacks (Armanto, 2000; Haji, 1994; Jailani, 1990) have made mathematics more difficult to learn and to understand and the students have become afraid of mathematics. In solving multiplication and division problems for instance, pupils conducted 'buggy' procedures (see section 2.5); the strategy pupils developed incorrectly. The strategy represented the effects of learning the subjects conventionally because they had nothing to fall back upon. Another example was coming from TIMSS reports (1999) that Indonesian students' scores was in the 33rd level from 37 countries in TIMSS evaluation.

Given all these shortcomings of the Indonesian current mathematics education, it may be concluded that there is a need of innovation in implementing mathematics curriculum for Indonesian schools. *Firstly*, good approach in teaching mathematics is needed so that pupils can understand and master mathematics facts, concepts, procedures, and operations are definitely needed. *Secondly*, good inservice education is needed to improve the teachers' competencies to teach mathematics in the classroom. *Thirdly*, a theoretical based formal curriculum and its prototypical materials should be structured as a basic foundation for the implementation of the representative teaching approach.

Based on studies and programs being developed in many countries (see Becker & Selter, 1996; Cobb, Wood, & Yackel, 1991; De Lange, 1994; Gravemeijer, 1997; Romberg, 1994; Treffers, 1987), it is believed that realistic mathematics education (RME) is an approach that can address the problems mentioned above.

RME theory is compatible with the idea of mathematics as a human activity (Freudenthal, 1983). In this philosophy, the mental activity of the learner is at the center. Mathematics ought not to be associated with mathematics as a well-organized deductive system, but with mathematics as an activity of doing and reinventing mathematics (mathematizing subject matter). The subject matter can be taken from reality and it must be organized according to mathematical patterns. Analyzing and reflecting ones' own mathematical activity is the main key principle of reinventing mathematics. In RME, teaching mathematics realistically starts with encountering contextual problems. Guided by the teacher, students develop their understandings by utilizing their former mathematics knowledge. Interactive discussions and negotiations of mathematical norms (effective and efficient frameworks) are the key elements in building students' understanding. Uniformity and formalism in pupils' mathematical strategies are not essential in this model. The key issue is on building students' understanding at a certain expected level of performances that allow them to develop their own effective mathematical frameworks to solve mathematical problems. The RME theory has been summarized in section 3.3.

By believing that the RME theory is an appropriate approach for Indonesia, this study constructed and provided the RME formal curriculum materials with which the teachers can practice the RME teaching approach under the guidance of the researcher. It is inline with the suggestion from Loucks-Horsley et al., (1996) and Feiter and Van den Akker (1995) that conducting a new formal curriculum implementation by constructing, learning, using, and refining a prototypical instructional material in the classroom is believed as an alternative effort to improve the conditions. The multiplication and division of multi-digit numbers was chosen as the subjects for this study, as it can be assumed that the Indonesian teachers understand its contents very well. Based on these elements the following research question was formulated for this study.

> *What are the characteristics of an RME prototype for teaching and learning of multiplication and division of multi-digit numbers in Indonesian primary schools?*

To address the research question mentioned this study utilized a developmental research approach (Freudenthal, 1991; Gravemeijer, 1996; Richey & Nelson, 1996; Van den Akker, 1999; and Van den Akker & Plomp, 1993) as the most suitable approach to investigate the development, implementation, and improvement of an RME prototypical product (see section 4.2). This study was aimed at determining characteristics of an RME prototype were investigated at two levels: the learning level and the curriculum level. In the learning level it concerned to the characteristics of the RME local instructional sequences for teaching multiplication and division of multi-digit numbers in Indonesia. The sequences were built in three components: (1) learning goals for pupils; (2) planned instructional materials; and (3) a conjectured learning activities (Gravemeijer & Cobb, 2001). In the curriculum level it referred to the substantive emphasis; the quality aspects of the prototypical instructional materials: validity, practicality, implementability, and effectiveness (see section 4.2.3 item b).

This study had been carried out in two phases of research activities. *The prototyping phase* was executed in three stages; focusing on developing, implementing, and revising the prototypical materials. This consisted of a process of the front-end analysis, expert reviews, teaching experiments, and reflections (see section 4.2.3 item c). The results obtained in one stage were used as input for the next stage. The prototyping phase resulted in a try-out version of the RME prototype, which was tested in assessment phase. *The assessment phase* focused on evaluating whether the RME prototypical materials were used as intended and whether the pupils performed on the expected level of performances after having engaged in the RME learning activities.

The next section of this chapter discusses the main findings of this study (section 10.2). It summarizes the characteristics of Indonesian local instructional sequences (section 10.2.1) and the RME prototype (section 10.2.2) for teaching multiplication and division of multi-digit numbers in primary schools. Then section 10.3 illustrates the reflections of this study, regarding pupils' learning performances, teachers' teaching performances, and further developmental research work. In the next section (10.4) recommendations are given. At the end of this chapter an epilogue is illustrated concerning the need of reforms in Indonesian mathematics education.

10.2 MAIN FINDINGS

This study reports its main findings on two insights: the characteristics of the local instructional theory and the quality aspects of the RME prototype. Each of these insights is elaborated in the next sections.

10.2.1 The characteristics of local instructional theory

In this study, the characteristics of local instructional theory referred to the explicit formulation of instructional theory in teaching multiplication and division of multi-digit numbers for Indonesian primary schools. The theory has three components: (1) learning goals for pupils; (2) conjectured learning route; and (3) planned instructional materials (Gravemeijer & Cobb, 2001).

This study structured the learning goals for pupils in learning multiplication and division of multi-digit numbers as in Table 10.1 follows. The RME experts had judged these objectives during the prototyping phase.

Table 10.1
The objectives of teaching multiplication and division

Contents in the RME prototype	Objectives and sub-objectives	Hours
Multiplication	*The pupils can understand, explore, and justify the conventional algorithm for multiplication in terms of repeated addition and the decimal numbers*	
▪ Repeated additions of ten numbers	▪ *Pupils can use repeated additions of ten numbers* ▪ *Pupils can use multiplication by 10*	2
▪ Multiplication by 10	▪ *Pupils can use multiplication by multiples of ten*	2
▪ Multiplication by multiples of ten	▪ *Pupils can use standard multiplication algorithm*	2
▪ Standard multiplication algorithm		2
Division	*The pupils can understand, explore, and justify the conventional algorithm for division in terms of repeated subtraction and the decimal numbers*	
▪ Unstructured repeated subtraction	▪ *Pupils can use the unstructured repeated subtraction*	2
▪ Limited structured repeated subtraction	▪ *Pupils can use the limited unstructured repeated subtraction*	2
▪ Structured repeated subtraction	▪ *Pupils can use the structured repeated subtraction*	2
▪ Standard division algorithm	▪ *Pupils can use the standard division algorithm*	2

The objectives suggest that pupils have to understand, explore, and justify the standard multiplication and division algorithm in terms of repeated addition and subtraction of multi-digit numbers. It means that the starting point for learning standard algorithm of multiplication and division is by grasping the idea of repeated addition and subtraction. In the RME approach the chance to encounter contextual problems that stand for the concepts of multiplication and division can facilitate this. The problems motivate pupils to use the repeated addition and subtraction for finding the solution. Having discussions with teachers, pupils can develop a curtailment of the repeated addition and subtraction for the next efficient strategy until they figure out the standard algorithm. This is a process of mathematizing contextual problems, in which the pupils actively engage in reinventing strategies individually or by discussion with their peers. Whenever the teacher guides them then it was guided reinvention process, a key principle of RME theory (Gravemeijer, 1999).

In learning multiplication of multi-digit numbers, pupils need three other basic operations: repeated addition and supplementary multiplication (multiplication facts and multiplication by 1-digit numbers). It was found that pupils lacked of either one or both operations (see section 9.3). These drawbacks made their engagement were not as intensive as it was intended. Pupils with good prerequisites involved more active than those who had difficulties. However, interactive discussions during the learning activities showed its potential use of improving pupils' understanding. As proposed in the RME prototype the instructional activities (see Table 10.2 below) started with the preliminary games before the actual learning process. The games had become an important practical issue to acknowledge pupils' former prerequisites in this study. Practicing multiplication facts, multiplication by 10 and by multiples of ten were aimed at attracting pupils' attention and motivation, as well as reforming pupils' understanding.

Table 10.2
The learning sequences of teaching multiplication

Objective: After engaging in this multiplication section the pupils can understand, explore, and justify the conventional algorithm for multiplication in terms of repeated addition and the decimal numbers			
Day 1 *Hours: 2 x 40 minutes*	***Day 2*** *Hours: 2 x 40 minutes*	***Day 3*** *Hours: 2 x 40 minutes*	***Day 4*** *Hours: 2 x 40 minutes*
***Sub-objective*:** Pupils can use repeated additions	***Sub-objective:*** Pupils can use multiplication by 10	***Sub-objective:*** Pupils can use multiplication by multiples of ten	***Sub-objective:*** Pupils can use standard algorithm
Learning route: *Preliminary game* Multiplication facts Addition of multi-digit numbers	***Learning route:*** *Preliminary game* Multiplication facts Multiplication by 10	***Learning route:*** *Preliminary game* Multiplication facts Multiplication by 10 and multiples of ten	***Learning route:*** *Preliminary game* Multiplication facts Multiplication by 10 and multiples of ten
Solving problems ▪ Tiles ▪ To the zoo ▪ Skilful mason	*Solving problems* ▪ Potatoes ▪ Books ▪ The teacher	*Solving problems* ▪ Using waters	*Solving problems* ▪ A Fan ▪ A jumping frog ▪ Plane and car

The learning sequences of multiplication proposed in this study distributed in four days of teaching. Each day pupils met different sub-objective (repeated addition, multiplication by 10, by multiples of ten, and standard multiplication algorithm). Three contextual problems were provided for each day of learning. For instance, as mentioned earlier, the 'Tiles' problem attracted pupils to create different repeated addition strategies. They employed their counting abilities: adding each tile until the end, counting tiles by rows or columns, and later representing them in formal mathematics forms, i.e. the repeated addition strategies. They might use doublings, triple numbers, five numbers in a row, and ten numbers consecutively. Discussing pupils' strategies, and focusing on what strategy is the most efficient, while solving other problems ended up at figuring out the repeated addition of ten numbers (see also Gravemeijer, 1994). From there, pupils can be stimulated to reorganize their thinking by making them aware of the need of more effective strategy. A contextual problem, 'The skillful mason' for instance, that could be solved with a long repeated addition of 10 numbers, such as 204 for 52 times would be an interesting discussion to find another prospective, easier, less time consumed, reliable, and understandable strategy. It was the multiplication by 10. The teacher could facilitate this reinvention by strategically representing the amount of numbers in each column (that is 10) and the number being added in the column (the multiplied number), for instance 10 x 204 for five times. Then by realizing that the multiplication by 10 could be changed into multiplication by multiples of ten, it made the learning process understandable. Having learnt the multiplication by multiples of ten would guide the learning process of the multiplication algorithm. The teacher could facilitate the learning by comparing both strategies or by writing the multiplication by multiples of ten downward. Figure 10.1 below illustrates the curtailment of the strategies.

$$\begin{array}{rr}
10 \times 204 = & 2040 \\
10 \times 204 = & 2040 \\
10 \times 204 = & 2040 \\
10 \times 204 = & 2040 \\
10 \times 204 = & 2040 \\
2 \times 204 = & 408 \\
\hline
\multicolumn{2}{r}{+} \\
52 \times 204 = & 10608
\end{array}$$

$$\begin{array}{rr}
50 \times 204 = & 10200 \\
2 \times 204 = & 408 \\
\hline
\multicolumn{2}{r}{+} \\
52 \times 204 = & 10608
\end{array}$$

$$\begin{array}{r}
204 \\
52 \\
\hline
\times \\
408 \\
1020 \\
\hline
+ \\
10608
\end{array}$$

Figure 10.1
Various strategies of multiplication

In learning the division of multi-digit numbers, pupils supposed to use division strategies such as the unstructured subtraction, the limited structured subtraction, the structured subtraction, and the standard division algorithm. The teachers' role was to facilitate the learning activities and to encounter pupils' difficulties, such as to understand the problems and to carry out the calculations. At the beginning of each learning process the preliminary game was carried out to attract pupils' attention and to practice the multiplication and subtraction skills.

Table 10.3
The learning sequences of teaching division

Objective: After engaging in this division section the pupils can understand, explore, and justify the conventional algorithm for division in terms of repeated subtraction and the decimal numbers			
Day 1 *Hours: 2 x 40 minutes*	***Day 2*** *Hours: 2 x 40 minutes*	***Day 3*** *Hours: 2 x 40 minutes*	***Day 4*** *Hours: 2 x 40 minutes*
Sub-objective: Pupils can use the unstructured repeated subtraction	***Sub-objective:*** Pupils can use the limited structured repeated subtraction	***Sub-objective:*** Pupils can use the structured repeated subtraction	***Sub-objective:*** Pupils can use the standard division algorithm
Learning route: *Preliminary game* Multiplication facts Multiplication by 10 and 100 *Solving problems* ▪ Lebaran day ▪ Pupils in line ▪ Reading a book	***Learning route:*** *Preliminary game* Multiplication facts Multiplication by 10 and 100 *Solving problems* ▪ Chicken farm ▪ Jumping on the rope ▪ Graduation	***Learning route:*** *Preliminary game* Multiplication facts Multiplication by 10, 100, and 1000 *Solving problems* ▪ Using waters	***Learning route:*** *Preliminary game* Multiplication facts Multiplication by 10, 100, and 1000 *Solving problems* ▪ The zoo ▪ Stack of paper ▪ Kangaroo's jump

Sequences of learning division started with encountering the 'Lebaran day' problem. It was not easy to guide pupils toward the repeated subtraction. Their dependent attitude made them waiting the teachers' orders. However after having guidance questions the pupils started to understand the problem and reinvented the strategies. The pupils can use their own understanding of repeated subtraction (see section 9.4.2 item a). During the interactive discussions it was found that most pupils adopted the idea of multiplication by 10 as the starting point to find the solution. Then the next calculation conducted depending on their capability of multiplication facts; some used the doubling strategy and others applied the multiplication by 5. Considering these facts it seemed that the pupils had mixed understanding of unstructured repeated subtraction and the limited structured strategy.

Based on the learning route mentioned this study developed and implemented the planned instructional materials. It included the teacher guide and the pupil book (see Appendix A). The teacher guide consisted of the objective and its sub-objectives, the mathematical contents, the contextual problems involved, the evaluation appraisals being used (quizzes and tests), the learning activities, the teaching strategies in encountering pupils' difficulties, and time needed. The pupil book was provided in order to facilitate the learning activities. It consisted of the objectives and its sub-objectives, the contextual problems involved and the summary of strategies to be understood in the learning process (see appendix A).

Concluding summary

The RME local instructional sequences discussed represent a formal curriculum of multiplication and division of multi-digit numbers. It illustrates an intended curriculum of the subjects that was developed and implemented during the prototyping phase and being tested during the assessment phase in this study. The sequences were a reconstruction of theory in action (Gravemeijer, 1994) that was built by confronting the hypothetical learning trajectory with the actual pupils' learning trajectory took place in the classroom (see section 9.4). They illustrated the learning activities that were conducted by the teachers in the classroom with an analysis of how it was conducted, the reasons of doing the activities, what difficulties took place, and how the teacher dealt with those difficulties.

The local instructional sequences were an initial Indonesian local theory that is open for adjustment and functions as a guideline for others for the next developmental research study. The theory was not an ideal RME instruction. It was the intended curriculum for Indonesian circumstances that was developed based on teachers' conducting the learning activities in the classroom. The instructional sequences of multiplication begin with encountering problems that led the pupils toward discovering such strategies as repeated addition of ten numbers, multiplication by

10, multiplication by multiples of ten, and standard multiplication algorithm. In learning division, the contextual problems guide pupils toward the reinvention of the following strategies: unstructured repeated subtraction, limited structured repeated subtraction, structured repeated subtraction, and standard division algorithm.

The whole sequences of learning (see section 9.4.1 and 9.4.2) showed that teachers still dominated the learning process. Having teachers' guidance developed the discussions being conducted and the strategies pupils understood. The teachers used a reinvention route to structure the instruction activities; it was more like the Socrates' lesson (Polya, 1982). It was not an ideal RME learning activities; it was called the passive reinvention (Gravemeijer, 2002). However, during the whole activities it was also found that pupils worked individually or together, discussing the strategies and the mathematics tools that lead them to their own understanding.

In an ideal RME instruction, the pupils are expected to invent solution procedures for contextual problems. The procedures were emerged from pupils' discussions, answers, and questions. The pupils developed their own strategies; using their former knowledge, building their own understanding, and learning by their own face. The teacher facilitates the learning activities and leads them toward the norms relevant to the contents. The RME experts believe that conducting the discussions interactively will establish pupils' understanding. This is one of the core tenets of progressive mathematization as a representation of the domain specific instruction theory for RME (Treffers, 1987).

10.2.2 The characteristics of the RME prototype

In this study the characteristics of the RME prototype referred to the quality aspects of the instructional materials that are operationalized using a typology of curriculum representations: ideal, formal, perceived, operational, experiential, and attained curriculum (see Goodlad, Klein & Tye, 1979; adapted by Van den Akker, 1988, 1990). This study applied four quality criteria: validity, practicality, implementability, and effectiveness (see section 4.2.3 item b). Validity was defined as whether the components of the materials were developed based on the state-of-the-art knowledge (content validity) and all components were consistently linked to each other (construct validity). Practicality referred to whether the materials were usable and easy for Indonesian teachers and pupils (a consistency between the intended and perceived curriculum). Implementability is defined as whether the RME prototype can be applied as intended in the classroom (representing consistency between the intended and operational curriculum and the intended and experiential curriculum). Finally, the effectiveness is related to whether the desired learning takes place (and refers to the consistency between the intended and the attained curriculum). Each of these quality aspects will be elaborated in the next sections.

a. Validity

This study focused on analyzing the content and construct validity. To assure the existence of the content validity in the RME prototype this study took several considerations into account. *Firstly*, the materials had to suit the Indonesian education culture and circumstances. It meant that (1) the contents were subjected to the 1994 mathematics curriculum in primary schools and (2) the contextual problems involved were addressed to the familiar for the teachers and pupils. *Secondly*, the materials developed represent the RME theory and its instructional approach.

The consistence link in the materials (construct validity) was analyzed from whether the RME tenets (see section 3.3.2) could be found accessible in the materials. *Firstly*, the contextual problems in this study were constructed based on its familiarity with the pupils, the level of difficulty, the reasonableness of numbers included, and the length of the sentences. The Indonesian and RME experts agreed that the contexts and numbers were imaginable, rational and common. *Secondly,* those experts also agreed that the repeated addition strategies serve as vertical instruments for building pupils' understanding of multiplication (interview citation from Gravemeijer, 2002). And the 'Tiles' problem (see section 9.2) for instance was considered a reasonable problem for pupils to start the learning path in the first day of learning multiplication for pupils (see in section 9.4). *Thirdly,* the experts examined that pupils' solving problem strategies (from the pre-test and the learning activities) provided the actual learning trajectory of multiplication and division of multi-digit numbers (see section 9.4.1 and 9.4.2). *Fourthly,* the experts determined the availability of guidance questions and hints in the materials to facilitate interactivity in the learning process. They argued that the Indonesian pupils' attitude in learning and teachers' beliefs in teaching should be put into consideration. This study prepared such guidance in the Planning Instruction in the Teacher Guide (see Appendix A). Having those considerations, this study believed that the short time available for teaching the subjects was seen as a limitation for the existence of the validity.

b. Practicality

Practicality was defined as whether the RME prototype was useable and easy for the Indonesian teachers and pupils. It was analyzed from the teachers' and pupils' initial impression of the RME materials and the instructional activities. This study found that the RME prototype was practically usable and moderately easy to be applied in the classroom. The learning activities proposed in the exemplary materials were found useful to lead pupils toward its aim. It also guided teachers to conduct a proper teaching performance.

However, the teachers (as well as the experts) convinced that to meet the aim more time was needed to apply the RME approach. Adjusting with different learning activities and pupils' dependent attitude in learning were the aspects to be account. Weaknesses such as pupils' ability to read, to multiply 1-digit numbers, and to add and subtract multi-digit numbers influenced pupils' engagement in the learning process. Guidance from experts was very essential for improving teachers' competencies in introducing contextual problems, asking questions, guiding discussions, and defining pupils' performances.

c. Implementability

Implementability was defined as the degree to which the RME prototype was applied as intended. It verified if the teacher: (a) introduced the contextual problems as intended; (b) conducted an interactive teaching approach; and (c) established socio-mathematical norms. This study found that the teachers were able to introduce properly the contextual problems to the pupils in the learning process. Even so the teachers needed to practice more various and different activities of introducing the problems, such as asking the pupils (individually or together) to read the problems loudly. The teachers conducted the interactive teaching model in the classroom properly as being proposed in the RME prototype. The main weakness was in encouraging pupils to discuss the strategies with their peers and to give reasons for the strategy and the mathematical tools being utilized. They needed more time and guidance from experts to go through the activities in the RME actual classroom approach.

However, the teachers did not establish the socio-mathematical climate as intended in the classroom. The teachers hardly conducted the regulation activities (cognitive and affective), reflecting on pupils' learning progress, taking remediation actions, maintaining motivation, and generating feedback. They did not realize the essence of conducting regulation activities. They thought that by asking one pupil to write his/her answer on the blackboard and the others to compare their answers, pupils could learn and understand the differences without teachers' guidance. In fact, pupils still struggled to utilize mathematical tools (see Figure 8.4) to do calculation properly (see Figure 8.5 and Table 8.11). Teachers did not reflect on pupils' progress in learning one strategy to another and take recovery actions for improving and establishing their understanding. Discussions, such as what difficulties pupils still had, which strategy was understandable, and why or how (see other guidance questions in section 9.5), did not occur. This study found out that teachers could not analyze pupils' understanding, reasons, and their thinking process. It decreased pupils' motivation and self-confidence in applying the strategies.

d. Effectiveness

In this study the effectiveness of the RME prototype was established if the pupils: (a) reached the intended learning progress; (b) performed in the expected level of understanding; and (c) obtained better achievement.

In overall the pupils progressed significantly in daily learning. This progress was beyond the teachers' predictions (see Figure 8.3 in section 8.2.2 item a.). In solving the problems most pupils demonstrated that they proceeded toward a rational solution, but often a major error or misinterpretation (i.e. lack of supplementary multiplication, and careless addition and subtraction) obstructed their performances (see Figure 8.4 section 8.2.2 item a.).

Pupils' level of understanding is related to their performance on the daily quiz items (see section 9.4). It was found that most pupils approached the problem with meaningful work that indicated that they understood the problem both in multiplication and in division. They were capable of conducting the procedures, however minor and major errors (prerequisite aspects) in multiplying numbers obstructed them toward the valid final solution.

This study found that most pupils reached better achievement after engaging in the RME learning process, and the low-level pupils got the most benefit from this engagement (see Table 8.13 in section 8.2.2 item c.1). The pupils achieved better than those thought in the conventional approach (see section 8.2.2 item c.2). The pupils that engaged in the RME approach scored significantly better in solving contextual problems than those from the conventional approach. They reached the same level in solving the conventional problems.

Concluding summary

This study found out that the Indonesian teachers did not implement RME prototype properly in the classroom. The passive reinvention mentioned earlier indicates that the teachers still tended to hang on to the conventional teaching approach. In one hand, their willingness to apply the RME approach in their classroom denotes that they are motivated to experience a new approach or teaching and to improve their skill of teaching. On the other hand, their lack of knowledge and practice makes them conduct the RME teaching approach as if a resemblance of the conventional approach. It can be see that the teachers need a practical guidance for improving their skill of teaching using the RME approach.

10.3 REFLECTIONS

10.3.1 Reflections on pupils' performances

This study shows that pupils can build their understanding of multiplication and division based on repeated addition and subtraction strategies (for instance, see section 9.3 in figure 9.1). As found in this study and other studies (Mulligan & Mictchelmore, 1997), many pupils successfully solved multiplication problems by additive calculations, even though in solving conventional problems (see section 9.3 in Figure 9.11). In effect, these methods create an appropriate sequence of multiples. Repeated addition is an advance on direct counting because it takes advantage of equal-sized groups present in the problem situations. In other word, the repeated addition and subtraction is the starting point of building pupils' understanding of multiplication and division. The pedagogical implication of this finding is that: (1) teaching multiplication and division should start with repeated addition and subtraction, and (2) when multiplication and division problems are given, pupils should be allowed to solve them by their own ways. Some pupils will solve problems with multiplication, but others will use repeated addition (Kamii with Livingston, 1994; and Clark & Kamii, 1996). As Steffe and Cobb (1988) stated emphatically, pupils "must not be forced to do things they cannot do, such as learn their multiplication facts and algorithms for computation at the same time" (p. 136).

This study also showed that Indonesian pupils learnt multiplication and division actively; building their own understanding in the RME learning activities, reinventing strategies, and finding solution individually and together with their peers. The opportunities of learning in different situation motivated them to reformulate their thinking process. Pupils still had a dependent attitude toward teachers' orders, but this could also be seen as a way of respecting older people, rather than copying teachers' way of thinking. In this condition, a significant influence from Indonesian culture still attached strongly to pupils' attitude. The discussions during the learning activities (see section 9.4.1 and 9.4.2) showed this condition in several occasions. And it should be interpreted as an indication of that the role of teacher was still very important to guide and accelerate pupils toward their understanding.

During the learning activities it was found out that pupils progressed significantly in learning multiplication and division of multi-digit numbers. However their attitude toward learning (discussing the strategies, giving reasons, asking questions, and answering with rational explanation) did not yet improve much. Their hesitation of being laughed at by their friends and giving wrong answers distracted their active engagement in the learning activities. Even so, there was sufficient indication that pupils were comfortable in learning the subjects realistically. The RME approach accommodated them to understand the subjects properly; in terms of making sense of the contextual problems, making sense of the solution procedures, and understanding the relation between informal and formal solution procedures. The pupils also built up mathematical confidence; in terms of choosing the solution procedure that makes sense to them and with which they feel comfortable. They had confidence in developing informal and formal mathematical solution procedures, using and applying them as a tool for solving problems.

It was also found out that pupils often had difficulties in understand the contextual problems. Several reasons for this were found out, such as weak reading ability, and lack of familiarity with solving contextual problems. These reasons made pupils often solving problems blindly, resulting on constructing procedures that had mathematical errors. They did not make use of the contexts involved in doing the calculation. Questions to the teacher such as "Is this a multiplication or division problem" illustrated this condition (see section 7.2.1 item b). In these circumstances, teachers' guidance was the most constructive help for the pupils; asking to read problems carefully, finding relations of the numbers, discussing problem conditions, drawing pictures illustrating the problems, and asking for retelling the problems by their own words. For those pupils who lacked of reading ability, asking other pupils to read or reading problems together would be a good alternative to do. Considering pupils' dependency on teachers' explanation, teachers' encouragement is needed to discuss the problem situation with peers.

It was true that some pupils still had difficulties in comparing, analyzing, choosing, and applying the best and understandable strategy, however, most of them discussed their own reasons to choose one. It is believed that having learnt in the RME approach the pupils began to build their own understanding, developed their own thinking process, and had their own freedom to choose the best and effective strategy for their own. What might be essential is that by using the RME approach in the learning activities we educate pupils to what many people thinking of the mathematics education should be opportunities and freedom for building own understanding, developing a way of thinking mathematically, respecting other people opinions, and giving reasons constructively.

10.3.2 Reflections on teachers' performances

This study showed that the Indonesian teachers could introduce contextual problems and perform interactive teaching model. However, they were not able to establish mathematical norms in the classrooms as intended. This might be an indication of the need for more time to practice different teaching approaches, like the RME approach. Differences in the learning and teaching activities made teachers reconsider their former personal thinking and beliefs about teaching and learning mathematics. At the one hand they believed that the RME approach gave more opportunities for students to learn different strategies, but on the other hand it seemed that the teachers were afraid to loose their respects as teachers. In many occasions they still had beliefs that their former conventional teaching method was the best strategy to conduct the learning activities, however many studies showed contrary results (De Lange, 1994; Freiter & Van den Akker, 1996; Gravemejier, 1994; and Kamii & Dominick, 1998). As De Lange (1996) argues that the conventional approach is effective for the teacher in delivering the subjects, but it is ineffective for pupils to engage learning. This study also showed this result (see section 8.2.2 item c.2 p. 140).

The teachers that were involved in this study argued that they needed more time to adjust to the RME approach and the experts agreed upon it. It is believed that the Indonesian teachers could perform better, if they had more time to get used to this new approach. Complexity of learning activities during the implementation process and teachers' lack of pedagogical aspects of the RME approach were some reasons to be encountered. Teachers' shifting role from conductor of teaching to facilitator of learning was one of those complexities. Teachers' shifting approach from teaching by telling the algorithm to introducing contextual problems needed more time to manage the classroom, discussions, questions and answers, and reasons for solutions. Building pupils understanding toward the contextual problems was another aspect to be taken into account firstly before the learning accelerated well. Then teachers' shifting the context to connect informal and formal mathematics forms made them feel that there was a lot of work to be done before the learning progress was reached. All these activities seemed unexpected, unbearable, overwhelmed for the teachers. However, many teachers found out the RME approach usable and moderately easy because they had enormous experience in teaching the subjects. Considering these experiences, it is believed that the Indonesian teachers can perform the implementation of the RME approach as intended.

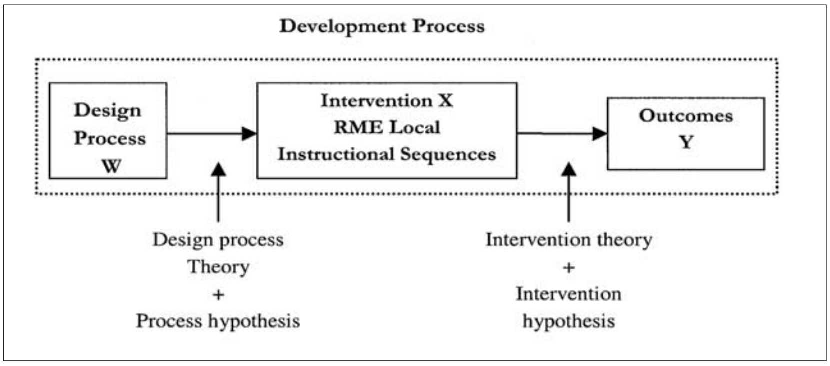
This study found that accessible time and support were extremely important during the implementation. The teachers needed more time to adjust and they also needed support from the experts. Emphasizing on pupils' making progress in understanding the contextual problem, structuring informal or formal mathematics forms, refining mathematical tools, and finding the solutions made the teachers perceive that the RME approach was difficult to apply. However, discussing the classroom situations with researcher, experts, and other sources would be a useful help for teachers to develop their own understanding. For this reason this study believes that teachers' learning by doing the RME approach in the actual classroom, with help from experts, becomes a practical alternative toward the steps to change the present formal curriculum into a RME basic curriculum.

Important as well in implementing the RME approach is that the teachers realize that characteristics of the change (Fullan, 1982): (1) the need of changing the teaching approach and (2) the clarity of differences of the two approaches. It is true that the RME approach is not the only answer for teaching all subjects in the curriculum. However, the RME approach will guide teachers toward a new meaning of teaching in general terms. Teaching is not for the test only, but should build pupils' understanding and knowledge of the subjects (cognitive domains). It also gives pupils opportunities to develop their attitude toward learning activities (affective domains). These domains are remarkably essential to develop pupils' attitude toward learning. As a consequence one may expect that they also guide the pupils toward the wholeness of life because a classroom represents a real life condition in which they can share opinions, asking questions, and delivering answers. It is believed that teachers play an important role to meet these objectives.

10.3.3 Reflections on the developmental research design

In summary, this study was built upon this sentence: "*The intervention consisting of the RME approach for teaching multiplication and division of multi-digit in Indonesian primary schools is characterized by four quality aspects (validity, practicality, implementability, and effectiveness) and carried out via procedural activities (front-analysis, expert reviews, teaching experiments, and reflections of the instructional sequences) because of the need of improving the Indonesian mathematics education and the RME theory as a prospective approach.*" This sentence shows that in this study the development of the RME prototype was based on elements of the design principles in the developmental research (substantive, procedural, and theoretical/empirical emphasis) mentioned by Van den Akker (1999). And during the teaching experiments, the researcher build a local instructional theory for teaching multiplication and division of multi-digit numbers that was based on the RME theory. The process was based on how the proposed instructional activities were realized during the learning interaction and what the pupils were learning as they engaged in the learning activities.

In conducting developmental research several theoretical assumptions are implied as can be illustrated with Figure 10.1 (Plomp, 2002).



Source: Plomp, 2002
Figure 10.2
Development process

This study developed the prototypical materials starting from the Indonesian 1994 mathematics curriculum in the primary schools and the RME theory. Conducting design process consisting of procedural activities such as front-end analysis, expert reviews, teaching experiments, and reflections toward the instructional sequences, this study built a RME local instructional theory for teaching multiplication and division of multi-digit numbers in Indonesia. Then, this study implemented the RME instructional sequences as intervention in primary schools.

This study indicated that implementing the RME approach for teaching multiplication and division of multi-digit numbers leads the pupils toward the expected level of understanding (see results on section 8.2.2). As a result from the teaching experiments, several conditions are assumed: (1) teachers are experienced in teaching the subjects, (2) they are willing voluntarily to apply the RME approach in their classroom. These conditions are essentially required for having an expected influence of implementing a new approach (Fullan, 1991). Theoretically, one can interpret this as an intervention theory implying that intervention X with certain characteristics leads to the expected and desired outcome Y.

On the other hand, one can also look at this study from a design process prospective as the RME local instructional theory that was developed based on procedural activities (how the researcher developed the local instructional sequences) and the link between RME theory and empirical evidence in the actual classroom activities. This can be perceived as a design process theory assuming that: *if we organize the designing of the RME local instructional sequences (Intervention X) according to design process W (design principles), then we expect that the resulting intervention X will result in outcome Y*. This study developed the instructional sequences using a cyclic design process (substantive, procedural, theoretical emphasis) resulting on prototypical materials that had RME characteristics (see section 10.2.1 and 10.2.2).

In building intervention X (RME local instructional sequences), procedural activities of developmental research in mathematics didactics are carried out. The activities begin with a preliminary design of prototypical activity, followed by conducting the teaching experiments that imply adjustments on a daily basis, and end up with a local instructional sequences for teaching multiplication and division of multi-digit numbers in Indonesian primary schools. It is an initial Indonesian local instructional theory that is open for adjustment and functions as a guideline for others for the next developmental study in RME approach.

10.4 RECOMMENDATIONS

This study is one of the first RME developmental research study in mathematics education in Indonesian. It is about developing, implementing, and evaluating local instructional theory for teaching multiplication and division of multi-digit numbers in Indonesian primary schools. The theory is developed based on the RME approach and the Indonesian 1994 mathematics curriculum in the primary schools. The results of this study are significant for Indonesian contexts; especially in improving mathematics education in primary schools. Considering the main findings of this study and the broad range of problems confronting Indonesian mathematics education, several recommendations are made.

First of all, the initial accomplishment of this study indicates a desirability of further implementation of the RME approach in Indonesian settings. Considering the fact that Indonesia is a country with a centralized system of education, a policy recommendation is given with respect to the policy of mathematics curriculum and development, teacher education curriculum, and a research institute for developing mathematics curriculum that suit the Indonesian circumstances.

Then, the shortcomings of the actual implementation of the RME approach in Indonesian primary schools provide recommendations related to the teacher education and the in-service training. Providing teachers (or prospective teachers) with deep understanding and practical guidance of the RME teaching approach is one of the suggestions.

And finally, much research is needed to make a good quality instruction happen. Not just research to develop all sorts of 'local instructional theories', but also research on how to bring about all the changes needed.

10.4.1 Recommendations regarding policy makers

The RME local instructional sequences developed in this study were used for teaching multiplication and division of multi-digit numbers in Indonesian primary schools. The sequences were developed based on the Indonesian 1994-mathematics curriculum and the RME theory. This study has shown that the sequences were effective for improving pupils' performances, even though the Indonesian teachers applied the RME approach in a limited fashion. Learning in the RME approach the pupils actively engaged in the learning activities and they performed on the expected level of understanding. This initial successful implementation experiments can be seen as inspiration for further implementation of the RME approach in Indonesian settings.

This initial successful implementation experiments can be seen as inspiration for further implementation of the RME approach in Indonesian primary schools. This also suggests that the Indonesian government conducts a curriculum reform for mathematics education so that a 'RME type' of mathematics curriculum is significant to be introduced in Indonesia. This policy recommendation has far-reaching consequences, as it calls for curriculum development, for textbook (or better curriculum materials) development, for teacher in-service training, for innovation of teacher training for mathematics education, for a campaign so that parents gets an understanding that math teaching is no longer drilling of formulas.

To realize this, a careful implementation strategy is needed, part of which might be the creation of something like an Indonesian 'Freudenthal Institute'. This institute would become a research and development organization for building mathematics curriculum that match Indonesian circumstances. Pusat Kurikulum (center of national curriculum) in the Department of National Education and other institutional research under affiliation with the universities suit this goal. Under the guidance from the RME experts and having several teachers and schools as working partner, these institutes develop, implement, and evaluate the mathematics curriculum for Indonesian schools. What might be important mentioned here is that beginning in the year of 2000, four universities in Indonesia (Education University of Bandung, State University of Yogyakarta, Sanata Dharma University of Yogyakarta, and State University of Surabaya) have been trying out the RME approach in several primary schools in Bandung, Yogyakarta, and Surabaya. They develop, apply, and evaluate the primary school mathematics curriculum in Grade 1, 2, and 3.

10.4.2 Recommendations relating teacher education and in-service training

From the teaching experiments using the RME approach conducted in Indonesian primary schools, several shortcomings were found out. Teachers' passive reinvention teaching model, teachers' persistence to the conventional teaching model, pupils' lack of multiplication facts, pupils' dependent attitude toward teachers' orders, pupils' attitude toward learning (stick to only one strategy, creativity, and self confidence) are the aspects to be improved. These facts verify that if Indonesia wants to introduce RME approach, then teachers need to be educated thoroughly in the RME approach, both via in-service and pre-service education. It is recommended that the teachers get a deep understanding and knowledge of the RME approach before they use it in the actual learning activities. Having a clear picture of the RME approach prepares the teachers to face the complexities of carrying out the approach in the classroom. Then, guided by experts they can practice it. The guidance helped them to encounter some difficulties and confusing matters during the learning activities. These activities are recommended to make a good transition from 'understanding' to 'enacting'.

The teachers did not establish the socio-mathematical norms as intended, and they did not encourage pupils to discuss with peers. It indicated that the Indonesian teachers did not realize the essential effects of learning from peers. Meanwhile, Kamii and Dominick (1998) found that pupils learn most from discussing with their peers. The other issue relates to the teachers' ability to ask guide questions for accelerating pupils' learning. It seemed that teachers' conventional teaching attitude still existed even though they applied the RME approach. Considering these teachers' shortcomings, it is recommended that more time have to spend to educate teachers in the skills of teaching according to the RME approach.

The teachers also did not maintain pupils' understanding, and did not take recovery actions of it. To establish pupils' understanding in the RME learning activities, it is recommended that the teachers apply several tactics in their mathematics teaching:

1. Discussing what strategy is effective will build pupils mathematical norms in the classroom. Pupils can learn, compare, analyze, and discuss which strategy is understood or difficult, easy or long enough, simple or too much work.
2. Practicing to apply the best and understood strategy for different types of contexts, situations, and problems is also recommended. These will help pupils realize the useful of the strategy in solving problem and guide them toward the meaning of having effective tools to find the solutions. This activity can be applied in several actions: giving homework and solving in the classroom different types of problems (including both contextual and conventional problems). These actions can be done during and after the classroom activities.
3. Relating different problem solving procedures is another aspect to be taken into consideration. Learning many strategies may be confusing for some pupils as they used to learn without understanding because the pupils used to learn only one mechanistic strategy. Relating those strategies to each other, connecting the mathematical concepts embedded in the strategies, and discussing its differences are the recommended activities in order to make pupils comprehend them.

After all, all teachers' shortcomings mentioned indicate that if Indonesia wants to introduce RME approach, then the teachers need to be educated thoroughly in the RME approach, both via in-service and pre-service education.

10.4.3 Recommendations for further research

For the future developmental studies, it is recommended that to develop local instructional theory by using the cumulative cyclic process of thought and teaching experiments *on daily basis* (Gravemeijer, 1999). By developing instructional activities on daily basis the researcher and the teachers can anticipate both how the proposed instructional activities (result from the thought experiments) might be realized, and what the pupils might learn as they engage in them (during the teaching experiments). However large a task this may be, this study suggests that developing

RME local instructional sequences for each mathematics subject would be a promising alternative program to improve pupils' performances and attitude as well as teachers' competencies.

Next to developmental research on instructional sequences (like this study and the research carried out by Fauzan, 2002), also research is needed on how to bring about all the changes needed for the whole mathematics curriculum. For instance, further research on optimal strategies to influence teachers' beliefs (see Hadi, 2002), and further research on learning environments on the RME approach for the teachers and pre-service teachers (see Zulkardi, 2002). The studies already carried out indicate that RME approach would be an effective approach to encounter the pupils' low performance problems in mathematics education in Indonesia that are caused by a number of factors, such as insufficiency of the teachers' mathematics knowledge and pedagogical approach, and the cultural aspects of the teaching and learning activity in the classroom.

10.5 EPILOGUE: A REFORM IN INDONESIAN MATHEMATICS EDUCATION

Most Indonesian teachers believe that teaching mathematics mechanistically was the most effective teaching model. However, De Lange (1996) said that it is easy and efficient for the teachers' side but not for the students' side. This study indicates that learning mathematics mechanistically makes pupils depend on the teachers' orders and leads them to the uniformity of their attitude (no questions during the learning activities, afraid of being different, hardly giving reasons on discussion, barely having different solution). In contrast, learning mathematics realistically improves pupils' achievement, enhances self-confidences, and builds learning attitude. The pupils are comfortable with the learning climate that motivates them to be creative in finding other strategies to solve problems. This initial successful teaching experiment can be seen as an encouragement for further implementation of the RME approach in Indonesia.

In the RME approach, pupils are at the centre. They construct their own knowledge and understanding (Cobb, 1994). In contrast, the conventional view says that knowledge is received from teachers when pupils listen to what they say. The RME theory argues that in line with mathematics as human activities, learning mathematics is a process that pupils need to do for themselves rather than one that is done to them by others (Gravemeijer, 1994). It does not mean that others are not influential in the process but recognizes the active role that pupils need to play to learn. Loucks-Horsley, et al. (1996) states that when pupils try to understand new information, they use their former knowledge and their own ways of learning. It means that the process of learning involves the construction of links between new ideas and what pupils already know to create meaning.

On the other hand, the teachers also have to take the initiative to reform their way of teaching. This study shows the potential and advantages of the use of the RME approach. Teachers' passive reinvention model of teaching would be an indication of the presence of teachers' willingness and deliberate beliefs toward the need of improvement of the Indonesian mathematics education in primary schools. However, one can also see that this is as a resistance toward the RME approach.

Based on the above viewpoint and after three decades of reforming the Indonesian mathematics curriculum since 1975 that resulted in a deep annoyance of pupils' performances, a change in the Indonesian teachers' way of teaching mathematics is needed to improve the condition; from mechanistic conventional to realistic, from teacher centered to pupil centered. This study has shown a successful indication of implementing the RME approach to improve pupils' performances and to build pupils' attitude toward learning. After all, it is important to realize the following sentences quoted from Loucks-Horsley, et al. (1996, p.28):

> "*Whenever individuals are learning effectively, they are deeply engaged in what they are doing and expect that it will make sense to them. They do not expect learning to be easy and instantaneous, but they have confidence that understanding will come from persistence, interaction with ideas and natural phenomena, dialogue with peers and teachers, attention to other possible ideas, and a willingness to change their view on the basis of compelling new evidence.*"

If this is true, why bother so much to teach pupils conventionally. Let them free to learn, to make sense what the real life is. As a wise man says*; "Let the kites fly away, but hold on the line. They go no where.*"

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ENGLISH SUMMARY
TEACHING MULTIPLICATION AND DIVISION REALISTICALLY IN INDONESIAN PRIMARY SCHOOLS: A PROTOTYPE OF INDONESIAN LOCAL INSTRUCTIONAL THEORY

Context of study

The implementation of the 1994 mathematics curriculum in Indonesian primary schools is focusing on the teaching and learning arithmetic. The goals are to prepare the students to use and apply their mathematics knowledge and mathematical way of thinking in solving problems in their life and in learning other different knowledge (Depdikbud, 1995). In conducting the learning process, the curriculum suggested to apply the student centered teaching model in which the teaching activities give opportunities for the pupils to develop their own understanding.

In contrast most teachers utilized the paper-and-pencil strategy combined with the concepts-operations-example-drilling approach (Suyono, 1996). This model of teaching is called the mechanistic way of teaching (Freudhental, 1973). The teachers teach mathematics with practicing mathematics symbols and emphasizing on giving information and application of mathematics algorithms (algorithmic mathematics education, Treffers, 1987). During the instruction process the typical teaching and learning in developing country (Feiter & Van Den Akker, 1995 and Romberg, 1998) progress regularly (see section 1.2.2).

There were many reasons behind this application. One of which was the low quality of teachers knowledge of mathematics (BPPN, 1996). The other was on teachers' pedagogical knowledge of teaching mathematics. Suyono (1996) found that teachers have low ability in using variety of teaching methods and teach using conventional methods without considering the logical thinking, critical and creativity aspects of the subject matter. The second aspect relates to the pupils' learning cognition. After all, it can be concluded that teachers' competence (knowledge of mathematics

contents, pedagogical aspects, and pupils' learning cognition) was the most essential element that should be improved progressively (see section 3.2).
Those teachers' weaknesses influence resulted in a terrible effect on the teaching and learning process of mathematics. The students' drawbacks (Armanto, 2000; Haji, 1994; and Jailani, 1990) have made mathematics more difficult to learn and to understand and the students become afraid of mathematics. In solving multiplication and division problems for instance, pupils conducted "buggy" procedures (see section 2.5) that represented the effects of learning the subjects in conventional approach since they had nothing to fall back upon. Another example was coming from TIMSS reports (1999) that shows Indonesian students score are in the 33th level from 37 countries in TIMSS evaluation.

Considering these aspects (teachers' lack of competencies and pupils' performances) this study convinced that the need of innovation in implementing mathematics curriculum for Indonesian schools becomes essential. Firstly, a representative approach in teaching mathematics in a way that all pupils can understand and master mathematics facts, concepts, procedures, and operations is needed indisputably. Secondly, an urgency of proper alternative improvement programs in order to improve the teachers' competencies to teach mathematics in the classroom is obvious.

Theoretical roots

Based on studies and programs being developed in many countries (see De lange, 1994; Romberg, 1994; and Becker & Selter, 1996), this study believed that a representative teaching approach that is beneficially pursued is "realistic mathematics education (RME)": mathematics education that is compatible with the idea of mathematics as a human activity (Freudenthal, 1983). In this philosophy, the mental activity of the learner is at the center. Mathematics ought not to be associated with mathematics as a well-organized deductive system, but with mathematics as an activity of doing and reinventing mathematics (mathematizing subject matter). The subject matter can be taken from reality and it must be organized according to mathematical patterns. Analyzing and reflecting own mathematical activity are the main key principle of reinventing mathematics. In RME, teaching mathematics realistically starts at encountering contextual problems. Guided by the teacher, students develop their understandings by utilizing their former mathematics knowledge. Interactive discussions and negotiations of mathematical norms (effective and efficient frameworks) are the key elements in building students' understanding. Uniformity and formalism in pupils' mathematical strategies are not essential in this model. The key issue is on building students' understanding on certain expected level of performances that allow them to develop their own frameworks to solve mathematical problems.

Aims and research question

Believing that the RME theory as the prospective approach to count on, this study analyzed proper alternative improvement program in order to improve the teachers' competencies to teach mathematics in the classroom. Loucks-Horsley (1998) and Feiter and Van den Akker (1995) suggest conducting a new formal curriculum implementation by constructing, learning, using, and refining a particular set of instructional materials in the classroom. This study constructed and provided the RME formal curriculum materials with which the teachers can practice the RME teaching approach under the guidance of the researcher. And the multiplication and division of multi-digit numbers were chosen as the subjects of this study as the teachers understood its contents and strategies very well. Based on these elements this study formulated the following research question:

> *What are the characteristics of the RME prototype for teaching multiplication and division of multi-digit numbers in Indonesian primary schools?*

The characteristics of the RME prototype are analyzed in two different aspects: local instructional sequences and quality aspects of the prototype. In the matter of intervention of teaching and learning mathematics, the characteristics refer to the explicit formulation of local instructional activities that is made up of three components: (1) learning goals for pupils; (2) planned instructional materials; and (3) a conjectured learning sequence (Gravemeijer & Cobb, 2001).

Meanwhile, the quality aspects of the RME prototype are defined as the degree to which the RME prototype is valid, practical, implementable, and effective. *Validity* of the RME prototype refers to the presence of the state-of-the-art knowledge of the Indonesian circumstances and the RME theory (content validity) and the consistent link of the components in the RME prototypical materials (construct validity). *Practicality* of the RME prototype referred to the initial satisfaction of the target groups (pupils and teachers) toward the materials and the teaching model suggested in the RME. *Implementability* of the RME prototype refers the proper teaching organization established by the teacher in teaching multiplication and division in Indonesian setting. These three quality aspects lead the study to the first sub-research question:

> *To what extent was the RME prototype valid, practical, and implementable for teaching multiplication and division of multi-digit numbers in Indonesian primary schools?*

The *effectiveness* of the RME prototype refers to the expected learning progress, understanding, and performance of the pupils in learning multiplication and division of multi-digit numbers. This lead to the second sub-research question:

> *To what extent was the RME prototype effective for teaching multiplication and division of multi-digit numbers in Indonesian primary schools?*

Research design

To address the research question and its sub-research questions discussed, a developmental research approach was chosen to analyze the development and improvement of the prototypical product. In the field of curriculum, it is a formative research design (Van den Akker, 1999 and Van den Akker & Plomp, 1996) and a type 1 of developmental research study (Richey & Nelson, 1996), in which the research activities were conducted, the products were analyzed during a cyclic developmental process, from exploratory phase through (formative and summative) evaluation phase.

In mathematics didactics, the developmental research was aimed at developing an instructional sequence for specific topic where the researcher constructs provisional instructional activities in an iterative process of designing and retesting. It is a "theory-guided bricolage" (Gravemeijer, 1994), which core is in the cyclic process of thought and teaching experiments (Freudhental, 1991). Like a handyman, the researcher can make use of all the domain specific knowledge concerning mathematics education: classroom experience, textbooks, exemplary instructional activities, relevant research, and educational psychology. The activities begin with a preliminary design of the prototypical instructional activities, followed by a cyclic teaching experiments, and end up with a retrospective analysis.

Based on the given types of developmental research, this study developed the RME prototypical materials in a cyclic process of front-end analysis, expert reviews, teaching experiments, and reflection to the local instructional sequences. These cyclic processes lead the study to build a conjectured local instructional theory of teaching multiplication and division of multi-digit numbers in Indonesian primary schools. The research design is illustrated as follow.

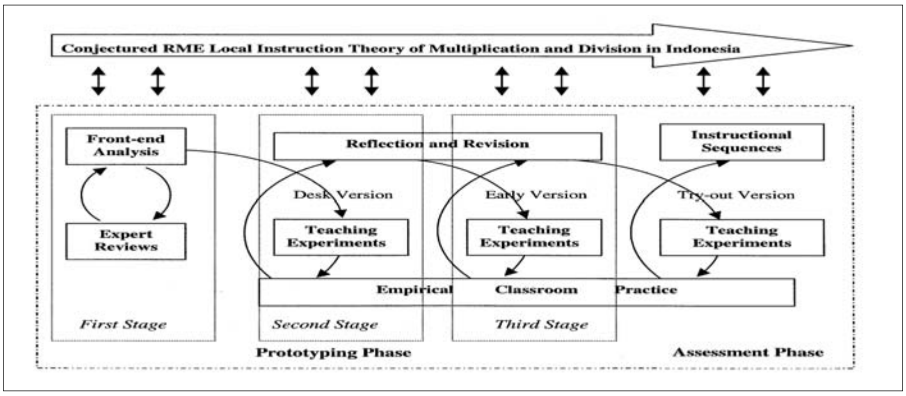


Figure ES. 1
The cyclic process of developmental research

This study was carried out in two-phase of cyclic activities. *The prototyping phase* that was executed in three-stage focused on developing, implementing, and revising the prototypical materials. It consisted of a cyclic process of the front-end analysis, expert reviews, teaching experiments, and reflections (see section 4.2.3 item c). Results obtained in one stage were used as input the next stage. The prototyping phase resulted in a try-out version of the RME prototype, which was tested in the assessment phase. *The assessment phase* focused on evaluating whether the RME prototypical materials were used as intended and whether the pupils performed on the expected level of performances after engaging in the RME learning activities.

Main findings

This study reports its main findings on two insights: characteristics of the local instructional sequences and the quality aspects of the RME prototype.

Characteristics of local instructional sequences

In this study, characteristics of local instructional sequences referred to the explicit formulation of instructional sequences in teaching multiplication and division of multi-digit numbers for Indonesian primary schools. It expressed the sequences concisely in three components: (1) learning goals for pupils; (2) conjectured learning sequence; and (3) planned instructional materials (Gravemeijer & Cobb, 2001). Each of these components is explained in the following sections.

The local instructional sequences for teaching multiplication and division of multi-digit numbers are an initial Indonesian local theory that is open for adjustment and function as a guideline for others in the next developmental research study. The theory was not an ideal RME instruction. It is the intended curriculum for Indonesian circumstances that were developed based on teachers' conducting the learning activities in the classroom. The instructional sequences of multiplication begin with encountering problems that lead pupils toward reinventing strategies: the repeated addition of ten numbers, the multiplication by 10, the multiplication by tens, and standard multiplication algorithm. In learning division, the contextual problems guide pupils toward the reinvention of the following strategies: the unstructured repeated subtraction, the limited structured repeated subtraction, the structured repeated subtraction, and the standard division algorithm.

The whole sequences of learning (see section 9.4.1 and 9.4.2) indicated that teachers still dominated the learning process. The discussions being conducted were built upon teachers' guidance questions. The strategies that pupils reinvented were developed by having guidance from the teacher. However, it was the guided reinvention process in which pupils worked individually or together, discussing the strategies and the mathematics tools, leading them to their own understanding.

Characteristics of the RME prototype

This study found that the RME prototype that developed in this study was effective for teaching multiplication and division of multi-digit numbers. The prototypical materials represented the Indonesian circumstances and the RME theory (content validity) and its components of the materials were consistently linked each other (construct validity). The RME prototype was usable in the classroom, but the teachers did not apply as intended because of its lack of establishing socio-mathematical norms. The teachers did not conduct the regulation activities (cognitive and affective) properly, reflecting on pupils' learning progress, taking recovery actions, maintaining motivation, and generating feedback. Learning in RME approach, pupils performed on the expected level of achievement. In one hand most pupils demonstrated that they proceeded toward a rational solution, but in another hand a major substantial error or misinterpretation obstructed the correct solution process. Lack of multiplication facts and careless subtractions interrupted their high performances. The pupils performed in the moderate level of achievement. However, they performed better than those pupils who learnt in the conventional approach.

Conclusion

The RME local instructional sequences developed in this study were used for teaching multiplication and division of multi-digit numbers in Indonesian primary schools. The sequences were developed based on the Indonesian 1994-mathematics curriculum and the RME theory. This study has suggested that the sequences were implementable in the classroom and effective for improving pupils' performances. The Indonesian teachers applied the RME sequences as intended, with a minor establishment of the socio-mathematical norms. Learning in the RME approach the pupils actively engaged in the learning activities and they performed on the expected level of understanding. This initial successful implementation experiments can be regarded as inspiration for further implementation of the RME approach in Indonesian settings.

INDONESIAN SUMMARY

PEMBELAJARAN PERKALIAN DAN PEMBAGIAN SECARA REALISTIK DI SEKOLAH DASAR INDONESIA: SEBUAH PROTOTIPE TEORI PEMBELAJARAN LOKAL

Kontek penelitian

Implementasi dari kurikulum 1994 di sekolah dasar Indonesia yang terfokus pada pembelajaran berhitung bertujuan untuk mempersiapkan siswa menggunakan dan mengaplikasikan pengetahuan matematika dan pola pikirnya untuk menyelesaikan masalah matematika sekolah dan dalam kehidupan sehari-hari (Depdikbud, 1994). Dalam mengelola proses belajar di dalam kelas, kurikulum menyarankan untuk menggunakan model pembelajaran berorientasi pada siswa dimana siswa mendapat kesempatan untuk mengembangkan pengertian matematikanya secara mandiri.

Namun demikian guru menggunakan strategi yang berbeda yaitu model kertas-dan-pinsil dikombinasikan dengan model konsep-operasi-contoh-latihan (Suyono, 1996). Model ini disebut model pengajaran secara mekanistik (Freudhental, 1973). Guru mengajarkan matematika dengan melatihkan simbol dan menekankan pada pemberian informasi dan penerapan prosedur (algorithmic mathematics education, Treffers, 1987). Pembelajaran berlangsung bercirikan seperti pengajaran di negara berkembang lainnya (Feiter & Van Den Akker, 1995 and Romberg, 1998).

Beberapa alasan yang terungkap antara lain adalah rendahnya kualitas guru dalam memahami konsep matematika (BPPN, 1996). Alasan lainnya adalah pengetahuan mendidik guru dalam pembelajaran matematika. Suyono (1996) menemukan bahwa guru (1) memiliki kemampuan yang rendah dalam menggunakan berbagai variasi model mengajar dan (2) mengajar dengan menggunakan model konvensional tanpa mempertimbangkan pola pikir logis, kritis, dan kreatif dari pelajaran matematika. Aspek kedua diatas berkaitan dengan aspek kognitif siswa. Dengan demikian dapat disimpulkan bahwa kompetensi guru (pemahaman materi matematika, aspek mendidik, and aspek kognitif siswa) adalah elemen penting yang harus diperbaiki secara berkesinambungan (lihat bab 3.2).

Kelemahan guru tersebut di atas telah mengakibatkan pembelajaran matematika di sekolah dasar Indonesia masuk ke dalam jurang kehancuran. Kelemahan siswa (Armanto, 2000; Haji, 1994; and Jailani, 1990) telah membuat matematika menjadi sangat sulit untuk dipelajari dan membuat mereka takut mempelajarinya. Dalam mengerjakan soal perkalian dan pembagian misalnya, siswa menggunakan strategi yang "asal jadi" (lihat bagian 2.5) karena mereka lupa cara menggunakan prosedur pengerjaannya. Hal ini melambangkan gagalnya pembelajaran matematika secara konvensional yang berfokus pada hafalan semata. Contoh lainnya dapat diperhatikan dari hasil laopran TIMSS (1997) dimana siswa Indonesia berada pada peringkat ke-33 dari 37 negara yang berpartisipasi.

Dengan mempertimbangkan hal di atas penelitian ini yakin bahwa pembaharuan pengajaran matematika di Indonesia harus segera dilakukan. Pertama, harus dipilih pendekatan yang sesuai dengan mana siswa dapat memahami fakta, konsep, prosedur, dan strategi dalam bermatematika. Kedua, harus diperoleh program perbaikan yang berkesinambungan yang membantu memperbaiki kompetensi guru dan siswa dalam bermatematika.

Kerangka Teori

Berdasarkan penelitian dan program perbaikan pendidikan yang telah dilakukan di beberapa negara (lihat De lange, 1994; Romberg, 1994; and Becker & Selter, 1996), penelitian ini percaya bahwa pendekatan mengajar yang sesuai adalah Pendidikan Matematika Realistik ("Realistic Mathematics Education" or RME): program pendidikan matematika yang berorientasi pada ide "matematika adalah aktifitas manusia". Dalam filsafat ini, pusat belajar matematika adalah pada aktivitas mental siswa. Matematika tidak dapat dihubungkan dengan "matematika sebagai sistem yang terorganisasi secara deduktif", melainkan dengan "matematika sebagai aktivitas dalam mengerjakan dan menemukan kembali materi matematika". Materi matematika dapat diperoleh dari lingkungan yang harus diorganisasi sesuai dengan pola-pola matematika. Menganalisa dan merefleksi aktivitas matematika secara mandiri merupakan prinsip kunci dalam memahami dan menemukan kembali materi matematika. Di dalam RME, pembelajaran matematika secara realistik dimulai dari menyelesaikan soal kontekstual. Dibimbing oleh guru, siswa mengembangkan pemahamannya dengan menggunakan pengetahuan matematika yang telah mereka ketahui. Diskusi secara interaktif dan negosiasi norma-norma matematika (kerangka matematika yang effektif dan efisien) merupakan elemen utama dalam menumbuhkembangkan pemahaman siswa. Kesamaan dan formalisme dari strategi matematika yang digunakan siswa tidaklah penting dalam pendekatan RME ini. Fokus utama adalah pada pengembangan pemahaman siswa dalam tingkatan keberhasilan tertentu yang memudahkan mereka menyelesaikan soal matematika.

Tujuan dan pertanyaan penelitian

Dengan mempercayai bahwa teori RME merupakan pendekatan yang menjanjikan, penelitian ini menganalisa program seperti apa yang dapat memperbaiki pendidikan matematika di Indonesia. Loucks-Horsley, et al., (1998) and Feiter and Van den Akker (1995) menyarankan untuk menyusun material yang akan digunakan dalam pengajaran di dalam kelas. Penelitian ini mengembangkan dan menerapkan formal kurikulum berdasarkan RME teori dengan mana guru dapat menggunakannya dalam mengajarkan materi dan melatih diri menggunakan material yang disusun di dalam kelas mereka. Pelajaran matematika yang dipilih adalah perkalian dan pembagian bilangan multi-angka dengan pertimbangan bahwa guru memiliki pemahaman yang baik dalam subjek ini. Jadi dalam penelitian ini perhatian guru dapat difokuskan pada proses pembelajaran saja. Dengan mempertimbangkan hal tersebut di atas, masalah penelitian ini dirumuskan sebagai berikut:

> *Bagaimanakah karakteristik prototipe pendekatan RME untuk mengajarkan perkalian dan pembagian bilangan multi-angka di sekolah dasar di Indonesia?*

Karakteristik prototipe pendekatan RME dapat dilihat dari dua aspek: alur pengajaran secara lokal (Indonesia) dan aspek kualitas dari prototipenya. Dalam pengajaran dan pembelajaran matematika, karakteristik alur pengajaran secara lokal disusun dalam tiga komponen: (1) tujuan pembelajaran untuk siswa, (2) material yang disusun secara terencana, dan (3) prediksi alur pembelajaran di kelas (Gravemeijer & Cobb, 2001).

Aspek kualitas dari prototipe RME didefenisikan dalam tingkat kesahihan, kepraktisan, penerapan, dan keefektifannya. Tingkatan kevalidan sebuah prototipe dapat dilihat dari keterwakilan (keberadaan) kondisi Indonesia dan teroi RME dalam material yang dikembangkan (validitas isi) dan kondisi keterikatan setiap komponen material yang disusun tersebut (validitas konstrak). Tingkatan kepraktisan ditinjau dari keterpakaian dan kemudahan guru dan siswa dalam menggunakan prototipe RME yang dikembangkan.tingkatan penerapan ditentukan oleh kemampuan guru dalam menerapkan pengajaran sesuai dengan yang disusun dalam prototipe RME. Ketiga aspek ini merupakan variabel ntuk menjawab sub pertanyaan penelitian berikut ini:

Sejauh manakah tingkatan kevalidan, kepraktisan, dan penerapan prototipe RME untuk mengajarkan perkalian dan pembagian di sekolah dasar di Indonesia?

Tingkatan keefektifan dari prorotipe RME ditinjau dari kemajuan belajar, pemahaman, dan prestasi belajar siswa dalam belajar perkalian dan pembagian bilangan multi-angka. Hal ini berkaitan dengan sub pertanyaan penelitian yang kedua:

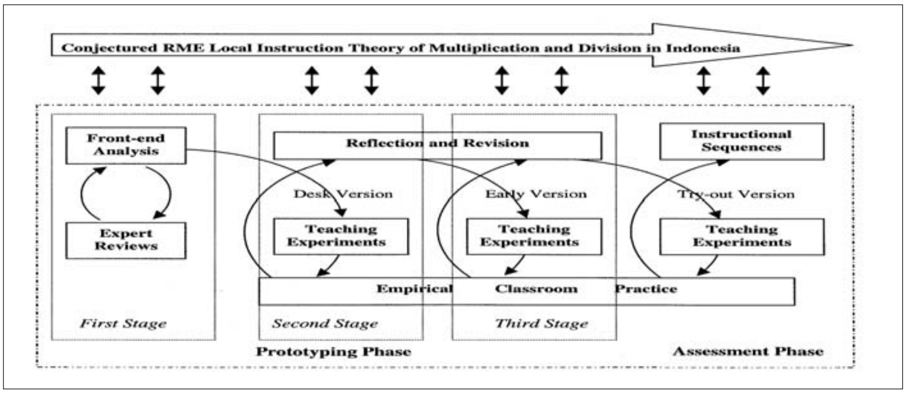
Sejauhmanakah tingkatan keefektifan prototipe RME untuk mengajarkan perkalian dan pembagian bilangan multi-angka di sekolah dasar di Indonesia?

Desain penelitian

Untuk menjawab pertanyaan penelitian di atas, riset pengembangan (developmental research approach) telah dipilih sebagai pendekatan penelitan yang digunakan untuk menganalisa proses pengembangan dan perbaikan prototipe RME. Di dalam lapangan kurikulam, penelitan ini disebut penelitian formatif (Van den Akker, 1999 dan Van den Akker & Plomp, 1996) dan termasuk dalam tipe pertama riset pengembangan (Richey & Nelson, 1996), dimana aktivitas penelitian dilaksanakan dan hasil penelitian dianalisa selama proses pengembangan sedang berlangsung dan diorganisasi sejak fase eksplorasi hingga fase evaluasi (formatif dan sumatif).

Di dalam pendidikan matematika, riset pengembangan ini bertujuan untuk mengembangkan alur pengajaran untuk topik tertentu dimana peneliti menyusun aktifitas pengajaran dalam proces pendisainan dan pengujian yang berulang. Proses penyusunan ini disebut “theory-guided bricolage” (Gravemeijer, 1994), dimana apspek utamanya terletak pada proses berulang dari melakukan eksperimen pemikiran dan pengajaran (Freudhental, 1991). Seperti halnya seorang tukang yang handal, peneliti menggunakan seluruh pengetahuannya yang dimilikinya dari: pengalaman mengajar, buku, aktivitas pengajaran, penelitian yang relevan, dan psikologi pendidikan. Aktivitas penelitian ini dimulai dengan desain awal dari prototipe aktivitas pengajaran, diikuti dengan eksperimen pengajaran, dan diakhiri dengan analisa retrospektif yang mengacu pada bagaimana kondisi awal dan akhir dari pengembangan prototipe tersebut.

Berdasarkan kedua tipe riset pengembangan di atas, penelitian ini mengembangkan prototipe material berdasarkan RME teori dalam sebuah proses berkelanjutan yang terdiri atas: analisa awal (front-end analysis), kajian ahli, eksperimen pengajaran, dan refleksi terhadap alur pengajaran lokal yang dikembangkan. Proses berulang ini menuntun penelitian ini untuk membangun sebuah teori pengajaran lokal untuk membelajarkan perkalian dan pembagian bilangan multi-angka di sekolah dasar di Indonesia. Desain penelitiannya disusun sebagai berikut:



Gambar IS. 1
Proses melingkar dari riset pengembangan

Penelitian ini dilaksanakan dalam dua fase. *Fase pengembangan* dilaksanakan dalam tiga tahap yang terfokus pada pengembangan, penerapan, dan perbaikan material. Aktivitas pelaksanannya terdiri dari analisa awal, kajian ahli, eksperimen pengajaran, dan refleksi terhadap alur pengajaran (lihat bagian 4.2.3 item c). Hasil dari setiap tahap digunakan pada tahapan berikutnya. *Fase pengembangan* ini menghasilkan versi uji-coba dari prototipe RME, yang diujicobakan pada *fase pengujian*. *Fase pengujian* difokuskan pada pengujian apakah prototipe RME dapat digunakan guru sesuai yang diharapkan dan apakah siswa berprestasi dalam tingkatan yang memuaskan setelah belajar dengan menggunakan pendekatan RME.

Hasil penelitian

Penelitian ini menghasilkan dua hal: pertama, karakteristik alur pengajaran lokal dan kualitas dari prototipe RME yang dikembangkan.

Karakteristik dari alur pengajaran lokal

Dalam penelitian ini, alur pengajaran lokal yang dikembangkan dirumuskan dalam tiga komponen: (1) tujuan pengajaran bagi siswa, (2) alur pengajaran yang diharapkan, dan (3) material pengajaran yang direncanakan (Gravemeijer & Cobb, 2001).

Alur pengajaran lokal untuk membelajarkan perkalian dan pembagian bilangan multi-angka merupakan teori lokal Indonesia yang terbuka untuk diaplikasikan lebih lanjut dan berfungsi untuk memandu peneliti lain dalam melakukan riset pengembangan selanjutnya. Teori yang dikembangkan ini belumlah sempurna dan belum seideal seperti yang diharapkan dalam teori RME. Alur pengajaran lokal yang dikembangkan untuk pengajaran perkalian dimulai dengan menyelesaikan soal kontekstual yang mengarahkan siswa menemukan kembali: penjumlahan berulang 10 angka berurutan, perkalian 10, perkalian puluhan, dan perkalian cara pendek. Sedangkan dalam alur pengajaran pembagian, soal kontekstual mengarahkan siswa menemukan kembali pengurangan berulang yang tidak terstruktur, yang berstruktur terbatas, yang berstruktur, dan pembagian cara bersusun ke bawah.

Keseluruhan alur pengajaran yang telah berlangsung (lihat bagian 9.4.1 dan 9.4.2) menunjukkan bahwa guru masih mendominasi proses belajar. Diskusi yang terjadi dan strategi yang ditemukan siswa merupakan hasil panduan dari guru. Akan tetapi hal ini menunjukkan bahwa proses menmukan kembali telah terjadi dengan bantuan guru dan dengan ini siswa dapat bekerja secara mandiri atau secara bersama dalam kelompok kecil, mendiskusikan strategi dan konsep matematika yang digunakan, yang mengarahkan mereka membangun pemahaman sendiri.

Karakteristik prototipe RME

Penelitian ini menemukan bahwa prototipe RME yang dikembangkan dalam penelitian ini valid, praktis dapat diaplikasikan, dan effektif dalam membelajarkan perkalian dan pembagian bilangan multi-angka di sekolah dasar Indonesian. Prototipe material yang dikembangkan telah mewakili kondisi Indonesia dan teori

RME (validitas isi) dan komponennya saling berkaitan satu dengan yang lain (validitas konstrak). Prototipe RME dapat digunakan di dalam kelas. Namun demikian, guru tidak dapat mengaplikasikannya seperti yang diharapkan karena terdapat beberapa kelemahan dalam menerapkan norma-norma matematika di dalam kelas. Guru belum dapat mengelola aktivitas regulasi (kognitif dan afektif) secara benar, yang merefleksikan kemajuan belajar siswa, mengambil tindakan perbaikan, meningkatkan motivasi, dan memanfaatkan masukan dari siswa. Belajar dalam pendekatan RME, siswa menunjukkan hasil belajar sesuai dengan yang diharapkan. Meskipun banyak siswa dalam satu sisi menunjukkan bahwa mereka dapat menyelesaikan soal dengan prosedur yang rasional, namun di lain pihak mereka juga melakukan kesalahan yang substantif atau salah menginterpretasi soal yang mengganggu mereka menyelesaikan soal dengan benar. Kelemahan dalam menghafal perkalian dan mengurangkan secara tidak terarah telah menurunkan kemampuan dan hasil belajar mereka. Hasil siswa berada pada tingkatan menengah. Namun demikian, hasil belajar ini lebih baik secara signifikan jika dibandingkan dengan siswa yang belajar secara konvensional.

Kesimpulan

Teori tentang alur pengajaran lokal secara RME dalam penelitian ini digunakan untuk membelajarkan perkalian dan pembagian bilangan multi-angka di sekolah dasar di Indonesia. Alurnya dibangun berdasarkan kurikulam 1994 untuk sekolah dasar dan teori RME. Penelitian ini menunjukkan bahwa alur yang dikembangkan tersebut dapat diterapkan di dalam kelas dan dapat memperbaiki hasil belajar siswa. Guru-guru Indonesia dapat menerapkannya dengan baik, dengan beberapa kelemahan pada membangun norma matematika di dalam kelas melalui diskusi kelas yang interaktif. Dalam aktivitas pembelajaran yang dilakukan dengan pendekatan RME, siswa belajar dan berdiskusi secara aktif dan mereka menunjukkan kemajuan belajar yang berarti. Kesuksesan awal dari penerapan model belajar mengajar RME ini dapat menjadi inspirasi bagi penerapan pendekatan RME dan pelaksanaan riset pengembangan lebih lanjut di Indonesia.

DUTCH SUMMARY
REALISTISCH ONDERWIJS IN VERMENIGVULDIGEN EN DELEN OP INDONESISCHE BASISSCHOLEN: EEN PROTOTYPE VAN EEN INDONESISCHE LOKALE INSTRUCTIE-THEORIE

Onderzoekscontext

Het wiskundecurriculum van 1994 dat op Indonesische basisscholen geïmplementeerd is, richt zich op het onderwijzen en leren van rekenkunde. Het curriculum heeft tot doel de leerlingen zo voor te bereiden, dat ze hun wiskundige kennis en wiskundige denkwijze kunnen gebruiken en toepassen bij het oplossen van dagelijkse problemen en bij het verwerven van nieuwe kennis (Depdikbud, 1995). Dit nieuwe curriculum gaat uit van een leerlinggericht onderwijsmodel. In dit model worden de leeractiviteiten zo vormgegeven, dat de leerlingen de mogelijkheid krijgen om op hun eigen manier tot beheersing van de stof te komen.

In tegenstelling tot de gewenste onderwijsmethode, blijken de meeste docenten echter de 'chalk-and-talk' methode te hanteren, gecombineerd met een benadering die kan worden gekarakteriseerd met 'voordoen-nadoen' (Suyono, 1996). Deze doceermethode wordt de mechanistische manier van lesgeven genoemd (Treffers, 1987). De leerkrachten wijden hun wiskundelessen vooral aan het oefenen van wiskundige symbolen, waarbij de nadruk ligt op informatieoverdracht en het toepassen van wiskundige algoritmen. De hier geschetste methode van lesgeven wordt algemeen aangetroffen in ontwikkelingslanden (zie paragraaf 1.2.2) (Feiter & Van Den Akker, 1995; en Romberg 1998)

Er is een aantal redenen aan te wijzen waarom docenten deze mechanistische aanpak hanteren. Eén daarvan is de gebrekkige kennis van wiskunde bij de leerkrachten (BPPN, 1996). Ook de kennis van de wiskundedidactiek is ontoereikend. Suyono (1996) concludeert dat docenten niet bekwaam zijn in het flexibel gebruik van lesmethodes. Ze hanteren conventionele onderwijsmethodes, zonder aandacht te besteden aan logisch denken en de kritische en creatieve aspecten van het onderwerp. Het tweede aspect heeft te maken met de kennisverwerving van de leerlingen. Samenvattend kan worden geconcludeerd dat de verbetering van de competenties van de docent (met betrekking tot de inhoudelijke kennis van wiskunde, pedagogische aspecten en de kennisverwerving van de leerlingen) zeer essentieel is voor de algehele verbetering van het wiskundeonderwijs (zie paragraaf 3.2).

De genoemde zwakke punten in de competenties van de leerkrachten, hebben een negatief effect op het onderwijs- en leerproces in de wiskunde. Het lage niveau van de leerlingen (Armanto, 2000; Haji, 1994; en Jailani, 1990) maakt het leren en begrijpen van wiskunde moeilijker, met als gevolg dat de leerlingen tegen het vak gaan opzien. Een voorbeeld van het negatieve effect van de conventionele benadering is dat leerlingen bij het oplossen van vermenigvuldigingen en deelsommen foutieve oplossingsstrategieën hebben ontwikkeld (zie paragraaf 2.5). Dit wordt veroorzaakt doordat leerlingen onvoldoende basiskennis hebben om op terug te vallen. Een indicatie voor de ernst van de problemen kan worden gevonden in de TIMSS-publikaties (1999), waarin de gemiddelde prestaties van Indonesische leerlingen op de 33e plaats komen, in een groep van 37 landen.

De bovenstaande aspecten (het gebrek aan competenties bij leerkrachten en de lage prestaties van de leerlingen), maken het noodzakelijk dat er een vernieuwd wiskundecurriculum wordt ontwikkeld en geïmplementeerd. In de eerste plaats moet er een benadering voor wiskundeonderwijs worden ontwikkeld, die het voor alle leerlingen mogelijk maakt om wiskundige feiten, concepten, procedures en vaardigheden te begrijpen en beheersen. In de tweede plaats is het essentieel dat nascholingsprogramma's worden ontwikkeld, gericht op het verbeteren van de wiskundecompetenties van de docenten.

Theoretische basis

Gebaseerd op onderzoek en ontwikkelingen in verschillende andere landen (zie: De Lange, 1994; Romberg, 1994; en Becker & Selter, 1996), wordt er in deze studie vanuit gegaan dat het concept 'realistisch wiskunde onderwijs' [in het vervolg wordt voor deze term de Engelse afkorting RME gehanteerd; 'realistic mathematics education'] een benadering is die voor Indonesië tot bevredigende resultaten kan leiden. Binnen dit concept wordt wiskunde gezien als een menselijke activiteit (Freudenthal, 1983). De mentale activiteit van de lerende staat daarbij centraal.

Wiskunde moet volgens deze benadering dan ook niet worden gezien als een structureel deductief systeem, maar als het actief hanteren en heruitvinden van wiskundige principes. Als startpunt kunnen onderwerpen uit het dagelijks leven worden gekozen, die moeten worden gemathematiseerd. Het analyseren van de wiskundige activiteit en de reflectie op het eigen handelen, vormen de basisprincipes van het proces. Door deze activiteiten kan de leerling wiskundige principes heruitvinden. Realistische wiskunde begint dan ook met de confrontatie van de leerling met een probleem in een voor hem of haar betekenisvolle context. Begeleid door de docent ontwikkelen leerlingen hun wiskundig begrip, daarbij gebruik makend van hun wiskundige voorkennis. Interactieve discussies en het leren onderscheiden van wiskundig waardevolle oplossingen, creëren een effectief en efficiënt kader om het begrip van de leerlingen te ontwikkelen. Uniformiteit en het formaliseren van de strategieën die leerlingen hanteren zijn niet belangrijk bij deze benadering. Het gaat er vooral om het begrip van de leerlingen op zo'n niveau te brengen, dat zij in staat zijn hun eigen kaders te ontwikkelen om wiskundige problemen op te lossen.

Doel en onderzoeksvraag

Ervan uitgaande dat de RME-theorie de benadering is met de beste vooruitzichten, wordt in dit onderzoek gezocht naar een passend onderwijsverbeteringsprogramma om de competenties van de docenten in het onderwijzen van het vak wiskunde te verbeteren. Loucks-Horsley (1998) en Feiter en Van den Akker (1995) stellen voor een nieuw formeel curriculum te ontwikkelen en te implementeren, door het construeren, bestuderen, gebruiken en bijschaven van een specifieke set instructiemateriaal. In het kader van het voorliggende onderzoek is curriculummateriaal ontwikkeld, waarmee leerkrachten de RME-benadering konden oefenen onder begeleiding van de onderzoeker. Cijferend vermenigvuldigen en delen is het onderwerp dat is uitgekozen voor dit onderzoek, omdat er vanuit kan worden gegaan dat docenten zeer vertrouwd zijn met de inhoud en de strategieën van dit onderdeel van de wiskunde. Gebaseerd op de voorgaande aspecten, is de volgende onderzoeksvraag geformuleerd:

> *Wat zijn de kenmerken van het RME-prototype voor lesgeven in cijferend vermenigvuldigen en delen op de Indonesische basisschool?*

De kenmerken van het prototype zijn geanalyseerd op twee verschillende aspecten, namelijk de onderwijsactiviteiten en de kwaliteit van de lokale onderwijstheorie. Vanuit het perspectief van het lesgeven en leren van wiskunde refereren de kenmerken van het prototype aan de expliciete formulering van de lokale onderwijstheorie, bestaande uit drie componenten: (1) leerdoelen voor leerlingen; (2) gestructureerd instructiemateriaal; en (3) een gepostuleerde leervolgorde (Gravemeijer & Cobb, 2001).

In dit onderzoek worden de kwaliteitsaspecten van het RME-prototype gedefinieerd als de mate waarin het prototype valide, bruikbaar, implementeerbaar en effectief is. *Validiteit* van het prototype verwijst enerzijds naar het toepassen van 'state-of-the-art'kennis van de Indonesische omstandigheden en van de RME-theorie (inhoudsvaliditeit), en anderzijds naar de consistente samenhang tussen de componenten van het prototypische lesmateriaal (constructvaliditeit). De *bruikbaarheid* van het RME prototype verwijst naar de aanvankelijke tevredenheid van de doelgroep (docenten en leerlingen) met de voorgestelde materialen en de onderwijsmethode. Het RME prototype is *implementeerbaar* wanneer de leerkracht de onderwijsleersituatie kan inrichten zoals die is bedoeld en op een manier die past binnen de Indonesische onderwijssituatie.
Deze drie kwaliteitsaspecten leiden tot de eerste sub-vraag van dit onderzoek:

> *In hoeverre is het RME-prototype valide, bruikbaar en te implementeren voor wat betreft het onderwijs in cijferend vermenigvuldigen en delen op Indonesische basisscholen?*

De effectiviteit van het RME-prototype heeft betrekking op de verwachte vooruitgang in het leerproces van de leerlingen, en op het begrip en de prestaties van de leerlingen in het cijferend vermenigvuldigen en delen. Dit leidt tot de tweede sub-vraag van het onderzoek:

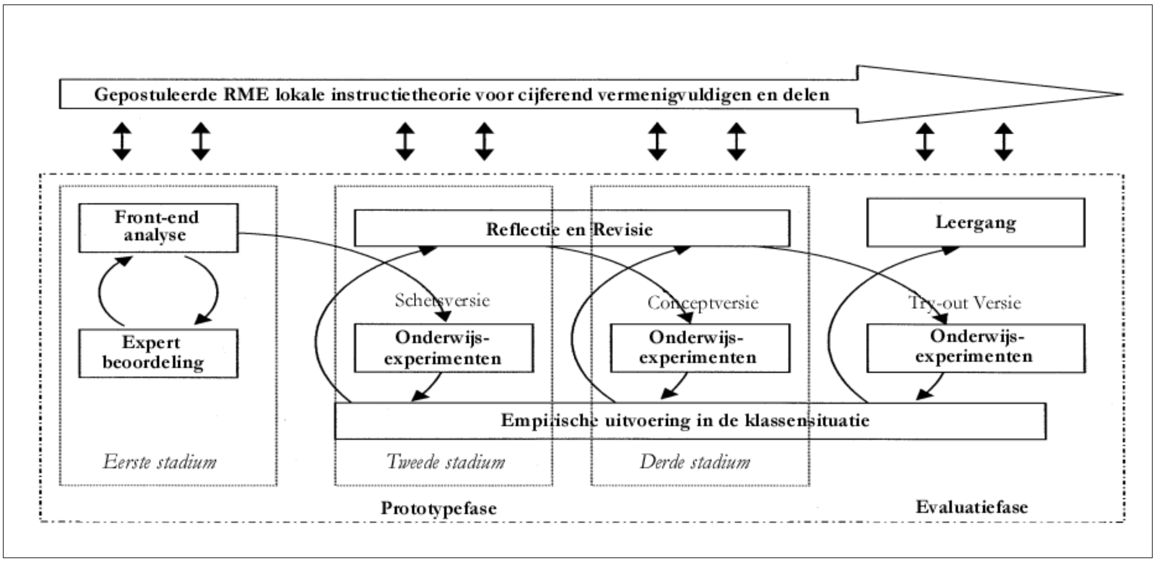
> *In hoeverre is het RME-prototype effectief voor het onderwijs in cijferend vermenigvuldigen en delen op Indonesische basisscholen?*

Onderzoeksopzet

Om de hiervoor genoemde onderzoeksvraag en bijbehorende sub-vragen te beantwoorden, is gekozen voor een ontwerpgerichte onderzoeksbenadering in het kader waarvan het prototype is ontwikkeld, beproefd en verbeterd. Op het terrein van curriculum en curriculumonderzoek spreekt men wel van een formatief onderzoeksdesign (Van den Akker, 1999 en Van den Akker & Plomp, 1996) of van een type 1 ontwerpgericht onderzoek (Richey & Nelson, 1996). Hierbij wordt onderzoek gedaan door de producten te analyseren gedurende een cyclisch ontwikkelingsproces, van een exploratiefase tot aan de (formatieve en summatieve) evaluatiefase.

Op het terrein van wiskundeonderwijs is het ontwerpgerichte onderzoek gericht op het ontwikkelen van een leergang voor een specifiek wiskundig onderwerp, waarbij de onderzoeker voorlopige instructieactiviteiten ontwikkelt in een iteratief proces van ontwerpen en beproeven.
Het is een theoriegestuurde '*bricolage*' (Gravemeijer, 1994), waarvan de kern ligt in het cyclische proces van doordenken en uitvoeren van onderwijsexperimenten (Freudenthal, 1991). Als een 'handyman' zo kan de onderzoeker gebruik maken van alle domeinspecifieke kennis, betreffende wiskundeonderwijs: lespraktijkervaring, lesmethodes, exemplarische onderwijsactiviteiten, relevant onderzoek en leerpsychologie. De activiteiten beginnen met een voorlopig ontwerp van de prototypische instructieactiviteiten, gevolgd door een aantal cycli van onderwijsexperimenten en die worden afgesloten met een retrospectieve analyse.

Gebaseerd op beide vormen van ontwerpgericht onderzoek is het prototypische materiaal in dit onderzoek ontwikkeld in een cyclisch proces van front-end analyse (vooronderzoek), expertbeoordelingen, onderwijsexperimenten en reflectie op de leergang. Deze cyclische processen leiden tot de constructie van een gepostuleerde lokale instructietheorie voor het onderwijs in cijferend vermenigvuldigen en delen op Indonesische basisscholen. Het onderzoeksdesign kan als volgt worden weergegeven:



Figuur DS. 1
Het cyclische ontwerpproces

Dit onderzoek bestaat uit twee fases van cyclische activiteiten. De prototypefase is onderverdeeld in drie stadia, gericht op ontwikkelen, implementeren en reviseren van het prototypische materiaal. Deze fase is, zoals gesteld, een cyclisch proces van front-end analyse, expertbeoordelingen, onderwijsexperimenten en reflectie (zie paragraaf 4.2.3 item c). De resultaten die in een bepaald stadium verkregen zijn, vormen de input voor het volgende stadium. De prototypefase heeft geresulteerd in een try-out versie van het RME-prototype, en deze is getoetst in de evaluatiefase. In deze fase wordt geëvalueerd of het RME prototypisch materiaal wordt gebruikt zoals bedoeld door de ontwikkelaars, en of de prestaties van de leerlingen naar verwachting zijn.

Belangrijkste bevindingen

In deze studie worden de belangrijkste bevindingen vanuit twee invalshoeken gerapporteerd: de kenmerken van de leergangen en de kwaliteitsaspecten van het RME-prototype.

Kenmerken van de lokale onderwijstheorieën

In dit onderzoek wordt met de kenmerken van de lokale onderwijstheorieën bedoeld de expliciete formulering van de volgorde van de instructie bij het onderwijs in cijferend vermenigvuldigen en delen op Indonesische basisscholen. De volgorde wordt beknopt weergegeven in drie componenten: (1) leerdoelen voor de leerlingen; (2) gepostuleerde leervolgorde; (3) het geplande instructiemateriaal

(Gravemeijer & Cobb, 2001). Deze componenten worden in de volgende paragrafen nader toegelicht.

De leergang voor het onderwijs in cijferend vermenigvuldigen en delen kan worden gezien als een eerste versie van een Indonesische lokale onderwijstheorie die zeker nog verbetering behoeft. Hij is bedoeld als een startpunt voor anderen in een volgend ontwikkelingsonderzoek. De ontwikkelde leergang blijkt nog niet tot een 'ideale' RME-instructie te leiden.
Het is een curriculum dat is aangepast aan de Indonesische omstandigheden, en dat is gebaseerd op de manier waarop de docenten de onderwijsactiviteiten in de praktijk organiseren. De leergang voor vermenigvuldigen begint met een aantal problemen, die leerlingen brengen tot het heruitvinden van wiskundige strategieën: herhaald optellen van tientallen, vermenigvuldiging met 10, vermenigvuldiging van tientallen en het standaard-vermenigvuldigingsalgoritme. Bij het leren van deelsommen, helpen problemen met een rijke context de leerlingen de volgende strategieën heruitvinden: ongestructureerd herhaald aftrekken, beperkt gestructureerd herhaald aftrekken, gestructureerd herhaald aftrekken en het standaardalgoritme voor deling.

Het onderwijsexperiment (zie paragraaf 9.4.1 en 9.4.2) laat zien dat het leerproces nog steeds gedomineerd wordt door de docenten. De discussies die tijdens de les door de leerlingen werden gevoerd, zijn gestuurd door richtvragen van de docent. De strategieën die door de leerlingen werden heruitgevonden, zijn tot stand gekomen onder leiding van de docent. Echter, het proces van geleid heruitvinden, waarbij leerlingen individueel of samen aan wiskundige problemen werkten en discussieerden over de strategieën en de wiskundige 'hulpmiddelen', zorgde er wel voor dat de leerlingen op hun eigen manier de stof leren begrijpen en beheersen.

Kenmerken van het RME prototype

Uit dit onderzoek is gebleken dat het ontwikkelde RME-prototype effectief is wat betreft het lesgeven in cijferend vermenigvuldigen en delen. Het prototypische materiaal was goed aangepast aan de Indonesische omstandigheden en de principes van de RME-theorie zijn juist toegepast (inhoudsvaliditeit). Ook sluiten de verschillende componenten in het lesmateriaal goed op elkaar aan (constructievaliditeit). Het RME-prototype was goed te gebruiken in de praktijk, maar de docenten hebben de RME-principes niet toegepast zoals bedoeld, gezien het ontbreken van een passend 'didactisch contract'. De docenten pasten een aantal regulerende (cognitieve en affectieve) activiteiten niet juist toe, zoals: reflecteren op het leerproces van de leerling, activiteiten bijsturen, vasthouden van de motivatie en het geven van feedback. De prestaties van de leerlingen blijken op het verwachte niveau te zijn, als geleerd wordt volgens de RME-methode. Aan de ene kant lieten de meeste leerlingen zien dat ze met hun aanpak op weg zijn naar een rationele oplossing. Maar aan de andere kant worden er nog belangrijke fouten en misinterpretaties gemaakt, waardoor het oplossingsproces wordt belemmerd. Onvoldoende kennis van de tafels van vermenigvuldiging en slordigheidfouten bij aftrekken, stonden hoge prestaties in de weg. Echter de leerlingen die met de RME-aanpak hebben gewerkt, presteren beter dan de leerlingen die les hebben gehad volgens de conventionele methode.

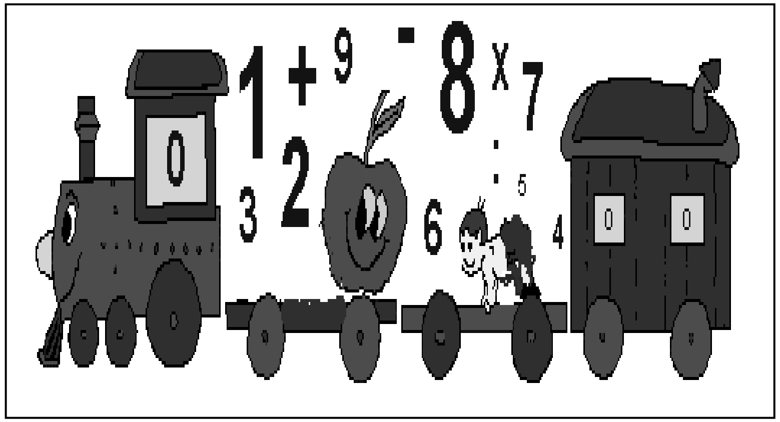
Conclusie

De RME-leergang, die in dit onderzoek is ontwikkeld, is gebruikt voor het lesgeven in cijferend vermenigvuldigen en delen op Indonesische basisscholen. De leerlijn is gebaseerd op het Indonesische wiskundecurriculum van 1994 en principes van de RME-theorie. De resultaten van dit onderzoek duiden erop, dat het praktisch mogelijk is de leerlijn in Indonesië te implementeren en dat deze effectief is wat betreft de verbetering van de prestaties van de leerlingen. De Indonesische docenten hebben de RME-leergang op de beoogde manier toegepast, waarbij echter het beoogde didactisch contract niet werd gerealiseerd. Bij het leren in de RME-aanpak, nemen de leerlingen actief deel aan de leeractiviteiten en de prestaties zijn naar verwachting. Aangezien deze experimentele implementatie succesvol is verlopen, kan dit onderzoek worden beschouwd als inspirerend voor toekomstige implementatie van de RME-aanpak in Indonesië.

Appendix A
The Teacher Guide and Pupil Book

The teachers guide and pupil book in this appendix has been reorganised for this book. It is not included the Section One (Introduction to numbers between 50.000 - 100.000) and Section Four (Multiplication and Division).

TEACHER GUIDE
UNIT 4: NUMBERS (PART 3)



GRADE 4 OF PRIMARY SCHOOL
THE SECOND TRIMESTER
1999/2000

REINVENTING MULTIPLICATION AND DIVISION OF NUMBERS GRADE 4 OF PRIMARY SCHOOL IN INDONESIA A TEACHER GUIDE

The Numbers

One of the biggest inventions in the world is the number system. Working with ten digit numbers (0 – 9), people can verify, compare, analyze and evaluate variety of quantity from 0 until infinity. The concepts and procedures of mathematics begin with the number systems. Four basic operations in the number systems develop many aspects of life such as economic, industry and everyday life as well.

In this unit, students engage in the learning process focuses on 2 basic operations, multiplication and division of numbers. The students will learn to understand the properties of numbers such as place value, concepts and procedures of number system. They will learn those properties by solving conceptual problems where the mathematics concepts and procedures embedded in the problems. It is very useful if the students use and apply their knowledge of multiplication and division they have learnt in the first trimester.

They will learn and develop their own understanding of multiplication and division of numbers by comparing and discussing their knowledge while they are solving problems with their peers. They work in the groups so that they can have opportunity to share their preferences of mathematics concepts and procedures. They will enhance their way of thinking to be more thorough, logic, and meaningful.

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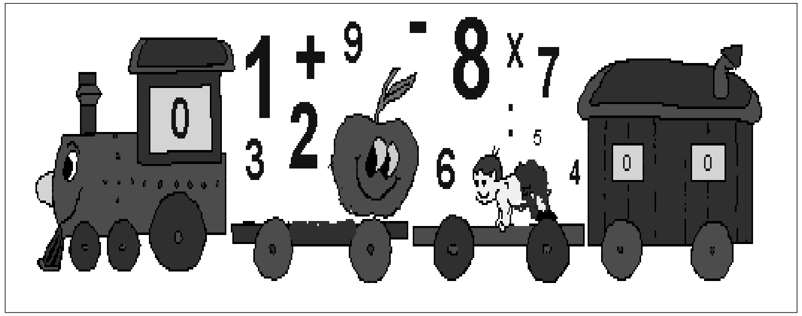
OVERVIEW



MULTIPLICATION AND DIVISION IN MULTI-DIGIT NUMBERS

How to use this guide

This guide refers to the Unit 4: Numbers (Part 3) of the 1994 mathematics curriculum for Grade 4 of Primary School in Indonesia. This unit is one of 11 units in the curriculum for Grade 4 and of 5 units in the second trimester. It is designed in order to become guidance for the teacher to teach multiplication and division of numbers using the RME Model. There is a teacher guide and a student workbook as well.

The teacher guide comprises elements that help teacher to present the multiplication and division concepts and procedures in the classroom. Using variety of contextual problems the teacher introduces the multiplication and division of numbers by describing the problems. The students, working together with their peers in the groups, analyze, evaluate and solve the problems by formalizing and/or informalizing the mathematical forms of the problems.

The teacher guide consists of overview, student workbook and teaching notes, and assessment activities.

Overview
Before beginning this unit, read the overview carefully in order to understand that strategies to develop and facilitate the instruction. The overview provides helpful information about the focus of the unit, objectives, planning, preparation, pace, and assessment.

Student Workbook and Teaching Notes
This guide contains all the student workbook pages. In each page this guide provide solutions, hints, and comments about how to facilitate the instruction in the classroom. Each Section begins with a page of information of the Section, materials needed, multiplication and division standard procedures and steps of development, homework and assessment.

Assessment Activities
Assessment activities in this guide provide formal assessment needed to measure the students' performance in learning multiplication and division of numbers. The assessments are pre-test, post-test, interactive response, and portfolios. To evaluate the process of teaching and learning in the classroom, this guide suggests to use protocol, logbook, and vignette as the tools to describe the ongoing process of teaching and learning in the classroom. General information about how to grade the items of the tests and the portfolios can be found in the page 32.

Unit Focus

Unit 4: Numbers (Part 3) introduces the numbers between 50.000 – 100.000 as part of the number strand in the 1994 mathematics curriculum of primary school in Indonesia. It focuses on the multiplication of 2-digit numbers and 3-digit numbers and on division of 4-digit numbers and 5-digit numbers by 2-digit numbers. The unit begins with students exploring the contextual problems from daily reality in which the concepts and procedures of multiplication and division of numbers are embedded in the problems. The students explore the problems together with their peers in the groups. They formalize and informalize the problems into mathematical forms and manipulate the forms to solve the problems using mathematics concepts and procedures they learnt. The notion of their understanding of the multiplication and division concepts and procedures can be verified by:

- Finding the solution of the problems
- Analyzing the structure of the concepts and procedures they use
- Looking at their way of formalizing and informalizing the mathematical form of the problems
- Comparing the solution and strategy they used to the standard form of multiplication and division of numbers.

Mathematical content and objectives

Contents in the RME prototype	Objectives and sub-objectives	Hours
Multiplication	*Pupils can understand, explore, and justify the conventional algorithm for multiplication in terms of repeated addition and the decimal numbers*	
• Repeated additions of ten numbers	• *Pupils can use repeated additions of ten numbers*	2
• Multiplication by 10	• *Pupils can use multiplication by 10*	2
• Multiplication by tens	• *Pupils can use multiplication by multiples of ten*	2
• Standard multiplication algorithm	• *Pupils can use standard multiplication algorithm*	2
Division	*Pupils can understand, explore, and justify the conventional algorithm for division in terms of repeated subtraction and the decimal numbers*	
• Unstructured repeated subtraction	• *Pupils can use the unstructured repeated subtraction*	2
• Limited structured repeated subtraction	• *Pupils can use the limited unstructured repeated subtraction*	2
• Structured repeated subtraction	• *Pupils can use the structured repeated subtraction*	2
• Standard division algorithm	• *Pupils can use the standard division algorithm*	2

Prior knowledge

This unit assumes that the student have knowledge about the followings:

- Ability to add, subtract, multiply and divide whole numbers between 1-digit numbers and 1-digit and 2-digit numbers.
- Ability to read
- Ability to place-value a number between 1 – 50.000

Planning, pacing, and preparation

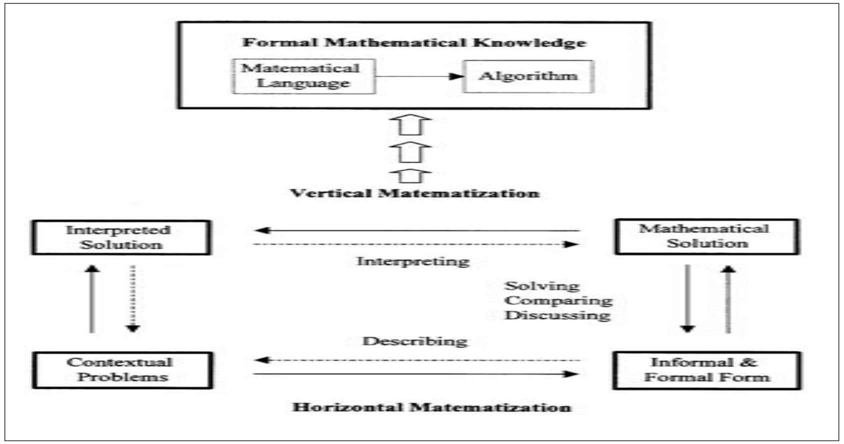
Planning and Pacing

Section	Activity Students Do	Materials
Section Two	• Multiply numbers between 2-digits and 3-digit numbers. • Interpret, recognize, and communicate the solutions of each problem • Analyze, formulate the problems into formal and informal mathematical forms • Compare and discuss the solutions with peers	• Student book • Portfolios • Homework • Students' logbook
Section Three	• Divide the 4-digit and 5-digit numbers with 2-digit numbers • Interpret, recognize, and communicate the solutions of each problem • Analyze, formulate the problems into formal and informal mathematical forms • Compare and discuss the solutions with peers	• Student book • Portfolios • Homework • Students' logbook

Preparation

Typically this unit takes approximately 20 – 25 hours of class periods, each lasting 40 minutes. Considering the usefulness of the contextual problems in introducing the mathematics concepts and procedures, the philosophy underlying both the content and the pedagogy of the unit (multiplication and division) is the realistic mathematics education (RME).

Realistic mathematics education (RME) is a mathematics teaching and learning approach that focuses on the use of contextual problems as the main aspects to introduce mathematics concepts and procedures. The students are given opportunity to reinvent mathematics by their own understanding guided by the teacher. The model is describes as follows:



The RME model of teaching and learning

From the model above the students learn mathematics in two different ways. The students encounter contextual problems in horizontal mathematization where they formalize and informalize the problems into mathematical form. After they find the mathematical forms, the students find the solution by manipulating the mathematics concepts and procedures in the form. The contextual problems chosen are the problems where the mathematics concepts and procedures embed in the problems.

The activities in the teaching process are as follows:

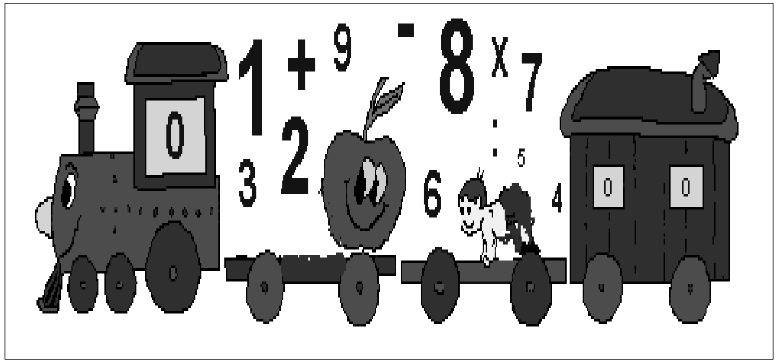
- *Understanding the contextual problems* (Teacher presents the mathematics story problems).
- *Describing* (Teacher discuss the situations of the problems by delivering clues, hints, or drawing pictures, and/or asking questions)
- *Solving problems* (Students learn individually or in their groups how to solve the problems. Differences in solutions are preferable. Using worksheet provided by teacher, students encounter varieties of problems in different difficulties. Teacher facilitates the discussion by asking questions, giving hints, and providing encouragement and rewards).
- *Comparing and discussing the solutions* (Teacher provides time and opportunity for students to distinguish solutions between groups in each problem).

There are some considerations in conducting the RME model:

- Work through the Section before teaching them
- Discuss your strategies with other teachers
- Invite students to use drawing, table, and graphic to illustrate their solution
- Allow students to work on their own strategies. Support them to share their strategies, this encourages students to take part in discussions and to understand the alternative ways to think about mathematics in the unit.
- In some cases, it may be necessary to read the problems to students or to facilitate students' understanding of the problems.

STUDENT BOOK

UNIT 4: NUMBERS (PART 3)

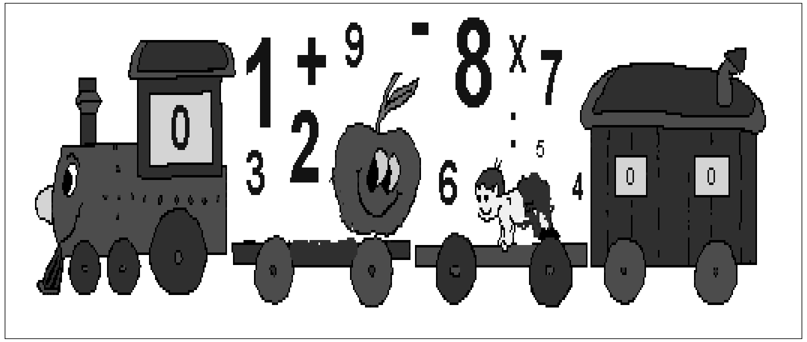


GRADE 4 OF PRIMARY SCHOOL
THE SECOND TRIMESTER
2000/2001

STUDENT BOOK AND TEACHING NOTES

STUDENT BOOK Table of Content

Section Two: Multiplication



Objectives:

After engaging in this Section the students can multiply numbers between 2-digits and 3-digit numbers by:

- Analyzing, formulating the problems into formal and informal mathematical forms and solving the problems
- Interpreting, recognizing, and communicating about the strategy used to find the solution
- Comparing and discussing the solutions with peers
- Representing the solutions of the numbers in words and in written

Section Two : Multiplication

Activity Students Do

Students encounter and examine several contextual problems in which they try to solve the problems by multiplying the 2-digit numbers and the 3-digit numbers. They analyze the problems and formulate the mathematical form of the problems. The students work in the groups.

Objectives

After engaging in this Section the students are able to multiply numbers between 2-digits and 3-digit numbers by:

- Analyzing, formulating the problems into formal and informal mathematical forms and solving the problems
- Interpreting, recognizing, and communicating about the strategy used to find the solution
- Comparing and discussing the solutions with peers
- Representing the solutions of the numbers in words and in written

Pacing

8 hours

About the Numbers

The ability to multiply the 2-digit and 3-digit numbers varies from person to person. By encountering the contextual problems from daily life, the students explore the multiplication procedures of the numbers. The activities help students to develop their understanding of numbers and to enhance their knowledge of the properties of the number system.

Materials

- Student book page 7-12.
- Portfolios
- Homework on each page
- Students' logbook

Homework

The last 3-4 problems on each page can be assigned as homework. Students' logbook would be a useful homework as well.

Planning Instruction

The teacher might introduce this Section with an introduction of a contextual problem about the tiles, to the zoo, skillful mason, and the oranges. Describing the problems might help students realize the context and the numbers involved in the problems. Ask students to write their idea or solution after they discuss the problems in the groups. The following question might help teacher to accelerate the discussion in the groups:

- How do you do that?
- How do you know that?
- How do you find that?
- Why do you think that?
- Why do you do that?

Students work in all problems individually and in the small groups. Ask them to compile all their answer in the folders. They also compare their solutions with other procedures in order to find the better solution to the problems

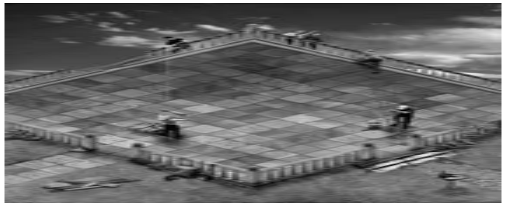
Planning assessment

The everyday quizzes can be used to assess the extent of the students' ability to multiply numbers. Students' portfolios would be an appropriate tool to be applied in observing the students' discussion and progress in understanding the multiplication.

STUDENT PAGE 8

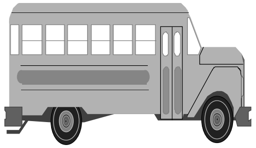
A. Tiles

1. How many tiles do they need to build the square as like in the picture?



B. To the zoo

2. In a weekend students from a school are going to the zoo using 35 buses. A bus can carry 42 students. How many students the buses can carry?

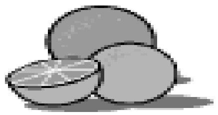


C. Skillful mason

3. Pak Budi is a skillful mason. He is asked to build a wall that needs 204 bricks in each layer. The wall contains 52 layers. How many bricks does the wall need?

D. Oranges

4. The price of an orange is Rp. 750. How much does Hakim have to pay if he buys 48 oranges?



E. Rambutan and papers

5. 1 kg of rambutan equals 98 pieces of paper. How many pieces of papers are there that equal to 37 kg of rambutan?

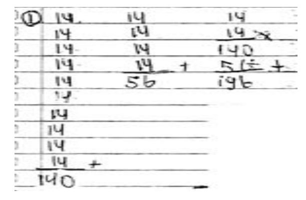
F. Riding a bike

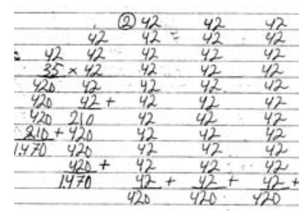
6. Hilma rides a bike to go to her grandmother's house. In a minute the wheels roll about 306 times. How many times do the wheels circulate if she rides about 49 minutes?

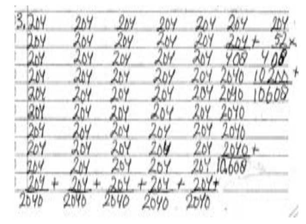
SOLUTIONS

1. 14 x 14 = 196 tiles
2. 42 x 35 = 1470 students
3. 204 x 52 = 10608 bricks
4. 750 x 48 = Rp. 36,000
5. 98 x 37 = 3,626 pieces
6. 306 x 49 = 14,994 times

PROCEDURES







HINTS AND COMMENTS

Sub-objective
The students are able to apply the addition of ten numbers to solve the problem

Overview
The students work individually and together to discuss the problems. The contexts will guide the students toward the procedures to be used to solve the problems.

About the problem
Problems 1-3 are to guide the students toward the learning process of the addition of 10 numbers. Problems 4-6 are for homework.

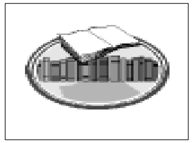
Planning

1. The teacher can read the problem and ask students to write using their own word or ask the students to read the problem.
2. To facilitate the learning process, the teacher guides the students by discussing the situation. This will lead them toward the informal math forms.
3. Usually students stick on one solution they are comfortable with. Asking the students to solve the problem using several procedures is the best way to understand the progress of the students' learning process
4. Comparing and discussing the students' procedures are the most important activity in order to share the students' learning process among their peers. It also gives chance to students to develop their ability to ask, to explain, and to understand other learning process in solving problems.

STUDENT PAGE 9

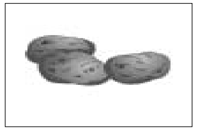
G. Books

7. A bookshop "CINTA BUKU" bought 96 boxes of Geography books. Each box contains 24 books. How many books are there?



H. Potatoes

8. If the price of 1-kg potatoes is Rp. 905, how much should you pay for 39 kg of potatoes?



I. The teacher

9. In a year there are 365 days. How many days do your teacher live if she is 54 year-old now?

J. War

10. There was a war in Kosovo. It was about 917 kosovars went out from Kosovo everyday to Albania as refugees. How many kosovars were there in Albania for 45 days of war?

11. A soldier got injured in a war and trapped in a jungle. He tries to escape from the jungle by walking through the woods. He walks very slowly that is about 76 m an hour. After 29 hours he finds a small village outside the jungle. How many meters does he walk?

K. Price and Weight

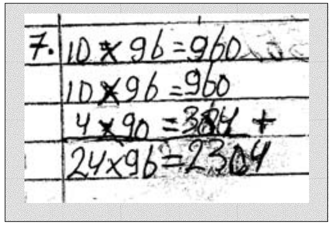
12. The price of a mathematics book is 26 times as much as the price of a pencil. A pencil costs Rp. 675. How much is the price of a mathematics book?

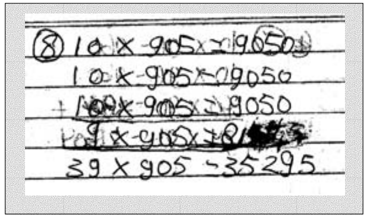
13. The weight of 1 box of tomatoes equals to 59 plastic bags of candies. The weight of 237 boxes of tomatoes equals to _______ plastic bags of candies.

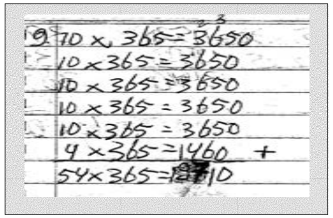
SOLUTIONS

7. 96 X 24 = 2304 books
8.. 905 X 39 = Rp. 35,295
9. 365 X 54 = 19710 days
10. 917 X 45 = 41,265 people
11. 74 X 29 = 2,146 meters
12. 675 X 26 = Rp. 17,550
13. 237 X 59 = 13,983 plastics

PROCEDURES







HINTS AND COMMENTS

Sub-objective
The students are able to apply the multiplication of 10 to solve the problems

Overview
The students work individually and together to discuss the problems. The contexts will guide the students toward the procedures to be used to solve the problems.

About the problem
Problems 7-9 are to guide the students toward the multiplication by 10. Problems 10-13 are for homework.

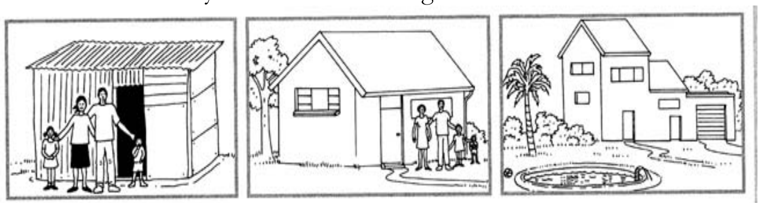
Planning
1. The teacher reads the problem and asks the students to write using their own word or the students read the problem.
2. To facilitate the learning process, the teacher guides the students by discussing the situation. This will lead them toward the informal math forms.
3. Usually students stick on one solution they are comfortable with. Asking the students to solve the problem using several procedures is the best way to understand the progress of the students' learning process
4. Comparing and discussing the students' procedures are the most important activity in order to share the students' learning process among their peers. It also gives chance to students to develop their ability to ask, to explain, and to understand other learning process in solving problems.

STUDENT PAGE 10

L. Using waters

Water is very important. The followings are the facts:

- One chicken from a chicken farm will need 1,545 liters of water before it is sold in the market.
- The water from one 10-minute shower could keep 200 children alive for a day.
- One toilet flush uses 9 liters of waters. This amount could keep 24 impala alive for a day.
- An adult can stay alive and clean using 35 liters of water a week.



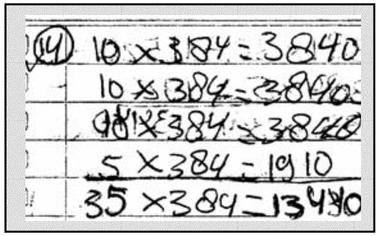
A family living in a house like this uses about 65 liters of water a day.	A family living in a house like this uses about 640 liters water a day.	A family living in a house like this uses about 7300 liters of water a day.

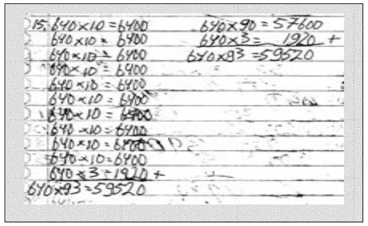
14. How many liters of water do 384 adults need to stay alive and clean every week?
15. In Desa Kelambir Lima Medan, there are 93 families live in a house like the middle one above. How many liters of water do the families need every day?
16. In a small village there are 209 families live in a house like the left one above. How many liters of water do the families need every day?
17. Ali has 39 chickens. How many liters of water do the chickens need before they are sold in the market?
18. If a family uses 78 toilet flushes in a day, how many impala can be alive using the amount of waters?
19. In Jalan Diponegoro Jakarta, there are 13 families live in a house like the right one above. How many liters of water do the families need every day?

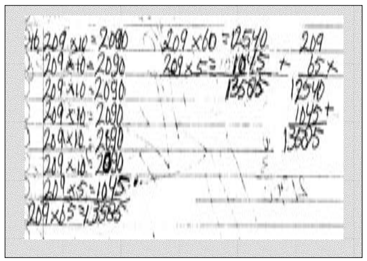
SOLUTIONS

14. 384 x 35 = 13,440 liters
15. 93 x 640 = 59,520 liters
16. 209 x 65 = 13,585 liters
17. 39 x 1545 = 60,255 liters
18. 78 x 200 = 16,600 impalas
19. 13 x 7300 = 94,900 liters

PROCEDURES







HINTS AND COMMENTS

Sub-objective

The students are able to apply the mental multiplication (using place value) to solve the problems

Overview

The students work individually and together to discuss the problems. The contexts will guide the students toward the procedures to be used to solve the problems.

About the problem

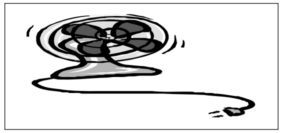
Problems 14-16 are to guide the students toward the multiplication by tens. Problems 17-19 are for homework.

Planning

1. The teacher reads the problem and asks students to write using their own word or the students read the problem.
2. The teacher discusses the situation to lead the students toward the math forms. *The teacher and the students compare the procedures of the multiplication by 10 and the multiplication by tens.*
3. Usually students stick on one solution they are comfortable with. Asking the students to solve the problem using several procedures is the best way to understand the progress of the students' learning process
4. Comparing and discussing the students' procedures are the important activity in order to share their learning process. It also gives chance to develop their ability to ask, to explain, and to understand other learning process in solving problems.

STUDENT PAGE 11

M. A Fan



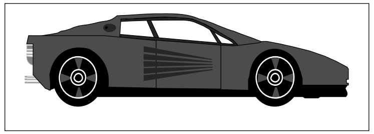
20. A fan has been used for 130 weeks and turned on for 84 hours a week. How many hours has the fan been used?

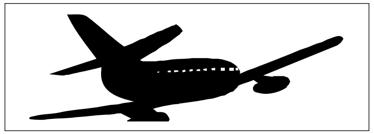
N. A Jumping Frog

21. A frog can jump 27 cm long each time. If the frog jumps 106 times, how long does he jump?

O. Plane and Car

22. Going by a plane is 35 times as quick as going by a car. If the speed of a car is 64 km for an hour, how much is the speed of the plane?





P. Football

23. There are going to be World Cup in Japan and South Korea in 2002. There are 48 teams in the competition with 24 players in each team. How many players are there in the championship?

24. 37 buses go to Semarang from Yogyakarta to see a football match between PSIM Yogyakarta and PSIS Semarang. Each bus can carry 46 supporters. How many supporters are there all altogether?

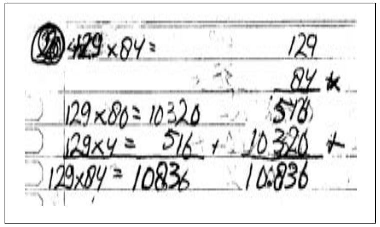
Q. Chicken farm

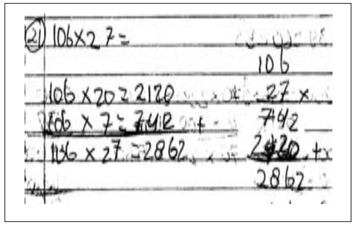
25. A chicken farm has 76 chickens. When they are fed well each chicken lays 14 eggs a month.
    a. How many eggs does the farm have every month?
    b. How many eggs does the farm get after a year?

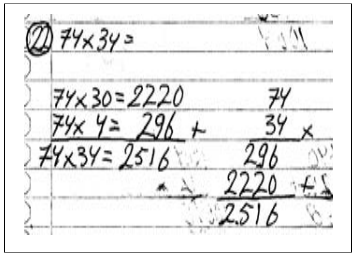
SOLUTIONS

20. 129 x 84 = 10,836 hours
21. 106 x 27 = 2,862 cm
22. 35 x 64 = 2240 in an hour 23.
48 x 24 = 1,152 players
24. 37 x 46 = 1,702 supporters

PROCEDURES







HINTS AND COMMENTS

Sub-objective
The students are able to apply the standard multiplication to solve the problems

Overview
The students work individually and together to discuss the problems. The contexts will guide the students toward the procedures to be used to solve the problems.

About the problem
Problems 20-22 are to guide the students toward the learning process of the addition of 10 numbers. Problems 23-25 are for homework.

Planning
1. The teacher can deliver the problems by reading the problem and asking students to write using their own word or by asking the students to read the problem.
2. *The teacher and the students discuss the procedures of the multiplication by 10 and the multiplication by the tens. Comparing these procedures will lead the students to the standard algorithm of multiplication.*
3. Usually students stick on one solution they are comfortable with. Asking the students to solve the problem using several procedures is the best way to understand the progress of the students' learning process
4. Comparing and discussing the procedures are the most important activity to share the students' learning process among their peers. It also gives chance to develop their ability to ask, to explain, and to understand other learning process in solving problems.

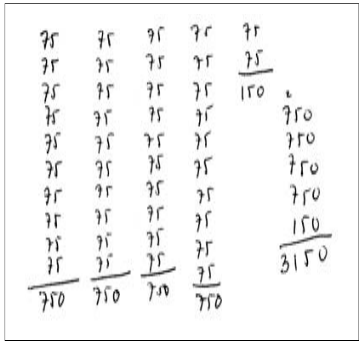
STUDENT PAGE 12

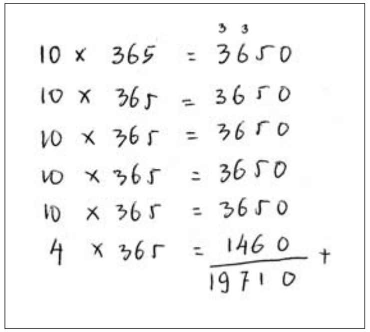
R. Miscellaneous problems

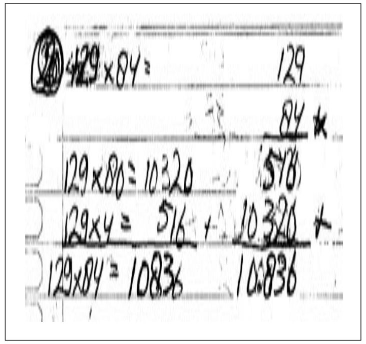
26. Each of 57 Klompencapir in several villages is about to get 309 sacks of urea donated and distributed by the government. How many sacks of urea are there?
27. Pak Adi bought 75 boxes of potatoes that contains 68 potatoes each. How many potatoes does he get?

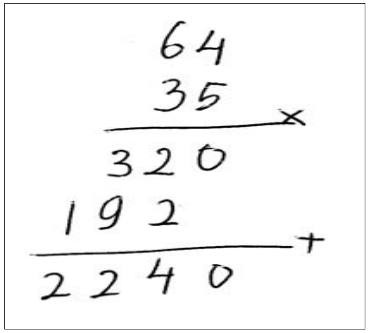
S. SUMMARY

- Formal mathematical knowledge of multiplication
  1. ***Multiple addition of ten numbers*** 2. ***Multiplication by 10***





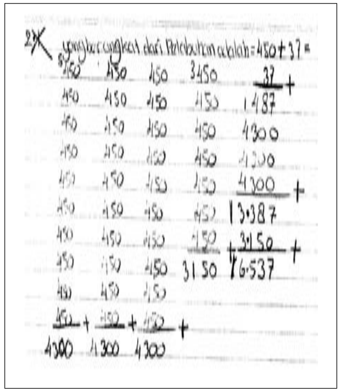


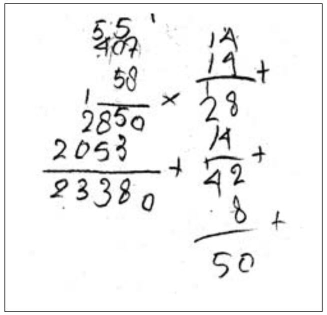


SOLUTIONS

25. 309 x 57 = 17,613 sacks
26. 75 x 68 = 5,100 potatoes

STUDENTS' MISTAKES



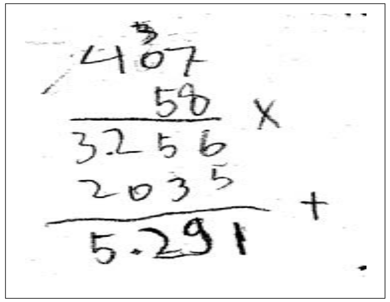


HINTS AND COMMENTS

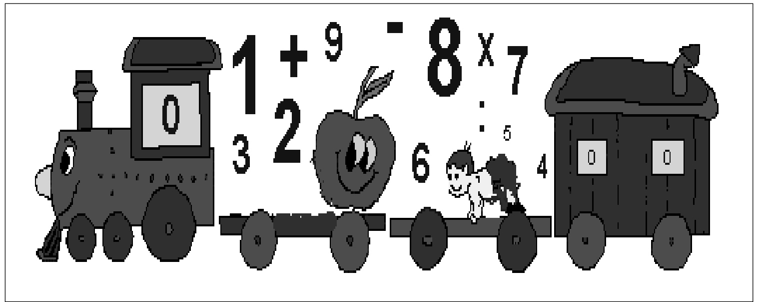
Sub-objective
The students are able to apply several procedures to solve the problems

Overview
The students work individually and together to discuss the problems. The contexts will guide the students toward the procedures to be used to solve the problems.
The problems are assigned as the summary problems in which the teacher is able to use them as the items to assess the progress of the students' understanding after learning the multiplication

About the problem
Problems 26-27 aim at analyzing the students' learning progress in the multiplication. The students can use the procedures they prefer and understand. They can also apply several procedures they understand to solve the problems.



Section Three : Division



Objectives:

After engaging in this Section the students are able to divide the 4-digit and 5-digit numbers with 2-digit numbers by:

- Analyzing, formulating the problems into formal and informal mathematical forms and solving the problems
- Interpreting, recognizing, and communicating about the numbers by comparing the solutions of each problems
- Representing the solutions in words and in written
- Comparing and discussing the solutions with peers

Section Three : Division

Activity Students Do

ıts encounter and examine several contextual problems in which they try to solve the problems by dividing the 4-digit and 5-digit numbers with the 2-digit numbers. They analyze the problems and formulate the mathematical form of the problems. The students work in the groups.

Objectives

After engaging in this Section the students can divide the 4-digit and 5-digit numbers with 2-digit numbers by:

- Analyzing, formulating the problems into formal and informal mathematical forms and solving the problems
- Interpreting, recognizing, and communicating about the numbers by comparing the solutions of each problems
- Representing the solutions in words and in written

Pacing

8 hours

About the Numbers

The ability to divide the 4-digit and 5-digit numbers with 2-digit numbers varies from person to person. By encountering the contextual problems from daily life, the students explore the division procedures of the numbers. The activities help students to develop their understanding of numbers and to enhance their knowledge of the properties of the number system.

Materials

- Student book page 13 - 18
- Work book (portfolios)
- Homework on each page
- Students' logbook

Homework

The last 3-4 problems can be assigned as homework. Students' logbook would be a useful homework as well.

Planning Instruction

The teacher might introduce this Section with an introduction of the contextual problems about the lebaran day, reading books, and students in line. Discussing the situation of the problems might help students understand the in the problems. Ask students to write their idea or solution after they discuss the problems. The following question might help teacher to accelerate the discussion in the groups:

- How do you do that?
- How do you know that?
- How do you find that?
- Why do you think that?
- Why do you do that?

Students work individually or in the small groups. Ask them to compile all their answer in the folders. They also compare their solution with other students in order to find the better solution to the problems

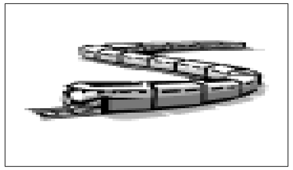
Planning assessment

The everyday quizzes can be used to assess the extent of the students' ability to divide numbers. Students' portfolios would be an appropriate tool to be applied in observing the discussion and the student progress in understanding the division procedures.

STUDENT PAGE 14

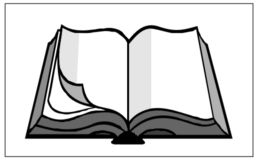
A. Lebaran Day

1. A day before the Lebaran day, there are 1400 people crowded in the Gambir Station to go back (mudik) to Surabaya. How many train-wagon do they need to go back to their place if each wagon can carry 86 people?



B. Reading a book

2. Nita is reading a book that is 410 pages long. She always reads 16 pages a day. How many days will it take for her to finish the book? How many pages does she read in the last day?



C. Students in line

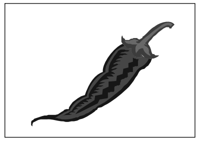
3. There are 5232 students in the football field to make several lines. Each line contains 48 students. How many lines do they make in the field?

D. Grapes

4. 1242 pieces of grapes are equal to 69 kg. How many pieces of grapes are for 1 kg?

E. Pepper and potatoes

5. A sack of red pepper contains 36 times as much as a sack of potatoes. If a sack of red paper contains 7416 pieces, how many pieces are the contents of a sack of the potatoes?



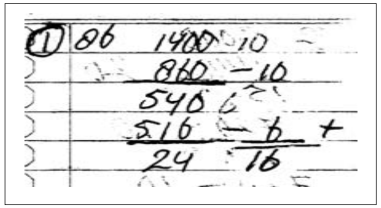
F. Computers

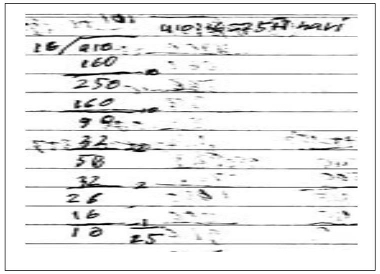
6. A computer can be used 70,794 hours before it is broken. If Adi uses his computer for 75 hours a week, how many weeks can his computer be used? How many years is it?

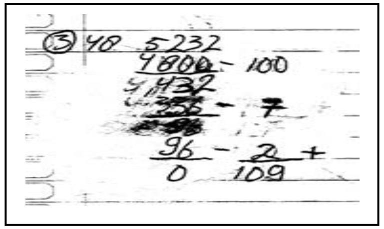
SOLUTIONS

1. 1400 : 86 = 16 + 1 wagons
2. 410 : 16 = 25 + 1 days
3. 5232 : 48 = 109 lines
4. 1242 : 69 = 18 pieces
5. 7416 : 36 = 206 pieces
6. 70794 : 75 =1242 weeks
   1242 : 52 = 23 years 48 weeks

PROCEDURES







HINTS AND COMMENTS

Sub-objective
The students are able to apply the multiplication by 1-digit numbers, by 10, 100, and 1000.

Overview
The students work individually and together to discuss the problems. The contexts will guide the students toward the procedures to be used to solve the problems.

About the problem
Problems 1-3 are to guide the students toward the multiplication by 1-digit, by 10, 100, and 1000. Problems 4-6 are for homework.

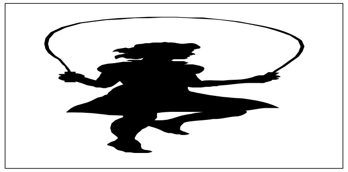
Planning

5. The teacher reads the problem and asks students to write it using their own word or the students read the problem.
6. To facilitate the learning process, the teacher and the students discuss the situation. This will lead them toward the informal math forms.
7. Usually students stick on one solution they are comfortable with. Asking the students to solve the problem using several procedures is the best way to understand the progress of the students' learning process
8. Comparing and discussing the procedures are the most important activity in order to share the students' learning process. It also gives chance to students to develop their ability to ask, to explain, and to understand other learning process in solving problems.

STUDENT PAGE 15

G. Graduation

7. There are 4630 people in the graduation ceremony. The OC should serve them with water. A bottle of instant water serves 23 person. How many bottles must be prepared for the people?



H. Jumping on the rope

8. The record for the greatest number of consecutive jumping on the rope is 8960 jumps. There were 56 jumps in a minute. How many minutes does the record take place?

I. Buying candies

9. A candy costs Rp. 75,-. How many candies do you get if you have Rp. 20000?

J. Chicken farm

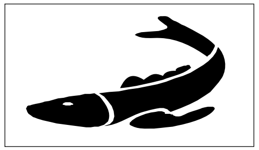
10. In the chicken farm there are 58 chickens. They need 81,722 kg of corns every month. How many kg of corns does a chicken need in average?

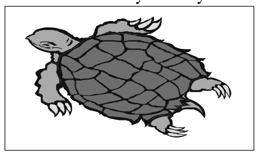
K. Marlioboro Street

11. Every 64 hours, there are 69,568 cars passed by the Jalan Marlioboro Yogyakarta. How many cars are there passed by the street every hour?

L. Eggs

12. A fish can lay eggs 43 times as many as that of a turtle. In a year a fish can lay 92,149 eggs, how many eggs can a turtle lay in a year?

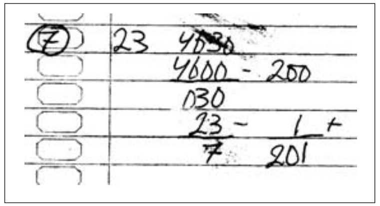


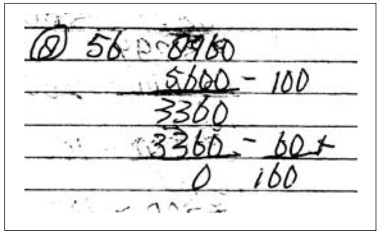


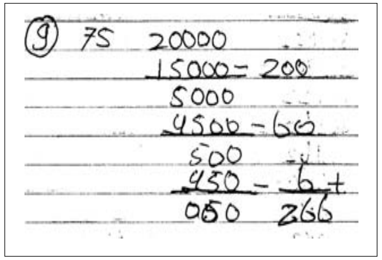
SOLUTIONS

7. 4630 : 23 = 201 + 1 bottles
8. 8960 : 56 = 160 minutes
9. 20000 : 75 = 266 with Rp. 50
10. 81722 : 58 = 1409 kg
11. 69568 : 64 = 1087 cars
12. 92149 : 43 = 2143 eggs

PROCEDURES







HINTS AND COMMENTS

Sub-objective
The students are able to apply the table of proportion or of the multiplication

Overview
The students work individually and together to discuss the problems. The contexts will guide the students toward the procedures to be used to solve the problems.

About the problem
Problems 7-9 are to guide the students to create the table of multiplication. Problems 10-12 are for homework.

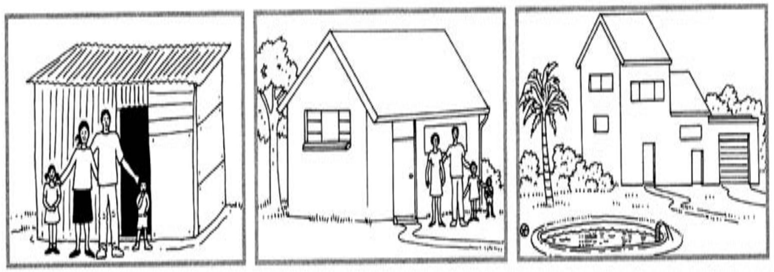
Planning
1. The teacher reads the problem and asks students to write it using their own word or the students read the problem.
2. The teacher and the students discuss the situation. This will lead them toward the math forms. By creating the table of the proportion or the multiplication the students can solve the problem. The students develop the table by multiplying the divisor with several numbers.
3. Usually students stick on one solution they are comfortable with. Asking the students to apply several procedures guides them to understand the procedures.
4. Comparing and discussing the procedures are the important activity in order to develop the students' learning process. It also gives chance to students to develop their ability to ask, to explain, and to understand other learning process in solving problems.

STUDENT PAGE 16

M. Using waters

Water is very important. The followings are the facts:

- One chicken from a chicken farm will need 1,545 liters of water before it is sold in the market.
- The water from one 10-minute shower could keep 200 children alive for a day.
- One toilet flush uses 9 liters. This amount could keep 4 impala alive for a day.
- An adult can stay alive and clean using 35 liters of water a week.



A family living in a house like this uses about 65 liters of water a day.	A family living in a house like this uses about 640 liters water a day.	A family living in a house like this uses about 7300 liters of water a day.

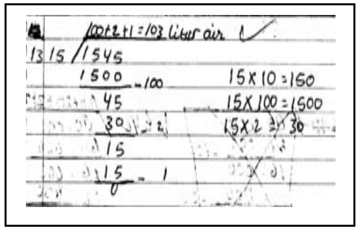
13. If a chicken stays for 15 days before it can be sold, how many liters of waters do a chicken need for a day?
14. How many weeks can an adult stay alive and clean using 8610 liters of water?
15. A water company can produce 80000 liters of water for a day.
    a. How many families can live using the amount of water in the house like the left one in a day?
    b. How many families can live using the amount of water in the house like the middle one in a day?
    c. How many families can live using the amount of water in the house like the right one in a day?

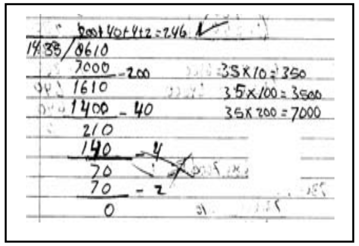
    Give your opinion of the three solutions you have.

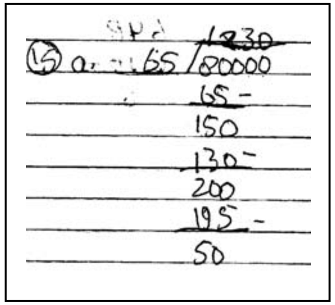
SOLUTIONS

13. 1545 : 15 = 103 liters
14. 8610 : 35 = 246 weeks
15. 80000 : 65 = 1230 r 50 liters

PROCEDURES







HINTS AND COMMENTS

Sub-objective

The students are able to apply the guess-multiply-subtract procedure

Overview

The students work individually and together to discuss the problems. The contexts will guide the students toward the procedures to be used to solve the problems.

About the problem

Problems 7-9 are to guide the students to create the table of multiplication. Problems 9.b-c are for homework.

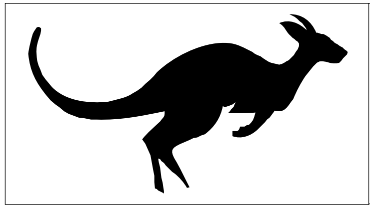
Planning

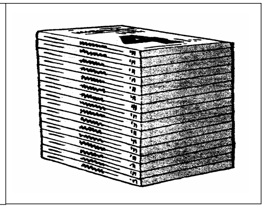
1. The teacher reads the problem and asks students to write it using their own word or the students read the problem.
2. The teacher and the students discuss the situation. This will lead them toward the math forms. By creating the table of the multiplication the students develop the guess-multiply-subtracting procedure.
3. Usually procedures.
4. Comparing and discussing the procedures are the important activity in order to develop the students' learning process. It also gives chance to students to develop their ability to ask, to explain, and to understand other learning process in solving problems.

STUDENT PAGE 17

N. Kangaroo's Jump

16. Kangaroos live in Australia. A kangaroo can jump 72 cm long. How many times he should jump in order to pass a street that is 7700 cm long?





O. Stack of paper

17. The following stack of paper equals 29 times as many as a book. If the stack contains 45240 pieces of papers, how many pieces of papers are in the book?

P. The Zoo

18. Last year the zoo is open in 38 weekends. There are 87,742 people visit the zoo. How many people visit the zoo every weekend in average?

Q. Durians

19. Bu Armani sells 35 durians for Rp. 47250. How much is the price of each durians if the durians has the same weight?

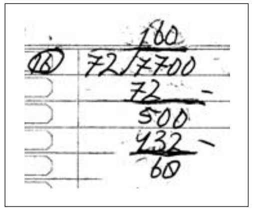
R. Factory

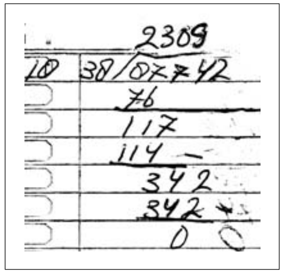
20. A Company can manufacture 84,672 shuttlecocks in 28 weeks. How many cocks can the company produce every week?
21. A medicine factory can produce 52,809 tablets in 87 minutes. How many tablets do the fabric produce every minute?

SOLUTIONS

16. 7700 : 72 = 106+ 1 times
17. 45240 : 29 = 1560 pieces
18. 87742 : 38 = 2309 people
19. 47250 : 35 = Rp. 1350
20. 84672 : 28 = 3024 cocks
21. 52809 :87 = 607 tablets.

PROCEDURES





HINTS AND COMMENTS

Sub-objective
The students are able to apply the standard algorithm of the multiplication

Overview
The students work individually and together to discuss the problems. The contexts will guide the students toward the procedures to be used to solve the problems.

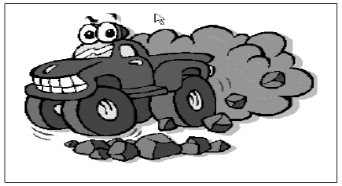
About the problem
Problems 16-18 are to guide the students toward the standard algorithm of division. Problems 19-21 are for homework.

Planning
1. The teacher reads the problem and asks students to write it using their own word or the students read the problem.
2. The teacher and the students discuss the situation. This will lead them toward the math forms. By discussing and comparing the procedure the students learn how to divide the numbers using the standard algorithm of division.
3. Usually students stick on one solution they are comfortable with. Asking the students to apply several procedures guides them to understand the procedures.
4. Comparing and discussing the procedures are the important activity in order to develop the students' learning process. It also gives chance to students to develop their ability to ask, to explain, and to understand other learning process in solving problems.

STUDENT PAGE 18

S. Miscellaneous Problems

22. How old would you be in years if you lived 95,040 hours?
23. Jagi has 3,400 stamps in his album. There are two dozen stamps on each page. How many pages are there?

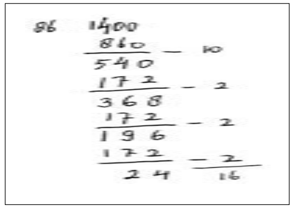


24. Each tire for my monster truck will last exactly 84 km. How many spares do I need so I can use the fewest number of tires in a total of 62,664 km?

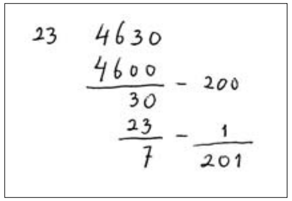
T. SUMMARY

Formal mathematical knowledge (mathematical language to algorithm)
Standard algorithm form of division:

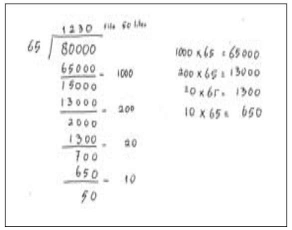
1. The Unstructured multiple subtraction



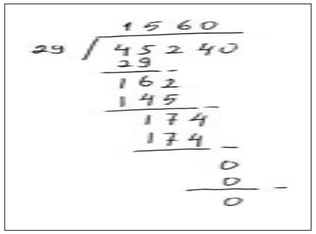
2. The limited structured subtraction



3. The structured subtraction



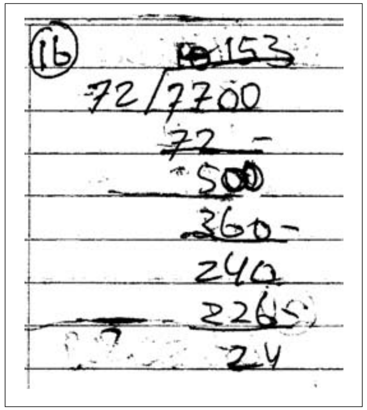
4. The standard algorithm

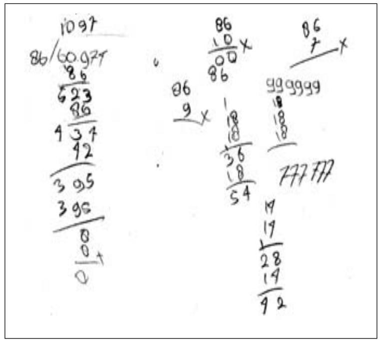


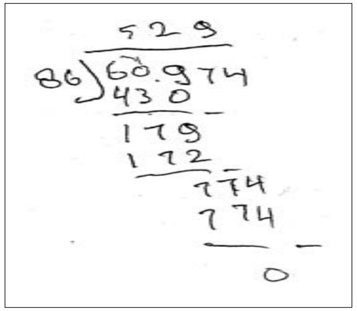
SOLUTIONS

22. 95040 : 24 = 3960 days
3960 : 30 = 132 months
132 : 12 = 11 years
23. 3400 : 24 = 141 + 1 pages
24. 62664 : 84 = 746

STUDENTS' MISTAKES







HINTS AND COMMENTS

Sub-objective
The students are able to apply several procedures to solve the problems

Overview
The students work individually and together to discuss the problems. The contexts will guide the students toward the procedures to be used to solve the problems.
The problems are assigned as the summary problems in which the teacher is able to use them as the items to assess the progress of the students' understanding after learning the division.

About the problem
Problems 22-24 aim at analyzing the students' learning progress in the division. The students can use the procedures they prefer and understand. They can also apply several procedures they understand to solve the problems.

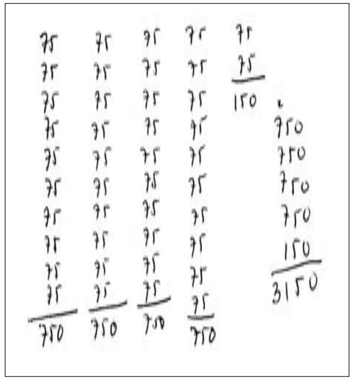
STUDENT PAGE 24

M. Summary

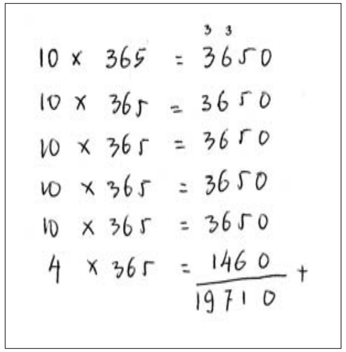
Formal mathematical knowledge (mathematical language to algorithm)

1. Multiplication

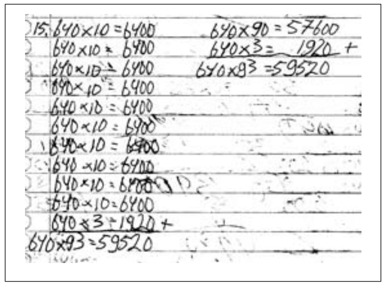
a. ***Multiple addition of ten numbers***



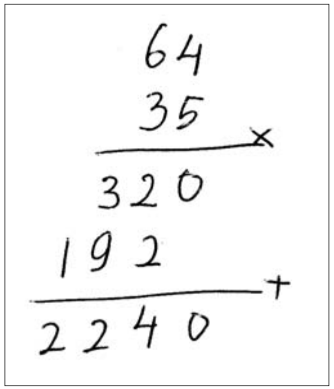
b. ***Multiplication by 10***



c. ***The multiplication by tens***

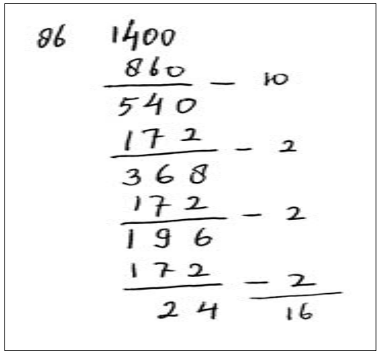


d. ***The standard algorithm***

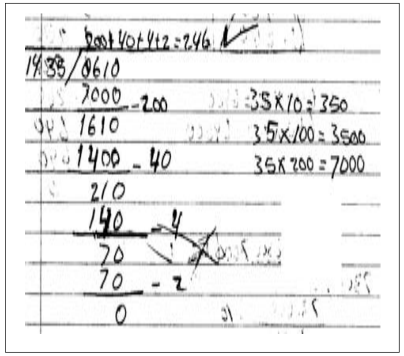


2. Division

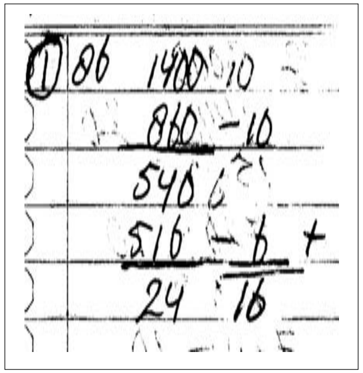
a. The Unstructured multiple subtraction



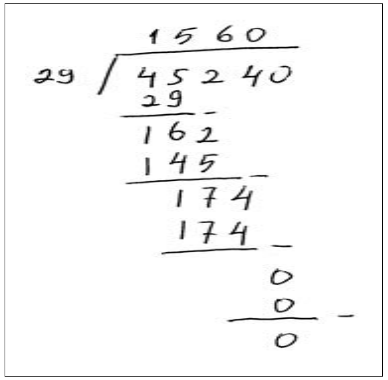
b. The limited structured subtraction



c. The structured subtraction



d. The standard algorithm



C. ASSESSMENT

PLANNING THE ASSESSMENT

Valid assessment should be based on evidence drawn from several sources. This guide suggests that the assessment plan may draw from the following resources:

- *Pre-test items and post-test items* of multiplication and division of numbers
- *Daily and weekly quizzes for* analyzing the student learning progress
- *Observations*
  The teacher looks, listens, records, analyzes the observable behavior of the students
- *Interactive response*
  The teacher records how students work with their peers in the groups when they respond, quest, clarify, revise, and extend their thinking
- *Portfolios*
  From the collections of students' pieces of works, the teacher looks for the concepts and procedures used in solving problems, the structure of the thinking process, the mathematics formal/informal forms applied, and the solutions found.

ONGOING ASSESSMENT

- Problems within the Section

To evaluate the ongoing process, this guide introduce the informal tools that suggest the teacher to describe the situations in the classroom using protocol, logbook, and vignette. The example of the tools can be found on page 40.

- Summary

This guide provides the summary of each Section. Each summary describes the example of standard form of concepts and procedures of the multiplication and division of numbers. The students can find the summary at the end of every section.

End-of-Section assessment

In this guide there are four assessments that can be used to measure the ability of the students in understanding the concepts and procedures of multiplication and division and the students' ability to solve the mathematics problems. There are three or four items in each Section. The teacher can choose one or both of them to verify the students' understanding. The teacher also is suggested to design his/her own items.

Objectives and assessment

This teacher guide provides the objectives of each Section considering the 1994 mathematics curriculum for primary school. The items are chosen and designed in order to find the development of students' understanding of the concepts and procedures of multiplication and division in numbers. It is important to note that the objectives in each Section correlate significantly with each other but it does not mean that the objectives are a prerequisite to attaining those in other Sections. The students should advance continuously towards all objectives in each Section.

Assessment criteria of scoring and analyzing response

Students may answer the problems using various level of mathematical understanding. Response should be considered for the mathematics that it shows, not for the expected standard response. Descriptive judgement would be preferable and would be a helpful information/feedback of understanding the students' progress in learning the concepts. The following table is the scoring criteria for the tests.

Score	**Solution Stage**
0	**Noncommecement** The student is unable to begin the problem or hands in work that meaningless.
1	**Approach** The student approaches the problem with meaningful work, indicating some understanding of the problem, but an early impasse is reached.
2	**Substance** Sufficient detail demonstrates that the student has proceeded toward a rational solution, but major errors or misinterpretations obstruct the correct solution process.
3	**Result** The problem is very nearly solved; minors errors produce an invalid final solution.
4	**Completion** An appropriate method is applied to yield a valid solution.

The following table is the categorization of the students' ability in solving problems

Score Interval (*)	Categorization	Interpretation Values
00 – 13	Low	Low problem solving ability
14 – 26	Mediocre	Mediocre problem solving ability
27 – 40	High	High problem solving ability

Self assessment

Self-assessment encourages students to describe their progress in learning multiplication and division, their ability to solve problems, and their attitude towards mathematics. This guide suggests to use several tools, such as:

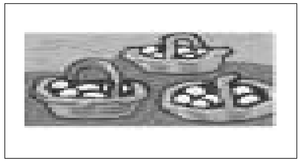
- Questionnaire of students' backgrounds (page 37)
- Students' log about learning multiplication and division (page 36)
- In-depth interview to individual or small group students to elicit what they have learned, what they think is important, and why.

D. APPENDICES

Quizzes

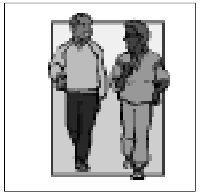
Multiplication

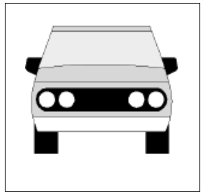
1. Buying eggs



Hamim buys 78 eggs. An egg costs Rp. 475. How many rupiahs does he have to pay?

2. Biking and walking

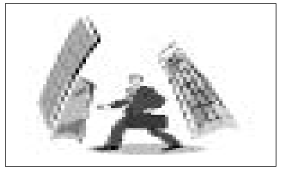




Riding a car is 207 times faster than walking on foot. What is the speed of the bike if a man walks 86 m per minute?

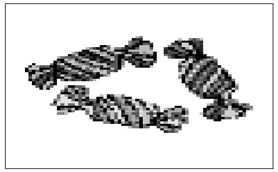
Division

1. Walking



Pak Amat always walks to his office. The distance is 1800 m from his house. If he walks for 82 m in a minute, how many minutes does he walk to his office?

2. Candys

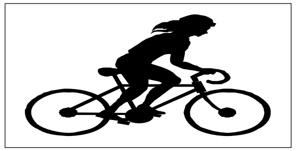


The price of a "COKELAT" candy is Rp. 75. If you have Rp. 9800, how many candies do you get using all your money?

Everyday quizzes

T. The price
The price of a math book is 36 times as much as that of a pencil. A pencil costs Rp. 675. How much is the price of a math book?

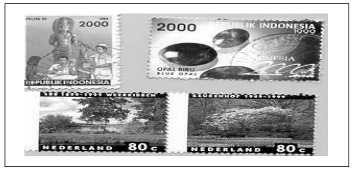
U. Riding a bike
Hilma rides a bike to go to her grandmother's house. She rides about 49 minutes. In a minute she reaches 306 meters. How many meters is the distance she reaches?



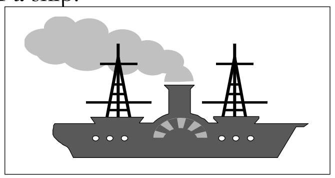
V. Riding a bike
Each of 57 Klompencapir in several villages is about to get 309 sacks of urea donated and distributed by the government. How many sacks of urea are there?

W. The supporters
3000 supporters are going to Jakarta from Bandung by bus to see the football match. Each bus can carry 72 supporters. How many buses are needed?

X. The price
Jagi has 5712 stamps in his album. There are two dozen stamps on each page. How many pages are there?

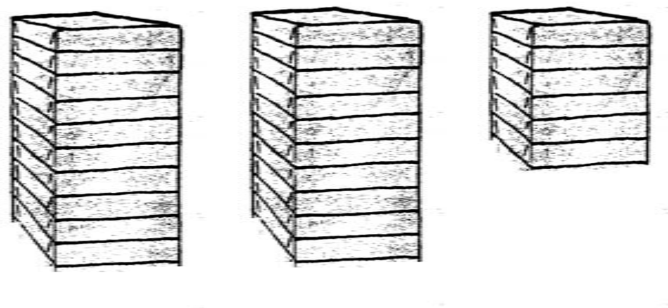


Y. Speed and eggs
The speed of a plane is 37 times as fast as that of a ship. If the speed of a plane is 11,396 km per hour, how much is the speed of a ship?



Pre-test

1. Students



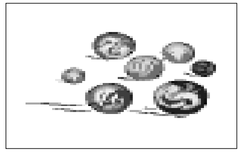
26 boxes in the picture contain 125 playing cards in each box. How many cards are there all?

2. Drinking water

Budi drinks 35 glasses of water every week. How many glasses of water does he drink for 108 weeks?

3. Marbles

Haqim has 230 marbles and Hilmy has 46 times as many. How many marbles does Hilmy has?



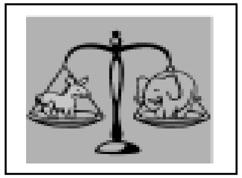
4. Changes

The length of a Rp. 1000-curency is 14 cm. Khalid lines up all his Rp. 1000-money together. The length of all his money is 1330 cm. How many changes does he have?

5. Good reader

To be a good reader, one could read 3404 words in 57 minutes. How many words should one read in a minute?

6. Elephant and goat



An elephant weights 94 times as much as the weight of a dog. If the weight of an elephant is 5452 kg, how much is the weight of a dog?

7. Multiplication

Find the answer for the multiplication below.

86	608
37	45
-------- x	------- x

8. Division

Find the answer for the division below.

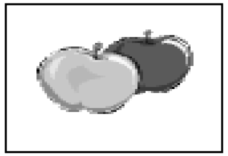
24 504   18 5544

9. How many correct answers do you think you made in answering all problems above? Why?

10. Do you think you are good in mathematics? Why or why not?

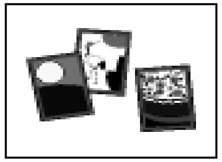
Post-test

1. Buying apple



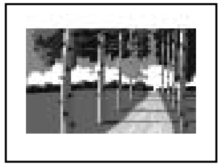
If you need Rp. 375 to buy an apple, how many rupiahs do you need to buy 48 apples?

2. Playing cards



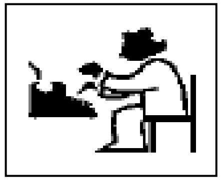
Hany has 208 playing cards and Hilda has 36 times as many. How many playing cards does Hilda has?

3. Trees



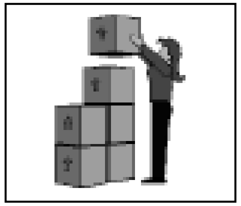
In a rubber plantation, there are 420 lines of rubber trees. Each line has 59 trees. How many trees are there?

4. Typing



Pak Amin is a good typewriter. In each minute he can type 86 words. If the speed of his typing is the same in every minute, how many minutes does he need to type 3870 words?

5. Donation



10000 pieces of the instant noodle are sent to Ambon. The noodle is set in several boxes. Each box contain 48 pieces. How many boxes are needed?

6. **Buying lollipops**

If you buy 28 ice cream lollipops and you pay Rp. 8540, how much is the price of each lollipop?

7. **Multiplication**

Find the result of the following multiplication.

```
   97          305
   48           86
------- x   -------- x
```

8. **Division**

Find the answer for the division below.

27 918   38 7904

9. **How many correct answers do you think you made in answering all problems above? Why?**

10. **Do you think you are good in mathematics? Why or why not?**

Student characteristics

General directions

In this questionnaire, you will find questions about yourself. Some questions ask for facts while other questions ask for your opinion.

Read each question carefully and respond as accurately as possible. You may ask for help if you do not understand something or are not sure how to respond.

Some of questions will be followed by a few possible choices indicated with a letter next to or below it. For these questions, circle then letter next to or below your choice as shown in example 1.

Example 1

	Yes	No
1. I attend school .	A	B

The letter "A" has been circled because you attend school.

If you decide to change your response to a question, put an "X" over your first choice and then put a circle around your new choice as shown in Example 2.

Example 2

	Strongly Agree	Agree	Disagree	Strongly disagree
1. I like mathematics	A	B	C	D

For other question you will be asked to write a number or date in the space provided in your questionnaire. For these questions, you may use words and numbers in your answers. When you write, please be sure that your handwriting is clear

1. On what date were you born? _______ day _______ month _______ year

	Girl	Boy
2. Are you a girl or a boy?	A	B

	Always	Sometimes	Never
3. How often you speak Indonesian at home?	A	B	C

4. Outside of school do you ever	Yes	No
a. have extra lessons/cramming school before or... after school in mathematics?	A	B
b. participate in mathematics clubs?	A	B

5. On normal day, how much time before or after school do you spend doing each of these things?

	No time	Less than 1 hour	1 – 2 hours	3 – 4 hours	More than 4 hours
a. watching television and videos .	A	B	C	D	E
b. playing computer games	A	B	C	D	E
c. playing or talking with friends outside of school	A	B	C	D	E
d. doing jobs at home	A	B	C	D	E
e. playing sports	A	B	C	D	E
f. reading a book for enjoyment ...	A	B	C	D	E
g. studying mathematics or doing mathematics homework after school........	A	B	C	D	E
h. studying or doing homework in school subject other than mathematics	A	B	C	D	E

6. Do each of these people live at home with you most or all of the time?

	Yes	No
a. mother	A	B
b. father	A	B
c. one or more brothers	A	B
d. one or more sisters	A	B
e. stepmother	A	B
f. stepfather	A	B
g. one or more grandparents	A	B
h. another relative or relatives such as uncles, aunts, cousins	A	B
i. another person or persons (not relatives)	A	B

7. Altogether how many people live in your home? ________ people

8. About how many books do you have in your home?
(Do not count magazines, newspapers, or your school books)

None or very few (0 – 10 books)	A
Enough to fill one shelf (11 – 25 books)	B
Enough to fill one bookcase (26 – 100 books)	C
Enough to fill two bookcases (101 – 200 books)	D
Enough to fill three or more bookcase (more than 200 books)	E

9. Do you have any of these items in your home?

	Yes	No
a. calculator	A	B
b. computer	A	B
c. study desk/table for your use	A	B
d. dictionary	A	B
e. study room	A	B

10. Does your mother think it is important for you to	Yes	No
a. do well in mathematics at school?	A	B
b. be good in sports?	A	B
c. have time to have fun?	A	B

11. Does your father think it is important for you to	Yes	No
a. do well in mathematics at school?	A	B
b. be good in sports?	A	B
c. have time to have fun?	A	B

12. Do you think it is important you to	Yes	No
a. do well in mathematics at school?	A	B
b. be good in sports?	A	B
c. have time to have fun?	A	B

13. How well do you usually do in mathematics at school?

	Strongly Agree	Agree	Disagree	Strongly disagree
a. I usually do well in mathematics	A	B	C	D
b. Sometimes I don't know what to do when I start a problem.........	A	B	C	D
c. I usually give up when a problem is really hard	A	B	C	D

14. To do well in mathematics at school you need ..

	Strongly Agree	Agree	Disagree	Strongly disagree
a lots of natural talent/ability	A	B	C	D
b good luck...................................	A	B	C	D
c lots of hard work studying at home..	A	B	C	D
d to memorize the textbook or notes...	A	B	C	D

15. Do you like mathematics?

	Like a lot	Like	Dislike	Dislike a lot
a How much do you like mathematics	A	B	C	D
b I like mathematics because I can figure things out.................	A	B	C	D
c The harder the problems, the better I like to work on them ...	A	B	C	D
d I like the memorizing part of mathematics	A	B	C	D
e I like to do a lot of problems of the same kind rather than have different kinds all mixed up......	A	B	C	D
f I liked mathematics when I was younger, but now it is too hard	A	B	C	D

16. What do you think?

	Strongly agree	Agree	Disagree	Strongly disagree
a I enjoy learning mathematics ...	A	B	C	D
b Mathematics is boring..............	A	B	C	D
c. Mathematics is an easy subject	A	B	C	D
d. There is more mathematics than just getting the right answer.....	A	B	C	D
e. I think mathematics is not really useful in everyday living	A	B	C	D
f. I would rather work alone than With a group.............................	A	B	C	D
g. There is always a best way to solve a problem..................................	A	B	C	D

17. How often does this happen in your mathematics lessons?

	Most lessons	Some lessons	Never
a. the teacher shows us how to do mathematics problems..................................	A	B	C
b. We copy notes from the board.....................	A	B	C
c. We have a quiz or test	A	B	C
d. We work from worksheets or textbooks on our own	A	B	C
e. We work on mathematics projects	A	B	C
f. We use calculator	A	B	C
g. We use computers	A	B	C
h. We work together in pairs or small groups .	A	B	C
i. We use things from everyday life in solving mathematics problems	A	B	C
j. The teacher gives homework	A	B	C
k. We can begin our homework in class	A	B	C
l. The teacher checks homework	A	B	C
m. We check each other's homework	A	B	C
n. We discuss our completed homework	A	B	C

STOP

THERE ARE NO MORE QUESTIONS IN THIS QUESTIONNAIRE

If you have finished answering this questionnaire before the time is over then you may go back and review your answers. Thank you for taking the time to answer these questions carefully.

TEACHER LOGBOOK

Lesson number & Title : ______________________
Lesson date : ______________________
Teacher name : ______________________
School name : ______________________

I General impression of the Lesson :

Useful	5	4	3	2	1	Not useful
Interesting	5	4	3	2	1	Not interesting
Easy to use	5	4	3	2	1	Not easy to use
Enjoyable	5	4	3	2	1	Not enjoyable

II Performance of the Lesson

❑ Please indicate whether the estimated time for the various activities in the teachers' guide is ok or not. If it is not, please indicate the amount of time you think necessary. If other activities took lesson time, please indicate these as well. If you skipped an activity, please fill in "0 minutes'.

Activities	Time (minutes)	Problems
- Introducing the problem	______	______
- Describing the situation of the problem	______	______
- Finding the informal/formal math form	______	______
- Solving the mathematical form	______	______
- Discussing the solution in the group	______	______
- Comparing the solution with other groups	______	______

❑ What do you think about the teaching and learning process using the RME materials?

Easy to apply	5	4	3	2	1	Not easy to apply
Ran smoothly	5	4	3	2	1	Many problems
Time estimation ok	5	4	3	2	1	Time is too optimistic
Lesson aims met	5	4	3	2	1	Lesson aims not met

III Students

❑ How was the participation of the students during the lesson?

Active						*Passive*
- asking questions	5	4	3	2	1	
- finding the procedures	5	4	3	2	1	
- applying the procedures	5	4	3	2	1	
- giving opinions	5	4	3	2	1	
Independent						*Dependent*
- working individually	5	4	3	2	1	
- finding and reinventing	5	4	3	2	1	
- thinking individually	5	4	3	2	1	
- discussing with peers	5	4	3	2	1	
Interested	5	4	3	2	1	*Uninterested*

IV Teachers' Materials

❑ What is your judgement of the teacher's guide for this lesson?

Content clear	1	2	3	4	5	Content unclear
Lay-out clear	1	2	3	4	5	Lay-out unclear
Information provided	1	2	3	4	5	Information not provided
Easy to use	1	2	3	4	5	Not easy to use
Easy to apply	1	2	3	4	5	Not easy to apply
Text too extensive	1	2	3	4	5	Text too concise

V Students' Materials

❑ What is your judgement of the students' guide for this lesson?

Content clear	1	2	3	4	5	Content unclear
Level of exercises ok	1	2	3	4	5	Level of exercises too high
Language usage ok	1	2	3	4	5	Language usage too difficult
Lay-out clear	1	2	3	4	5	Lay-out unclear
Text too extensive	1	2	3	4	5	Text too concise

VI Any other remarks

____
____

Thank you very much for your cooperation

Teacher characteristics

This questionnaire is addressed to classroom teachers, who are asked to supply information about their academic and professional backgrounds, instructional practices, and attitudes towards teaching mathematics. Your responses are very important in helping to describe the teaching of mathematics in Indonesia.

Some of the questions in this questionnaire ask about your class. This is the class, which is identified at the top of this page.

It is important that you answer each question carefully so that the information provided reflects your situation as accurately as possible. It is estimated that it will require approximately 60 minutes to complete this questionnaire.

Your cooperation in completing this questionnaire is greatly appreciated.

General direction

1. Identify a place and a time when you will be able to complete the questionnaire without being interrupted. The questionnaire has been designed to be completed within 60 minutes by most teachers. However, the amount of time you will need may be either more or less. To make it as easy as possible for you to respond, most items may be completed simply by checking the appropriate box.

2. There are no “right” or “wrong” answers to any of these items.

3. Several items ask you to think of a recent class (hour/period) as you respond. In responding to these items, choose a recent class (hour/period) with your class which you can recall in some detail and which was fairly typical of what occurs in your classroom i.e. a class (hour/period) which was not affected by special events such as assemblies, guests, student testing other than short quizzes, or any other unusual circumstances.

4. More specific instructions to assist you in responding are found in *italics* for each item.

Again, thank you for your time, effort and thought in completing this questionnaire.

1. How old are you?
   a. under 25 .... ❑
   b. 25 – 29 .... ❑
   c. 30 – 39 .... ❑
   d. 40 – 49 .... ❑
   e. 50 – 59 .... ❑
   f. 60 or more .... ❑

2. Are you female or male? ❑ Female ❑ Male

3. What is the highest level of formal education you have completed?
   Teacher training W/O completing secondary .... ❑
   Secondary school only .... ❑
   Secondary + 1 or 2 year teacher training .... ❑
   Secondary + 3 or 4 year teacher training .... ❑
   BA or equivalent + no teacher training .... ❑
   BA or equivalent + teacher training .... ❑
   MA/PHD with no teacher training .... ❑

4. How many years will you have been teaching al together? _____ years.

5. Approximately how many hours per week do you normally spend on each of the following activities outside the formal school day?

		No time	Less than 1 hour	1 – 2 hours	3 – 4 hours	More than 4 hours
a.	preparing or grading student tests or exams	❑	❑	❑	❑	❑
b.	reading and grading other student work	❑	❑	❑	❑	❑
c.	planning lessons by yourself ...	❑	❑	❑	❑	❑
d.	meeting with students outside classroom time	❑	❑	❑	❑	❑
e.	meeting with parent	❑	❑	❑	❑	❑
f.	professional reading and development activity	❑	❑	❑	❑	❑
g.	keeping students’ records up to date	❑	❑	❑	❑	❑
h.	administrative tasks including staff meetings	❑	❑	❑	❑	❑

6. About how often do you have meetings with other teachers in to discuss and plan curriculum or teaching approaches?

Never	❑
Once a week	❑
Almost every day	❑

7. To be good in mathematics at school, how important do you think it is for students to

	Not important	Somewhat important	Very important
a. remember formulas and procedures	❑	❑	❑
b. think in a sequential and procedural manner	❑	❑	❑
c. understand mathematical concepts, principles, and strategies	❑	❑	❑
d. be able to think creatively	❑	❑	❑
e. understand how mathematics is used in the real world	❑	❑	❑
f. be able to provide reasons to support their solutions	❑	❑	❑

8. To what extent do you agree or disagree with each of the following statements?

	Strongly agree	Agree	Disagree	Strongly disagree
a. Mathematics is primarily an abstract subject	❑	❑	❑	❑
b. Mathematics is primarily a formal way of representing the real world	❑	❑	❑	❑
c. Mathematics is primarily a practical and structured guide for addressing real situation	❑	❑	❑	❑
d. If students are having difficulty, an effective approach is to give them more practice by themselves during the class	❑	❑	❑	❑
e. Some students have a natural talent for mathematics and others do not	❑	❑	❑	❑
f. More than one representation (picture, concrete material, symbol set, etc.) should be used in teaching a mathematics topic	❑	❑	❑	❑
g. Mathematics should be learned as sets of algorithms and rules that cover all possibilities	❑	❑	❑	❑
h. Basic computational skills on the part of the teacher are sufficient for teaching	❑	❑	❑	❑
i. A liking for and understanding of students are essential for teaching mathematics	❑	❑	❑	❑

9. To what extent do you agree with each of the following statements?

Mathematics is _____

Dynamics	1	2	3	4	5	Fixed
Predictable	1	2	3	4	5	Surprising Absolute
	1	2	3	4	5	Relative
Doubtful	1	2	3	4	5	Certain
Applicable	1	2	3	4	5	Aesthetic

Learning mathematics requires mostly _______

Practice	1	2	3	4	5	Insight
Independent work	1	2	3	4	5	Group work
Good teachers	1	2	3	4	5	Strong students
Trying hard	1	2	3	4	5	Being good at math
Memorizing	1	2	3	4	5	Understanding

Good mathematics teaching entails, or depend on ______

A good textbook	1	2	3	4	5	Use manipulatives
Teacher direction	1	2	3	4	5	Student participation
Teacher efforts	1	2	3	4	5	Student efforts
Explicit planning	1	2	3	4	5	Flexible lessons
Helping students to like mathematics	1	2	3	4	5	Helping students see mathematics as useful

In many of the following questions, a reference is made to your class.

10. How many students are in your class? ______ students, _____ boys ______ girls
11. How many minutes per week do you teach mathematics? __________ minutes
12. What textbook do you use in teaching mathematics to your class?

Title : ____________________

Author (Publisher) : ____________________

Year : ____________________

Other : ____________________

13. In your view to what extent do the following limit how you teach your class?

	Not at all	a little	quite a lot	a great deal
a. students with different academic abilities	❑	❑	❑	❑
b. students who come from a wide range of backgrounds (e.g. economic, language)	❑	❑	❑	❑
c. students with special needs (e.g. hearing, vision speech impairment, physical and mental disabilities)	❑	❑	❑	❑
d. uninterested students	❑	❑	❑	❑
e. disruptive students	❑	❑	❑	❑
f. parents interested in their children's learning and progress	❑	❑	❑	❑
g. shortage of computer	❑	❑	❑	❑
h. shortage of other instructional student equipment	❑	❑	❑	❑
i. shortage of teaching equipment	❑	❑	❑	❑
j. inadequate physical facilities	❑	❑	❑	❑
k. high student/teacher ratio	❑	❑	❑	❑
l. low morale among fellow teachers/ administrators	❑	❑	❑	❑
m. low morale among students	❑	❑	❑	❑

14. How many of your students have access to calculators during mathematics lessons?

	Almost every day	Once or twice a week	Once or twice a month	Never, or hardly ever
a. checking answers	❑	❑	❑	❑
b. tests and exams	❑	❑	❑	❑
c. routine computation	❑	❑	❑	❑
d. solving complex problems	❑	❑	❑	❑
e. exploring number concepts	❑	❑	❑	❑

15. When planning mathematics lessons, how much do you rely on:

	Not at all	a little	quite a lot	a great deal
a. your own previously prepare lessons	❑	❑	❑	❑
b. a written plan compiled by teachers in the school	❑	❑	❑	❑
c. other teachers or math specialists	❑	❑	❑	❑
d. students textbooks	❑	❑	❑	❑
e. other textbooks or resource books	❑	❑	❑	❑
f. teacher guides or teacher edition of textbook	❑	❑	❑	❑
g. external examinations or standardized tests	❑	❑	❑	❑

16. In planning mathematics lessons, what is your main written source information when

	Other resource books	Student edition of textbook	Teacher edition of textbook	School curriculum guide	National or regional curriculum guide	National or regional examination specification
a. deciding which topics to teach (goals)	❑	❑	❑	❑	❑	❑
b. deciding how to present a topic	❑	❑	❑	❑	❑	❑
c. selecting problems for work in class and homework	❑	❑	❑	❑	❑	❑
d. selecting problems and applications for assessment and evaluation	❑	❑	❑	❑	❑	❑

17. How long did you spend teaching the Numbers (Part 3) topic?
Have taught this topic last year completely for :
❑ 1-5 ❑ 6-10 ❑ 11-15 ❑ >15 hours

18. Was this lesson

	Yes	No
a. the introduction of this topic...	❑	❑
b. a continuation of a previous lesson on the same topic	❑	❑
c. the end of the coverage of this topic.................................	❑	❑

19. How many years have you been teaching this subject? _____ years

20. How did the lesson proceed?

Order

- Review of previous lesson(s) ______
- A short quiz or test to review previous lesson ______
- Oral recitation or drill (students responding aloud) ______
- Review or correction or previous lesson's homework ______
- Introduction of a topic (class discussion, teacher explanation /demonstration, film, video, use of concrete materials etc.) ______
- Development of a topic (class discussion, teacher explanation/demonstration, group problem solving, film, video, etc.) ______
- Small group activities (with or without teacher) ______
- Students do paper-and-pencil exercises related to topic (not as same as homework) ______
- Assignment of student homework ______
- Students work on homework in class ______
- Student laboratory or data collection activity (not a separate laboratory hour) or hands-on session ______

21. In this class (hour/period) did the students work in small groups?
None of the time........................ ❑
Some of the time........................ ❑
All of the time .... ❑

22. How often do you usually assign mathematics homework?
Never .... ❑
Less than once a week.............. ❑
Once or twice a week .... ❑
Every day................................... ❑

23. In your mathematics lessons, how often do you usually ask students to do each of the following?

	Never or almost never	Some lessons	Most lessons	Eevery lessons
a. explain the reasoning behind an idea...	❑	❑	❑	❑
b. represent and analyze relationships	❑	❑	❑	❑
using tables, charts, or graphs..............	❑	❑	❑	❑
c. work on problems for which there is no immediately obvious method of solution..	❑	❑	❑	❑
d. use computers to solve exercises or problems	❑	❑	❑	❑
e. write equations to represent relationships..	❑	❑	❑	❑
f. practice computational skills................	❑	❑	❑	❑

24. In your mathematics lessons, how frequently do you do the following when a student gives an incorrect response in a discussion?

	Never or almost never	Some lessons	Most lessons	Eevery lessons
a. correct the student's error in front the class	❑	❑	❑	❑
b. ask the student another question to help him or her get the correct response	❑	❑	❑	❑
c. call on another student who's likely to give the correct response.....................	❑	❑	❑	❑
d. call on other students to get their responses and then discuss what is correct	❑	❑	❑	❑

25. In mathematics lessons, how often do students

	Never or almost never	Some lessons	Most lessons	Eevery lessons
a. work individually without teachers' assistance	❑	❑	❑	❑
b. work individually with teachers' assistance	❑	❑	❑	❑
c. work together with teacher teaching the class	❑	❑	❑	❑
d. work together as a class with students responding to one another	❑	❑	❑	❑
e. work in groups without teachers' assistance	❑	❑	❑	❑
f. work in pairs or small groups with assistance from the teacher..................❑	❑	❑	❑	

26. If you assign mathematics homework, how often do you assign of the following kinds of tasks?

	Never	Rare-ly	Some-times	Al-ways	I do not assign home-work
a. worksheet or workbook	❑	❑	❑	❑	❑
b. problem/question sets in textbook	❑	❑	❑	❑	❑
c. reading in a textbook or supplementary materials	❑	❑	❑	❑	❑
d. writing definitions or other short writing assignment	❑	❑	❑	❑	❑
e. small investigation(s) or gathering data	❑	❑	❑	❑	❑
f. working individually on long term projects or experiments	❑	❑	❑	❑	❑
g. finding one or more uses of the content covered	❑	❑	❑	❑	❑
h. oral reports either indivi-dually or a small group	❑	❑	❑	❑	❑
i. keeping a journal	❑	❑	❑	❑	❑

27. If students are assigned written mathematics homework, how often do you do the following?

	Never	Rarely	Sometimes	Always	I do not assign homework
a. record whether or not the homework was completed	❑	❑	❑	❑	❑
b. collect, correct and keep assignments	❑	❑	❑	❑	❑
c. collect, correct assignments and then return to students	❑	❑	❑	❑	❑
d. give feedback on homework to whole class	❑	❑	❑	❑	❑
e. have students correct their own assignments in class	❑	❑	❑	❑	❑
f. use it as a basis for class discussion	❑	❑	❑	❑	❑
g. use it to contribute towards students' grades or marks	❑	❑	❑	❑	❑

STOP

THERE ARE NO MORE QUESTIONS IN THIS QUESTIONNAIRE

If you have finished answering this questionnaire before the time is over then you may go back and review your answers. Thank you for taking the time to answer these questions carefully.

The Teacher teaching Profile

Choose one of the options and give your opinion.

Very Very
worse good Reasons

Introduction to the lesson

1. Teacher introduces and formulates the problems ❑ ❑ ❑ ❑ ❑ ___
2. Teacher asks guided questions to introduce lesson ❑ ❑ ❑ ❑ ❑ ___
3. Teacher asks students for their own idea ❑ ❑ ❑ ❑ ❑ ___
4. Teacher responds to students ideas ❑ ❑ ❑ ❑ ❑ ___
5. Teacher encourages students to ask questions ❑ ❑ ❑ ❑ ❑ ___
6. Teachers guides the students to the conclusions ❑ ❑ ❑ ❑ ❑ ___

Body of the lesson

1. Students explore problems in groups or individually ❑ ❑ ❑ ❑ ❑ ___
2. Teacher allows students to choose own approach ❑ ❑ ❑ ❑ ❑ ___
3. Students actively make use of their knowledge ❑ ❑ ❑ ❑ ❑ ___
4. Students discuss operation employed in the problems ❑ ❑ ❑ ❑ ❑ ___
5. Teacher focuses students attention on crucial aspects ❑ ❑ ❑ ❑ ❑ ___
6. Teacher interacts with students during the activity ❑ ❑ ❑ ❑ ❑ ___
7. Teacher assists students when necessary ❑ ❑ ❑ ❑ ❑ ___
8. Teacher asks students guiding questions, but does not provide outcomes ❑ ❑ ❑ ❑ ❑ ___
9. Teacher encourages students to discuss with peers in their groups ❑ ❑ ❑ ❑ ❑ ___
10. Teacher allows learners to draw own conclusions ❑ ❑ ❑ ❑ ❑ ___

Conclusion of the lesson

1. Teacher asks several groups/individual to report their results to the class ❑ ❑ ❑ ❑ ❑ ___
2. Teacher invites and encourages students to comment on the outcomes ❑ ❑ ❑ ❑ ❑ ___
3. Teacher asks critical open-ended questions regarding the outcomes ❑ ❑ ❑ ❑ ❑ ___
4. Teacher compares students outcomes and its discrepancies ❑ ❑ ❑ ❑ ❑ ___
5. Teacher guides students to understand discrepancies in their results ❑ ❑ ❑ ❑ ❑ ___
6. Teacher draws conclusions from the activity with the students ❑ ❑ ❑ ❑ ❑ ___

General impression of the lesson

1. Teacher acknowledges students' ideas ❑ ❑ ❑ ❑ ❑ ___
2. Teacher uses and discusses students' ideas ❑ ❑ ❑ ❑ ❑ ___
3. Teacher summaries students answers ❑ ❑ ❑ ❑ ❑ ___
4. Teacher asks open-ended questions to individual students ❑ ❑ ❑ ❑ ❑ ___
5. Classroom atmosphere seems to encourage students to ask and answer questions ❑ ❑ ❑ ❑ ❑ ___

APPENDIX B
EXAMPLES FROM PUPILS'REINVENTED STRATEGIES

1. Daily quiz
2. Weekly quiz
3. Post-test

APPENDIX B.

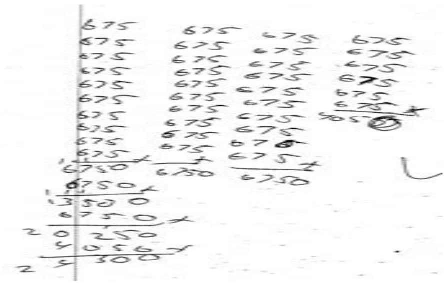
EXAMPLES FROM PUPILS' REINVENTED STRATEGIES

4. DAILY QUIZ
5. WEEKLY QUIZ
6. POST-TEST

1. DAILY QUIZ

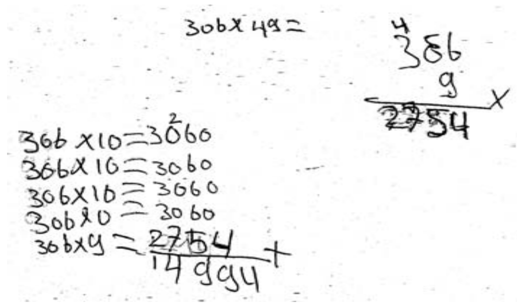
A. The price

The price of a math book is 36 times as much as that of a pencil. A pencil costs Rp. 675. How much is the price of a math book?



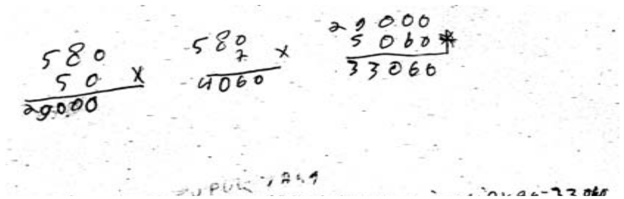
B. Riding a bike

Hilma rides a bike to go to her grandmother's house. She rides about 49 minutes. In a minute she reaches 306 meters. How many meters is the distance she reaches?



C. Riding a bike

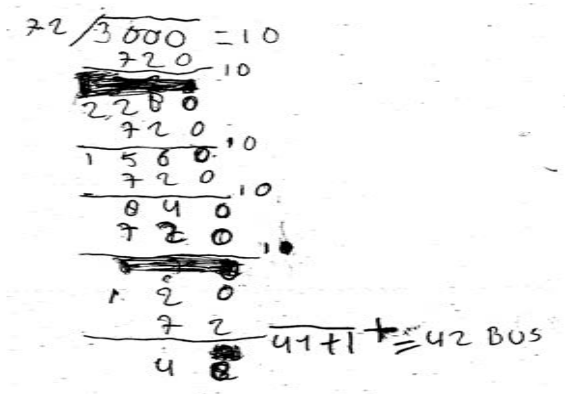
Each of 57 Klompencapir in several villages is about to get 309 sacks of urea donated and distributed by the government. How many sacks of urea are there?



Thus the amount of urea that are prepared are 33060 sacks

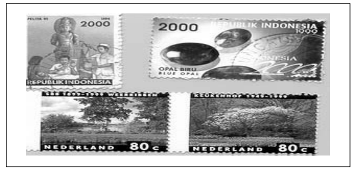
D. The supporters

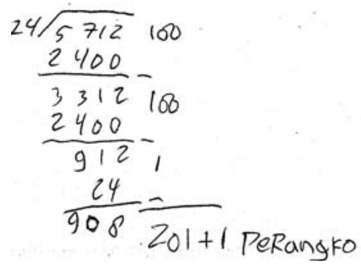
3000 supporters are going to Jakarta from Bandung by bus to see the football match. Each bus can carry 72 supporters. How many buses are needed?



E. The price

Jagi has 5712 stamps in his album. There are two dozen stamps on each page. How many pages are there?

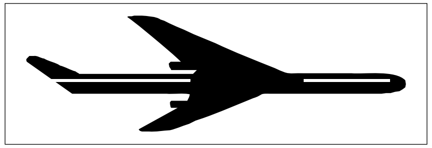


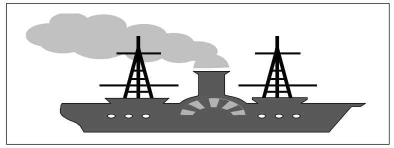


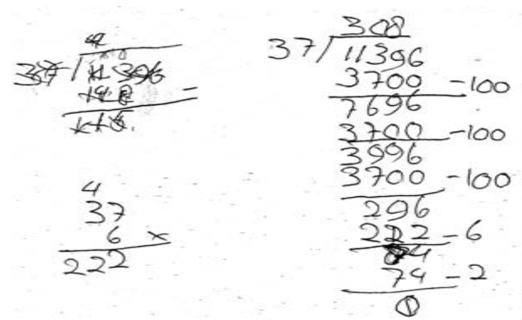
(stapms)

F. Speed and eggs

The speed of a plane is 37 times as fast as that of a ship. If the speed of a plane is 11,396 km per hour, how much is the speed of a ship?





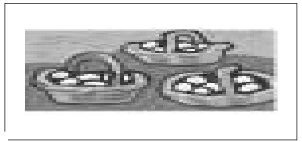


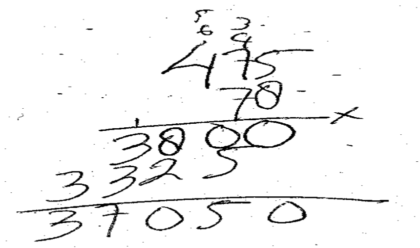
2. WEEKLY QUIZ

Multiplication

1. Buying eggs

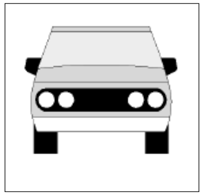
Hamim buys 78 eggs. An egg costs Rp. 475. How many rupiahs does he have to pay?

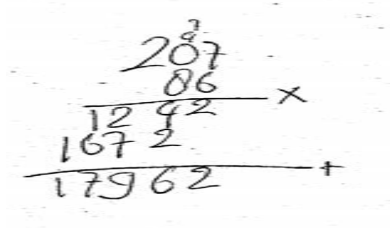




2. Biking and walking

Riding a car is 207 times faster than walking on foot. What is the speed of the bike if a man walks 86 m per minute?

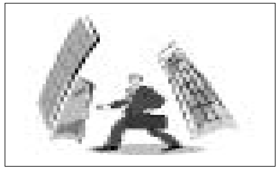


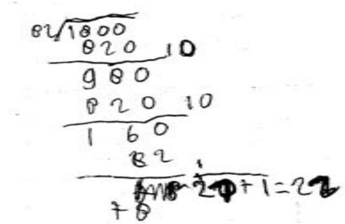


Division

1. Walking

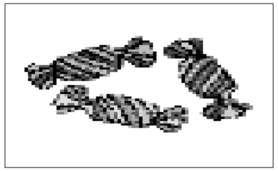
Pak Amat always walks to his office. The distance is 1800 m from his house. If he walks for 82 m in a minute, how many minutes does he walk to his office?

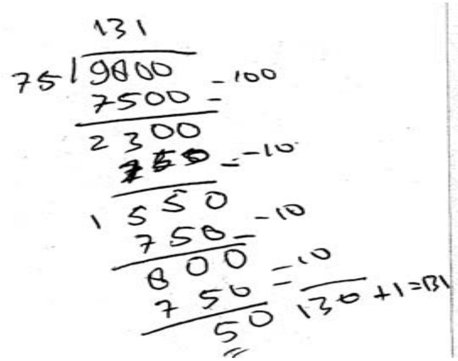




2. Candys

The price of a "COKELAT" candy is Rp. 75. If you have Rp. 9800, how many candies do you get using all your money?

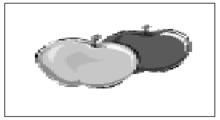




3. POST-TEST

1. Buying apple

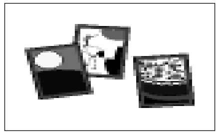
If you need Rp. 375 to buy an apple, how many rupiahs do you need to buy 48 apples?



```
      375
       48  x
    3000
  1505
  ----------- +
  18050
```

2. Playing cards

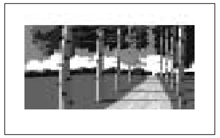
Hany has 208 playing cards and Hilda has 36 times as many. How many playing cards does Hilda has?



```
      420
       59  x
     3780
   2100
  ----------- +
   24780
```

3. Trees

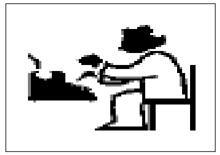
In a rubber plantation, there are 420 lines of rubber trees. Each line has 59 trees. How many trees are there?

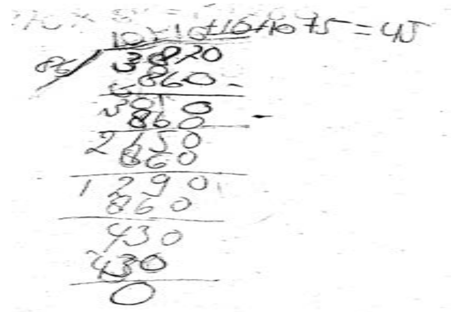


```
      208
       36  x
    1248
   624
  ----------- +
   7488
```

4. Typing

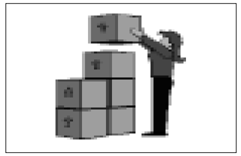
Pak Amin is a good typewriter. In each minute he can type 86 words. If the speed of his typing is the same in every minute, how many minutes does he need to type 3870 words?

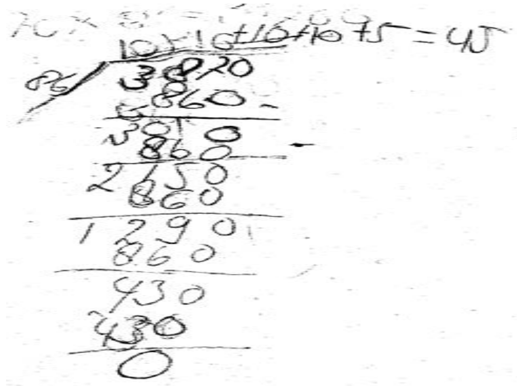




5. Donation

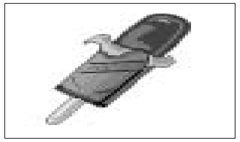
10000 pieces of the instant noodle are sent to Ambon. The noodle is set in several boxes. Each box contain 48 pieces. How many boxes are needed?





6. Buying lollipops

If you buy 28 ice cream lollipops and you pay Rp. 8540, how much is the price of each lollipop?



```
     305
28 / 8540
     84
    ----- +
     140
     140
    -----
       0
```

7. Multiplication

Find the result of the following multiplication.

```
  97            305
  48             86
------- x     -------- x
```

```
   97           305
   48            86
  ----- x      ----- x
   776          1830
  388          2440
  ----- +      ------ +
  4656         26230
```

8. Division

Find the answer for the division below.

```
27 / 918                38 / 7904
```

```
     10+10+10+4=4              100+100+2=307
27 / 918                 38 / 7904
     270 -                    3800 -
     ---                      ----
     648                      9104
     270 -                    3800 -
     ---                      ----
     378                       304
     270 -                     304
     ---                      ----
     108                         0
     108
     ---
       0
```

APPENDIX C
ANALYSIS OF THE INSTRUMENTS AND PUPILS' PERFORMANCES (USING SPSS)

1. Reliability of the quizzes and the tests
2. Analysis of pupils' performances

1. RELIABILITY OF THE QUIZZES AND THE TESTS

A. Reliability of Pre-test

Correlation Matrix

	ITEM1	ITEM2	ITEM3	ITEM4	ITEM5
ITEM1	1,0000				
ITEM2	,1570	1,0000			
ITEM3	,2917	,3735	1,0000		
ITEM4	,2269	,3547	,5038	1,0000	
ITEM5	,4486	,2475	,4507	,5165	1,0000
ITEM6	,2774	,2460	,2913	,5493	,6215
ITEM7A	,1983	,2123	,1623	,1974	,3328
ITEM7B	,2257	,0943	,1938	,1807	,2024
ITEM8A	-,0162	,3220	,3510	,2130	,3251
ITEM8B	,2673	,3250	,3490	,2502	,3692

	ITEM6	ITEM7A	ITEM7B	ITEM8A	ITEM8B
ITEM6	1,0000				
ITEM7A	,3349	1,0000			
ITEM7B	,1799	,3679	1,0000		
ITEM8A	,1796	,2387	,4157	1,0000	
ITEM8B	,2722	,2773	,3944	,6079	1,0000

R E L I A B I L I T Y   A N A L Y S I S - S C A L E (A L P H A)

Reliability Coefficients   10 items

Alpha = ,7999   Standardized item alpha = ,8121

B. Reliability of the post-test

Correlation Matrix

	ITEM1	ITEM2	ITEM3	ITEM4	ITEM5
ITEM1	1,0000				
ITEM2	,5921	1,0000			
ITEM3	,4756	,6379	1,0000		
ITEM4	,0919	,3251	,4053	1,0000	
ITEM5	,3268	,3997	,5170	,4482	1,0000
ITEM6	,3684	,3214	,3210	,3028	,3758
ITEM7A	,3385	,4663	,4436	,2961	,4582
ITEM7B	,3422	,3745	,3898	,1106	,4875
ITEM8A	,1899	,3945	,4694	,5275	,4825
ITEM8B	,1454	,2926	,4752	,3284	,6125

	ITEM6	ITEM7A	ITEM7B	ITEM8A	ITEM8B
ITEM6	1,0000				
ITEM7A	,3221	1,0000			
ITEM7B	,1969	,6406	1,0000		
ITEM8A	,2534	,4364	,4068	1,0000	
ITEM8B	,2600	,3595	,3518	,4627	1,0000

R E L I A B I L I T Y A N A L Y S I S - S C A L E (A L P H A)
Reliability Coefficients 10 items
Alpha = ,8573 Standardized item alpha = ,8611

C. Reliability of the weekly quiz

1. Multiplication

Correlation Matrix

	ITEM1WM	ITEM2WM
ITEM1WM	1,0000	
ITEM2WM	,2364	1,0000

R E L I A B I L I T Y A N A L Y S I S - S C A L E (A L P H A)
Reliability Coefficients 2 items
Alpha = ,3808 Standardized item alpha = ,3824

2. Division

Correlation Matrix

	ITEM1WD	ITEM2WD
ITEM1WD	1,0000	
ITEM2WD	,5962	1,0000

R E L I A B I L I T Y A N A L Y S I S - S C A L E (A L P H A)
Reliability Coefficients 2 items
Alpha = ,7468 Standardized item alpha = ,7470

D. Reliability of the daily quiz

1. Multiplication

Correlation Matrix

	ITEM1DM	ITEM2DM	ITEM3DM
ITEM1DM	1,0000		
ITEM2DM	,3716	1,0000	
ITEM3DM	,2420	,2681	1,0000

R E L I A B I L I T Y   A N A L Y S I S   -   S C A L E   (A L P H A)
Reliability Coefficients   3 items
Alpha =   ,5575   Standardized item alpha =   ,5553

2. Division

Correlation Matrix

	ITEM1DD	ITEM2DD	ITEM3DD
ITEM1DD	1,0000		
ITEM2DD	,5572	1,0000	
ITEM3DD	,5708	,6209	1,0000

R E L I A B I L I T Y   A N A L Y S I S   -   S C A L E   (A L P H A)
Reliability Coefficients   3 items
Alpha =   ,8050   Standardized item alpha =   ,8074

2. ANALISIS OF PUPILS' PERFORMANCES

A. Descriptive analysis of pupils' performances

Cumulative score of the tests

	N	Mean	Std. Deviation	95% Confidence Interval for Mean		Minimum	Maximum
				Lower Bound	Upper Bound		
Pre-test of the EG	291	8,44	5,90	7,76	9,12	0	37
Post-test of the EG	291	25,07	7,75	24,17	25,96	9	40
Pre-test of the CG	310	9,56	7,60	8,72	10,41	0	30
Post-test of the CG	310	20,42	9,98	19,30	21,53	0	38
Total	1202	15,85	10,62	15,24	16,45	0	40

B. ANOVA of pupils' performances in the pre-test and post-test

Test of Homogeneity of Variances

Cumulative score of the tests

Levene Statistic	df1	df2	Sig.
54,213	3	1198	,000

ANOVA

Cumulative score of the tests

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	59405,249	3	19801,750	311,597	,000
Within Groups	76131,969	1198	63,549		
Total	135537,2	1201			

Post Hoc Tests

Multiple Comparisons

Dependent Variable: Cumulative score of the tests
Scheffe

(I) The type of test	(J) The type of test	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
Pre-test of the EG	Post-test of the EG	-16,63*	,66	,000	-18,48	-14,78
	Pre-test of the CG	-1,12	,65	,397	-2,94	,70
	Post-test of the CG	-11,97*	,65	,000	-13,79	-10,15
Post-test of the EG	Pre-test of the EG	16,63*	,66	,000	14,78	18,48
	Pre-test of the CG	15,50*	,65	,000	13,68	17,33
	Post-test of the CG	4,65*	,65	,000	2,83	6,47
Pre-test of the CG	Pre-test of the EG	1,12	,65	,397	-,70	2,94
	Post-test of the EG	-15,50*	,65	,000	-17,33	-13,68
	Post-test of the CG	-10,85*	,64	,000	-12,64	-9,06
Post-test of the CG	Pre-test of the EG	11,97*	,65	,000	10,15	13,79
	Post-test of the EG	-4,65*	,65	,000	-6,47	-2,83
	Pre-test of the CG	10,85*	,64	,000	9,06	12,64

*. The mean difference is significant at the .05 level.

Homogeneous Subsets

Cumulative score of the tests

Scheffe[a,b]

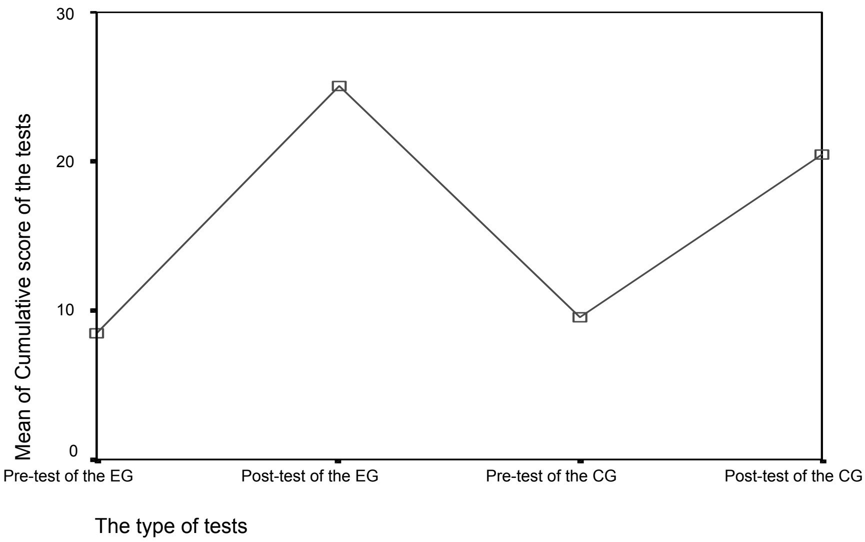
The type of tests	N	Subset for alpha = .05		
		1	2	3
Pre-test of the EG	291	8,44		
Pre-test of the CG	310	9,56		
Post-test of the CG	310		20,42	
Post-test of the EG	291			25,07
Sig.		,397	1,000	1,000

Means for groups in homogeneous subsets are displayed.

a. Uses Harmonic Mean Sample Size = 300,200.

b. The group sizes are unequal. The harmonic mean of the group sizes is used. Type I error levels are not guaranteed.

Means Plots



C. T-test of pupils' performances in pre-test and post-test in EG (experimental group)

Group Statistics

	The type of tests	N	Mean	Std. Deviation	Std. Error Mean
Cumulative score of the tests	Pre-test of the EG	291	8,44	5,90	,35
	Post-test of the EG	291	25,07	7,75	,45

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means				
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference
Cumulative score of the tests	Equal variances assumed	49,347	,000	-29,112	580	,000	-16,63	,57
	Equal variances not assumed			-29,112	541,577	,000	-16,63	,57

D. T-test of pupils' performances in the post-test of EG and CG

Group Statistics

	The type of tests	N	Mean	Std. Deviation	Std. Error Mean
Cumulative score of the tests	Post-test of the EG	291	25,07	7,75	,45
	Post-test of the CG	310	20,42	9,98	,57

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means				
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference
Cumulative score of the tests	Equal variances assumed	33,465	,000	6,355	599	,000	4,65	,73
	Equal variances not assumed			6,405	579,053	,000	4,65	,73

E. Oneway ANOVA of pupils' performances in solving problems

Descriptives

Score Differences of CT (contextual problems) & CV (conventional problems)

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
Pre-post CT in EG	291	11,27	6,24	,37	10,55	11,99	-13	24
Pre-post CT in CG	310	5,38	6,84	,39	4,62	6,15	-18	23
Pre-post CV in EG	291	5,08	5,17	,30	4,49	5,68	-11	16
Pre-post CV in CG	310	5,28	6,43	,37	4,56	6,00	-15	16
Total	1202	6,71	6,72	,19	6,33	7,09	-18	24

Test of Homogeneity of Variances

Score Differences of CT & CG

Levene Statistic	df1	df2	Sig.
8,281	3	1198	,000

ANOVA

Score Differences of CT & CG

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	8001,335	3	2667,112	69,044	,000
Within Groups	46277,913	1198	38,629		
Total	54279,248	1201			

Post Hoc Tests

Multiple Comparisons

Dependent Variable: Score Differences of CT & CG
Tukey HSD

(I) Pretest and Posttest in groups	(J) Pretest and Posttest in groups	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
Pre-post CT in EG	Pre-post CT in CG	5,89*	,51	,000	4,58	7,19
	Pre-post CV in EG	6,19*	,52	,000	4,87	7,51
	Pre-post CV in CG	5,99*	,51	,000	4,68	7,29
Pre-post CT in CG	Pre-post CT in EG	-5,89*	,51	,000	-7,19	-4,58
	Pre-post CV in EG	,30	,51	,934	-1,00	1,60
	Pre-post CV in CG	1,00E-01	,50	,997	-1,18	1,38
Pre-post CV in EG	Pre-post CT in EG	-6,19*	,52	,000	-7,51	-4,87
	Pre-post CT in CG	-,30	,51	,934	-1,60	1,00
	Pre-post CV in CG	-,20	,51	,979	-1,50	1,10
Pre-post CV in CG	Pre-post CT in EG	-5,99*	,51	,000	-7,29	-4,68
	Pre-post CT in CG	-1,00E-01	,50	,997	-1,38	1,18
	Pre-post CV in EG	,20	,51	,979	-1,10	1,50

*. The mean difference is significant at the .05 level.

Homogeneous Subsets

Score Differences of CT & CG

Tukey HSD[a,b]

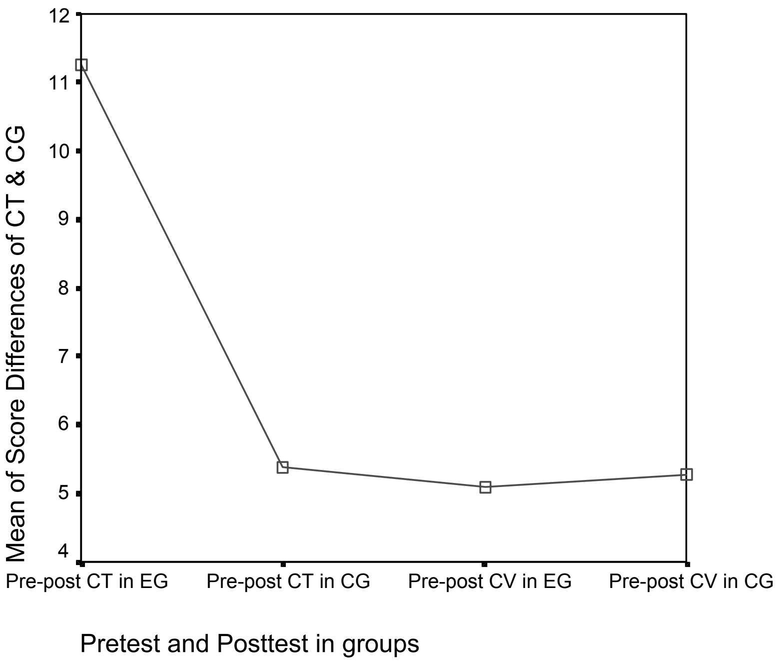
Pretest and Posttest in groups	N	Subset for alpha = .05	
		1	2
Pre-post CV in EG	291	5,08	
Pre-post CV in CG	310	5,28	
Pre-post CT in CG	310	5,38	
Pre-post CT in EG	291		11,27
Sig.		,934	1,000

Means for groups in homogeneous subsets are displayed.

a. Uses Harmonic Mean Sample Size = 300,200.

b. The group sizes are unequal. The harmonic mean of the group sizes is used. Type I error levels are not guaranteed.

Means Plots


12
11
10
9
8
7
6
5
4
Mean of Score Differences of CT & CG
Pre-post CT in EG
Pre-post CT in CG
Pre-post CV in EG
Pre-post CV in CG
Pretest and Posttest in groups